Lecture 7: Non-equilibrium Strategic Thinking

Till now: Game theoretic rationality with non-egoistic preferences

Now: models of boundedly rational behavior in games

Lecture 8: Learning models

This lecture: non-equilibrium behavior

models behavior in "new", unfamiliar strategic situations before players can learn from experience

1. Quantal Response (McKelvey and Palfrey 1995, Goeree, Holt, and Palfrey 2008)

Basic features

- Player choose "noisy" best response

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Basic features

- Player choose "noisy" best response
- In equilibrium the likelihood of a strategy is increasing its expected payoff, taking the noisy strategies of the other players into account

n player normal form game G (or normal form representation of an extensive form game)

S_i: finite set of pure strategies of player i, $s_i \in S_i$

S: set of pure strategy profiles, $s \in S$

s_{-i} strategies of all players but i

each player i endowed with payoff-function:

$$\pi_i : S \to \mathbb{R}$$

mixed strategy of player i, σ_i: probability distribution over S_i

$\sigma_i \in \Sigma_i$

$$\sigma = (\sigma_1, \sigma_2, ...\sigma_N),\ \sigma \in \Sigma = \prod_{i \in N} \Sigma_i$$

$i's$ strategic behavior can be summarized by a function P_i: $\mathbb{R}^{|S_i|} \rightarrow \Sigma_i$

$P_{ij}(u_i)$: For a vector $u = (u_{i1}, u_{i2}, ...u_{i|S_i|})$, $P_{ij}(u_i)$ denotes the probability that player i chooses pure strategy j, if his payoff of choosing his strategy 1 is u_{i1}, of choosing his strategy 2 is u_{i2}, etc.

Definition: P_i is a quantal-response function, if the following conditions holds:
Continuity: $P_{ij}(u_i)$ is continuously differentiable at all u_i
Interiority: $P_{ij}(u_i) > 0$ for all u_i.
Responisveness: $\frac{\partial P_{ij}(u_i)}{u_{ij}} > 0$ for all j at all u_i.
Monotonicity: $u_{ij} > u_{ik} \iff P_{ij}(u_i) > P_{ik}(u_i)$ for all $j, k \in S_i$

All conditions exclude that P_i is equavalent to a best response correspondence, but conditions 3 and 4 keep the flavor of a best response $\iff P_i$ models a smoothened, non-precise version of best response.

Definition: Take a normal form game G and let $P = (P_1, P_2 ... P_N)$ be a profile of quantal response functions. A mixed strategy profile $\sigma^* \in \Sigma$ is a quantal response equilibrium (QRE) iff $\sigma^* = P(\sigma^*)$.

Theorem: For any game G and any profile of quantal response functions P there exists a QRE.

Theorem: Let P^n be a sequence of profiles of quantal response functions that converges to a profile equivalent to the best responce correspondances. The resulting sequence of QREs converges to a Nash equilibrium.

QRE can explain many experimental results, but:
QRE depends on P which is of course not unique - too many degrees of freedom

2. Level-k-Models

2.1. Experimental evidence

Guessing game ("beauty contest") (Nagel 1995, Ho et al. 1998):

n players choose simultaneously a number between α and β; $\alpha \geq 0$, $\beta > \alpha$

The person whose number is closest to x times the average number wins.
Ties broken randomly

Unique Nash equilibrium:
$x < 1$: all players choose α.
$x > 1$: all players choose β.

This equilibrium also results from iterated elimination of strictly dominated strategies

Experimental results in early rounds:

Most players choose numbers larger than zero

Spikes at $\frac{\alpha+\beta}{2}x^k$, $k = 1, 2$, and 3.

Inconsistent with quantal response

2.2. The model (Nagel 1995, Stahl and Wilson 1995)

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S: set of pure strategy profiles, $s \in S$

s_{-i} strategies of all players but i

each player i endowed with payoff-function:

$$\pi_i : S \rightarrow \mathbb{R}$$

Players are heterogenous and charaterized by their sophistication level.

A L(evel)0 player i makes a random choice according to some probability distribution over S_i (for many applications equal distribution over feasible strategies assumed). Such $L0$ players do not actually exist, but they are needed to anchor the belief of all other players.

$L1$, $L2$, $L3$...players play best response, but differ in terms of beliefs about what the other players do.

A $L1$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L0$.

A $L2$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L1$.

A $L3$ player i chooses a strategy s_i that maximizes his payoff under the believe that all other players are $L2$.

etc.

Applied to the guessing game:

$L0$: random choice with equal distribution over numbers between α and β. choice of strategy is random

$L1$ players believe that othere players are $L0 \Rightarrow$ average is $\frac{\alpha+\beta}{2} \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x$

$L2$ players believe that othere players are $L1 \Rightarrow$ average is $\frac{\alpha+\beta}{2}x \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x^2$

$L3$ players believe that othere players are $L2 \Rightarrow$ average is $\frac{\alpha+\beta}{2}x^2 \Rightarrow$ best response is $\frac{\alpha+\beta}{2}x^3$

Level-k models predict well actual behavior in many economically important experimental games, e.g. auctions

Variant of level-k model: Cognitive Hierarchy Model (Camerer at al 2004):

Lk players do not play best response against belief that only $L(k-1)$ players exist, but against a belief given by a distribution over all $L(k-m)$ players with $m = 1, 2...k$.

Believed distribution of $L(k-m)$ players is such that the relative frequency of all types are correct - A players does not realize that other players might of the same or higher level than himself, but he correctly predicts the ratio between $L(k-m)$ and $L(k-n)$ players $(m, n \in \{1...k\})$

Shortcomings of level-k models:

Cannot capture learning well - "short term" predictions

In many games, predictions depend on assumption about $L(0)$, and on size of $L(1)$ players, etc.

Unclear, whether predictions are better than with QRE (Breitmoser 2012)