Lecture 3: Other regarding preferences in General Equilibrium

Suspicion: In large anonymous markets, effects of nonselfishness "wiped out" (e.g. Sobel 2005) $\Longrightarrow$

other-regarding preferences ignored e.g. in most micro-based macro models

Under which circumstances is it justified to ignore other-regarding preferences in the positive and normative analysis of large anonymous markets?

large anonymous markets: markets without strategic interaction, i.e. with price taking behavior $\Longrightarrow$

Arrow-Debreu GE economy framework

only models of distributional concerns investigated:

other forms of non-selfishness (e.g reciprocity, spitefulness) relate to personal relation in strategic settings, but distributional concerns can have impact on large markets with non-strategic behavior

not interested in impact of the number of players in strategic situations (like "competition" in ultimatum games, Roth et al 1991)

1. Multi-good Economy with Other-Regarding Preferences

$L = \{1, 2, ..., L\}$ goods

price p, $p \in S^{L-1}$, i.e.$\sum_{l \in L} p_l = 1$

$J = \{1, 2, ..., J\}$ firms, $Y_j \subset \mathbb{R}^L$ production set of j, $y_j \in Y_j$ production plan of j

$\pi_j(p)$: maximum attainable profit of price-taking firm j under p

$y = (y_1, ...y_J)$ production profile

$Y = \prod_{j=1}^{J} Y_j$ set of all feasible production profiles

$I = \{1, 2, ..., I\}$ consumers, consumption set of i is $\mathbb{R}_+^L$

initial endowment ε_i

consumption bundle $x_i = (x_{i1}, ...x_{iL})$

θ_{ij}: $i's$ share of firm j

w_i: wealth of i: $w_i = p\varepsilon_i + \sum_J \theta_{ij}\pi_j(p)$

$b_i = \{x_i \in \mathbb{R}_+^L : px_i \leq w_i\}$ budget set of i

$b = (b_1, ...b_I)$ budget set profile

B: set of budget set profiles

In multi-goods world, other-regarding preferences could refer to:

- distribution of actual consumption (well-being externalities)
- distribution of consumption possibilities (opportunity-based externalities)

$\Longrightarrow$

utility function

$$u_i : \mathbb{R}_+^{L \times I} \times B \to \mathbb{R}$$

$u_i(x_i, x_{-i}, b)$ is $i's$ utility from profile of consumption bundles (x_i, x_{-i}) and from budget set profile b.

preferences are assumed to be strictly convex and strictly monotone in own consumption bundle

Price taking behavior assumption in the context of other-regarding preferences

Consumers:

- Own decisions have no impact on the prices
- Nobody rationed at the going prices $\Longrightarrow$

actual consumption and consumption possibilities of others taken as given

Profit maximizing firms: justified by single ownership

2. Separability

problem of price-taking consumer i

$$\max_{x_i \in b_i} u_i(x_i, x_{-i}, b)$$

$\implies$ demand function $d_i(x_{-i}, b_i, b_{-i})$

Definition 1: A consumer i behaves as if classical if $d_i(x_{-i}, b_i, b_{-i})$ is independent (constant) of (in) x_{-i}, and b_{-i}.

Definition 2: The preferences of consumer i are separable if for all $x, x' \in \mathbb{R}_+^{L \times I}$, and for all $b, b' \in B$:

$$u_i(x_i, x_{-i}, b) \geq u_i(x_i', x_{-i}, b) \text{ iff } u_i(x_i, x_{-i}', b') \geq u_i(x_i', x_{-i}', b').$$

Separability trivially fulfilled for one-good models with monotonicity in own wealth.

Theorem 1:

i) If agent $i's$ preferences are separable, then he behaves as if classical.

ii) If agent i behaves as if classical and if his demand function is continuously differentiable, then his preferences are separable.

separable preferences are the most general class of preferences which generates as if classical behavior

non-separable preferences are generic

3. Equilibrium Equivalence

Definition 3: A Walrasian equilibrium is given by (p^*, x^*, y^*, b^*) such that for all $i = 1, \ldots, I$, $j = 1, \ldots, J$, $L = 1, \ldots, L$

$$\begin{aligned}
p^* y_j^* &\geq p^* y_j' \text{ for all } y_j' \in Y_j \\
x_i^* &= \arg\max_{x_i \in b_i^*} U_i(x_i, x_{-i}^*, b^*) \\
\sum_{i \in I} x_{il}^* &\leq \sum_{i \in I} \varepsilon_{il} + \sum_{j \in J} y_{jl}^* \\
b_i^* &= \{x_i : p^* x_i \leq p^* \varepsilon_i + \sum_{j \in J} \theta_{ij} p^* y_j^* \}
\end{aligned}$$

In equilibrium each firm maximizes its profits for given price p^*, each consumer i chooses her utility maximizing consumption bundle x_i^* for given profile of budget sets b^*, and the profile of budget sets b^* is compatible with p^* and y^*.

If all agents have separable preferences, we can define a "selfish counterpart economy" with selfish agents

Definition 4: For a given economy its selfish counterpart economy is characterized by:
(i) $L_S = L$, $I_S = I, J_S = J$;
(ii) $Y_{jS} = Y_j$ for all $j_S = j \in J$;
(iii) For all $i_S = i \in I$ and all $j_S = j \in J$ it holds that $\varepsilon_{iS} = \varepsilon_i$ and $\theta_{iSj} = \theta_{ij}$;
(iv) The preferences of each consumer $i_S \in I_S$, $\succeq_{iS}$, are defined over $\mathbb{R}^L_+$.
(v) For all $i_S = i \in I$, for all $x_{iS} = x_i \in X_i$, for all $x'_{iS} = x'_i \in X_i$, for all $x_{-i} \in \mathbb{R}^{L\times(I-1)}_+$, and for all $b \in B$ it holds:

$$u_{iS}(x_i) \geq u_{iS}(x'_i) \text{ iff } u_i(x_i, x_{-i}, b) \geq u_i(x'_i, x_{-i}, b)$$

Note: Definition requires separability

Theorem 2: If all agents have separable preferences that are strictly monotone in own consumption, the set of Walrasian equilibria of an economy with other-regarding preferences coincides with the set of Walrasian equilibria of its selfish counterpart economy.

