Lecture 10: Normal Form Games with Incomplete Information

Incomplete information: private information that is not common knowlegde, and that is relevant for play (e.g. information about payoffs, information about strategy spaces etc.)

To solve problem: game transformed

Example: Market entrance game (figure 6.2., 6.3)

1. Normal form game of incomplete information (Bayesian Game)

I : finite set of players; typical element i

each player i is of type θ_i; Θ_i finite set of possible types of player i.

θ_i summarizes all private information if $|\Theta_i| = 1$ for all i, then game with complete information

objective probability distribution of type-profiles

$$p : \prod_i \Theta_i \to \Delta^I$$

$p(\theta_1, ...\theta_I)$: probability, that type-profile $\theta_1, ...\theta_I$ realized

Assumed that for each θ_i there exists a θ_{-i} with

$$p(\theta_i, \theta_{-i}) > 0$$

Every player i knows own type, but not that of other players

$p(\theta_{-i} | \theta_i)$: conditional probability that all but i are of type profile θ_{-i}, if i is θ_i.

set of pure actions $\widetilde{S}_i$

payoff function $\widetilde{u}_i : \prod_i \widetilde{S}_i \times \prod_i \Theta_i \rightarrow \mathbb{R}$

Game with incomplete information transformed (expanded):

pure strategy $s_i : \Theta_i \rightarrow \widetilde{S}_i$ with $s_i(\theta_i)$ being the action choosen by i if he is of type θ_i

mixed strategy $\sigma_i : \Theta_i \rightarrow \Delta^{|\widetilde{S}_i|-1}$

payoffs function:

$$u_i(\sigma_1, ...\sigma_n) = \sum_{\theta \in \Theta} p(\theta)\widetilde{u}_i(\sigma_1(\theta_1), ...\sigma_I(\theta_I), \theta)$$

This expanded game is a standard normal form game.

2. Baysian equilibrium

Definition: The Bayesian equilibrium of a game with incomplete information is a Nash-equilibrium of the expanded game.

Existence guaranteed by the Nash's equilibrium existence theorem.

Example: Cournot game with unknown cost structure

2 firms with constant returns to scale technology choose simultaneously quantities q_1 and q_2

Firm 1: marginal costs c_1, common knowlegde

Firm 2: marginal costs c_2^l with probability $\frac{1}{2}$ and marginal costs c_2^h with probability $\frac{1}{2}$. $2\prime s$ cost type only known to 2.

Market demand: $p = (A - q_1 - q_2)$

One type of firm 1; two types of firm 2, $\Theta_2 = \{h, l\}$

Sets of pure strategies: $S_1 = \mathbb{R}_+$; $S_2 = \mathbb{R}_+ \times \mathbb{R}_+$

profit functions:

$$\pi_1 = \frac{1}{2}(q_1(A - q_1 - q_2(c_2^l) - c_1)) + \frac{1}{2}(q_1(A - q_1 - q_2(c_2^h) - c_1))$$

$$\begin{aligned} \pi_2 &= \frac{1}{2}(q_2(c_2^l)(A - q_1 - q_2(c_2^l) - c_2^l)) \\ &\quad + \frac{1}{2}(q_2(c_2^h)(A - q_1 - q_2(c_2^h) - c_2^h)) \end{aligned}$$

Profit maximization of 2:

$$\underset{q_2(c_2^l),q_2(c_2^h)}{Max} \quad \pi_2$$

FOC's

$$\begin{aligned} 0 &= A - q_1 - 2q_2(c_2^l) - c_2^l \\ 0 &= A - q_1 - 2q_2(c_2^h) - c_2^h \end{aligned}$$

$\Longrightarrow$

$$\begin{aligned} q_2(c_2^l) &= \frac{A - q_1 - c_2^l}{2} \\ q_2(c_2^h) &= \frac{A - q_1 - c_2^h}{2} \end{aligned}$$

Profit maximization of firm 1:

$$\underset{q_1}{Max}\frac{1}{2}(q_1(A-q_1-q_2(c_2^l)-c_1))+\frac{1}{2}(q_1(A-q_1-q_2(c_2^h)-c_1))$$

$\Longrightarrow$

$$0=\frac{1}{2}(A-2q_1-q_2(c_2^l)-c_1)+\frac{1}{2}(A-2q_1-q_2(c_2^h)-c_1)$$

$\Longrightarrow$

$$q_1=\frac{2A-q_2(c_2^l)-q_2(c_2^h)-2c_1}{4}$$

Equilibrium:

$$\begin{aligned}
q_1^* &= \frac{2A+c_2^l+c_2^h-4c_1}{6}\\
q_2^*(c_2^l) &= \frac{4A-7c_2^l-c_2^h+4c_1}{12}\\
q_2^*(c_2^h) &= \frac{4A-c_2^l-7c_2^h+4c_1}{12}
\end{aligned}$$