Lecture 5. Choice under Uncertainty II

1. Stochastic Dominance

Comparison of payoff distributions

First order stochastic dominance - two equivalent definitions:

i) $F(.)$ first-order stochastically dominates $F'(.)$ if for every nondecreasing function $u : \mathbb{R} \rightarrow \mathbb{R}$ it holds:

$$\int u(x)dF(x) \geq \int u(x)dF'(x)$$

"Every expected utility maximizer who appreciates money prefers $F(.)$ over $F'(.)$."

ii) $F(.)$ first-order stochastically dominates $F'(.)$ if

$$F(x) \leq F'(x) \text{ for all } x.$$

"The probability, that the realized payoff is above a certain threshold x, is larger for $F(.)$ than for $F'(.)$ for any such threshold."

Second order stochastic dominance:

For any two distributions $F(.)$ and $F'(.)$ with the same mean, $F(.)$ second-order stochastically dominates $F'(.)$ if for every nondecreasing concave function $u : \mathbb{R} \rightarrow \mathbb{R}$ it holds:

$$\int u(x) dF(x) \geq \int u(x) dF'(x)$$

"Provided that both distributions give the same expected monetary payoff, every risk averse agent prefers $F(.)$ over $F'(.)$."

2. State dependent utility

Agent cares not only about consequences, but also about reasons for consequences.

S : set of states of nature, finite; actual state unknown

$\pi_s > 0$: probability that s occurs; objective probabilities

Function $g : S \to \mathbb{R}_+$ maps states of nature into monetary outcomes.

Every $g(.)$ induces a lottery $F(.)$ with $F(x) = \sum_{\{s:g(s)\leq x\}} \pi_s$

Each g also represented by $(x_1 ... x_S)$. Set of (nonnegative) g is $\mathbb{R}_+^S$

$\succeq$: rational preferences defined on $\mathbb{R}_+^S$

Definition: $\succeq$ has an extended expected utility representation iff for every $s \in S$ there is a function $u_s : \mathbb{R}_+ \longrightarrow \mathbb{R}$ such that: $(x_1...x_S) \succeq (x_1'...x_S')$ if and only if $\sum\limits_{s\in S} \pi_s u_s(x_s) \geq \sum\limits_{s\in S} \pi_s u_s(x_s')$.

$u_{s:}$ state dependent utility function (before: state-independent or state uniform utility functions)

Furthermore, we allow that within each state the monetary payoff is not a certain amount, but a lottery with distribution $F_s(.)$.

Hence, alternative $L = (F_1, F_2....F_S)$.

Extended independence: $\succeq$ satisfies the extended independence axiom if for all $L, L', L"$ and $\alpha \in (0,1)$, we have:

$$L \succeq L' \text{ iff } \alpha L + (1-\alpha)L" \succeq \alpha L' + (1-\alpha)L"$$

Proposition: Suppose $\succeq$ is rational, continuous and satisfies the extended independence axiom. Then we can assign utility functions $u_s(.)$ for money in every state s such that for any $L = (F_1, F_2...F_S)$ and $L' = (F'_1, F'_2...F'_S)$ we have:

$$\begin{aligned} L &\succeq L' \text{ iff} \\ \sum_{s\in S} \pi_s(\int u_s(x_s)dF_s(x_s)) &\geq \sum_{s\in S} \pi_s(\int u_s(x_s)dF'_s(x_s)). \end{aligned}$$

3. Subjective Probability Theory

Till now: objective probabilities

Now: no objective probabilities - uncertainty

Goal: Derivation of subjective probabilities from preferences

"The agent has preferences over the uncertain alternatives as if he maximizes an expected utility function with a probability distribution that looks like... "

S: set of states without objective probabilities

g again represented by $(x_1 ... x_S)$. Set of (nonnegative) g is $\mathbb{R}^S_+$

Again we allow that within each state the monetary payoff is not a certain amount, but a lottery with distribution $F_s(.)$.

Hence, again alternative $L = (F_1, F_2....F_S)$

state dependent preferences $\succeq = (\succeq_1, \succeq_2, ... \succeq_S)$

Preferences $\succeq$ are assumed to be rational, continuous, and satisfy the extended independence axiom. Then we have $u_s(.)$ functions that are Bernoulli functions for every state.

Definition: The state preferences $(\succeq_1, \succeq_2, ... \succeq_S)$ are state uniform iff $\succeq_s = \succeq_{s'}$ for all $s, s' \in S$.

"The risk attitude towards money is the same in all states."

Proposition: Suppose $\succeq$ is rational, continuous, satisfies the extended independence axiom, and is state uniform. Then there are probabilities $(\pi_1, \pi_2 ... \pi_S) >> 0$ and a Bernoulli function $u(.)$ on amounts of money such that for any $(x_1 ... x_S)$ and$(x'_1 ... x'_S)$ we have:

$$\begin{aligned} (x_1 ... x_s) &\succeq (x'_1 ... x'_s) \text{ if and only if} \\ \sum_s \pi_s u(x_s) &\geq \sum_s \pi_s u(x'_s). \end{aligned}$$

Moreover, the probabilities are uniquely determined, and the utility function is unique up to a scalar transformation.