Proof: Immediate consequence of Theorem 1

Remark: If separability does not hold, we can define a selfish counterpart economy for any given equilibrium, and show that this equilibrium is also an equilibrum of the selfish counterpart economy.

Problem: Multiplicity of equilibria of the original as well as of the counterpart economy.

4. Efficiency

For separable preferences "internal" utility function $m_i : \mathbb{R}_+^L \to \mathbb{R}$ represents "internal" preferences over own consumption bundles.

4.1. Internal Efficiency

If all agents have separable, locally nonsatiated preferences, any Walrasian equilibrium allocation is efficient with respect to the internal preferences. (First Welfare Theorem)

If all agents have separable, locally nonsatiated preferences, any allocation efficient with respect to the internal preferences is a Walrasian equilibrium for an appropriate choice of the initial endowment. (Second Welfare Theorem)

4.2. Efficiency with Well-Being Externalities

Restriction on well-being externalities
Generically, equilibrium outcome is inefficient.
Restriction on "Bergsonian" preferences: $V_i(m_1(x_1), m_2(x_2), ...m_I(x_I))$

Social Monotonicity (SM): For any allocation x and any $z \in \mathbb{R}^L_+$, $z \neq 0$ there is a $(z_1, ...z_I) \geq 0$ with $\sum_I z_i = z$ such that for all i:

$$V_i(m_1(x_1 + z_1), ...m_I(x_I + z_I)) > V_i(m_1(x_1), ...m_I(x_I))$$

Theorem 3: If SM holds, then every Parto-efficient allocation can be achieved as a Walrasian equilbrium by a suitable lump sum transfer.
But: Even with SM, the Walrasian equilibrium might be inefficient.

4.3. Efficiency with Opportunity-Based Externalities

utility depends on own consumption bundle and on budget set profile
Which budget set profiles are feasible?
$p(b_i)$ denotes the price, $w(b_i)$ the wealth necessary to induce budget set b_i.
Definition 5: $(x, y, b) \in \mathbb{R}^{L \times I} \times Y \times B$ is feasible for a price p, iff for all $i \in I$, $j \in J$, and $l \in L$ it holds:

$$\begin{array}{rrcl}
\text{i)} & y_j & \in & Y_j \\
\text{ii)} & \sum_{i \in I} x_{il} & \leq & \sum_{i \in I} \varepsilon_{il} + \sum_{j \in J} y_{jl} \\
\text{iii)} & x_i & \in & b_i \\
\text{iv)} & p(b_i) & = & p \text{ for all } i = 1, \ldots I \\
\text{v)} & \sum_{i \in I} w(b_i) & = & \sum_{i \in I} p\varepsilon_i + \sum_{j \in J} py_j
\end{array}$$

Definition 6: In an economy $\mathcal{E}$ with distributional concerns a triple (x, y, b) is efficient with respect to a price vector p iff
i) (x, y, b) is feasible for p.
ii) there does not exist another triple (x', y', b') which is feasible for p, and for which it holds:

$$\begin{aligned} u_i(x_i, b) &\leq u_i(x_i', b') \text{ for all } i \in I \text{, and} \\ u_i(x_i, b) &< u_i(x_i', b') \text{ for at least one } i \in I \end{aligned}$$

$d_i(b_i)$ denotes $i's$ optimal consumption bundle for budget set b_i.

Redistributional Loser Property (RLP): RLP holds if for any $b, b' \in B$ with $b \neq b'$, $p(b) = p(b')$, and $\sum_{i \in I} w(b_i) \geq \sum_{i \in I} w(b'_i)$ there exists a consumer r such that

$$u_r(d_r(b_r), b) > u_r(d_r(b'_r), b')$$

Theorem 4: Under RLP, any equilibrium outcome (x^*, y^*, b^*) is efficient with respect to the equilibrium price vector p^*.

When is RLP fulfilled?
Example: i evaluates budget set b_k by $i's$ internal utility from $i's$ optimal consumption bundle in b_k.
Denote by $m_i(d_i(b_i))$ the indirect internal utility function of i, which is money proportional.

Theorem 5: RLP is fulfilled, whenever I is large enough and each consumer exhibits preferences represented by one of the following utility functions

i)

$$u_i(x_i, b) = m_i(x_i) + \frac{\beta}{I-1} \sum_{k \neq i} m_i(d_i(b_k))$$

with $\beta > -1$.

ii)

$$u_i(x_i, b) = m_i(x_i) + \beta \left| m_i(d_i(b_i)) - \frac{\sum_{k \in I} m_i(d_i(b_k))}{I} \right|$$

with $-1 < \beta < 0$.

iii)

$$u_i(x_i, b) = m_i(x_i) - \frac{\alpha}{I-1} \sum_{k \in I} \max\{m_i(d_i(b_k)) - m_i(x_i), 0\}$$
$$- \frac{\beta}{I-1} \sum_{k \in I} \max\{m_i(x_i) - m_i(d_i(b_k)), 0\}$$

with $\alpha \geq \beta \geq 0$, $\beta < 1$.

RLP also filfilled for the original one-good versions of these models
Same result for indirect utility functions, which are not money proportional, but with marginals bounded from above and away from zero.