Lecture 12a: Global Games

Global games: Uncertainty of actual game played and of knowledge of others about the game.

Each player receives noisy signal about the "real game", and signals are not perfectly correlated

Simpliest case: symmetric 2x2 game

	α		β	
α	x	x	x	0
β	0	x	4	4

$x > 4$: Unique NE in dominant strategies: (α, α)

$x \in [0, 4]$: 2 pure NEs: (α, α), (β, β) - coordination game

$x < 0$: Unique NE in dominant strategies: (β, β)

true x unknown to the players; random draw from $[\underline{x}, \overline{x}]$ with $\underline{x} < 0$ and $\overline{x} > 4$;

each player i receives noisy signal x_i uniformly distributed on $[x - \epsilon, x + \epsilon]$; draws independent, $\epsilon < \min(\frac{\underline{x}}{2}, \frac{\overline{x}-4}{2})$

distribution of x and x_i common knowledge $\Rightarrow$

if player i receives signal x_i, he considers x to be uniformly distributed on $[x_i - \epsilon, x_i + \epsilon]$

if player i receives signal x_i, he thinks that his opponent's signal x_j is uniformly distributed on $[x_i - 2\epsilon, x_i + 2\epsilon] \Rightarrow$

$\Rightarrow Prob(x_j > x_i | x_i) = Prob(x_j < x_i | x_i) = \frac{1}{2}$

pure strategy: an action for each possible signal

$$s_i : [\underline{x} - \epsilon, \overline{x} + \epsilon] \to \{\alpha, \beta\}$$

switching (cutoff) strategy:

$$s_i(x_i) = \begin{matrix} \alpha & \text{if } x_i > k_i \\ \beta & \text{if } x_i \le k_i \end{matrix}$$

with k_i being the cutoff point

Proposition: $k_i = k_j = 2$ is the unique equilibrium

Proof (skectch): If $x_i < 0$, for player i action α is strictly dominated by β. Same reasoning for $j \Rightarrow k_j \geq 0$. If $x_i = 0$, then for any $k_j \geq 0$ the probability that j will not play α is at least $\frac{1}{2}$, since $Prob(x_j < x_i \,|x_i\,) = \frac{1}{2}$. Therefore i gets at least an expected payoff of 2 from β, and 0 from α - α also dominated for $x_i = 0$. By continuity, the same argument holds for also for low, but strictly positive x_i. Take any such $2 > x_i > 0$. Because of symmetry, $k_j \geq x_i$. This further increases probalility that $x_j \leq k_j$ and therefore "slows down" the increase in the expected utility from choosing α. It can be shown that as long as $x_i < 2$, α gives a lower expected payoff that β when the iteratively strictly dominated actions of j are taken into account.

Same reasoning from above for $x_i > 2$ ■

Note:

Any uncertainty about the game, i.e. any $\epsilon > 0$ is enough to solve the coordination problem - unique equilibrium

Reason: In standard coordination game the equilibrium is determined by self-fulfilling higher order beliefs. Here impact of these beliefs is eliminated by uncertainty.

Resulting equilibrium coincides with the risk-dominant (and not pareto-dominant) equilibrium of the game with given x

Results can be generalized for

continuum of players

large strategy sets, if strategies are complements (best reply of i increases when strategy of j increases)

asymmetric games