Graduate Microeconomics I
Problem Set 5

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1. The purpose of this exercise is to illustrate how expected utility theory allows us to make consistent decisions when dealing small probabilities by considering relative large ones. Suppose that a safety agency is thinking of establishing a criterion under which an area prone to flooding should be evacuated. The probability of flooding is 1%. There are four possible outcomes: (i) no evacuation is necessary, and none is performed; (ii) an evacuation is performed that is unnecessary; (iii) an evacuation is performed that is necessary; and, (iv) no evacuation is performed and a flood causes a disaster. Suppose that the agency is indifferent between the sure outcome (ii) and the lottery of (i) with probability p and (iv) with probability $1-p$, and between the sure outcome (iii) and the lottery of (i) with probability q and (iv) with probability $1-q$. Suppose also that it prefers (i) to (iv) and that $p, q \in (0, 1)$. Assume that the conditions of the expected utility theorem are satisfied.

   (a) Construct a utility function of expected utility form for the agency.

   (b) Consider two different policy criteria:

   Criterion 1 This criterion will result in an evacuation in 90% of the cases in which flooding will occur and an unnecessary evacuation in 10% of the cases in which no flooding occurs.

   Criterion 2 It results in an evacuation in 95% of the cases in which flooding will occurs and an unnecessary evacuation in 15% of the cases in which no flooding occurs. First derive the probability distributions over the four outcomes under these criteria. Then, by using the utility function in a), decide which criterion the agency would prefer.

2. Assume that a firm is risk neutral with respect to profits and that if there is any uncertainty in prices, production decisions are made after the resolution of such uncertainty. Suppose that the firm faces a choice between two alternatives. In the first, prices are uncertain. In the second, prices are nonrandom and equal to the expected price in the first alternative. Show that a firm that maximizes expected profits will prefer the first alternative over the second.

3. An individual has Bernoulli utility function $u(\cdot)$ and initial wealth w. Let lottery L offer a payoff of G with probability p and a payoff B with probability $1-p$.

   (a) If the individual owns the lottery, what is the minimum price he would sell it for?

   (b) If he does not own, what is the minimum price he would be willing to pay for it?

   (c) Are buying and selling prices equal? Give an economic interpretation of your answer. Find conditions on the parameters of the problem under which buying and selling prices are equal.

4. Suppose that an individual has a Bernoulli utility function $u = \sqrt{x}$.

   (a) Calculate the Arrow-Pratt coefficients of absolute and relative risk aversion the level of wealth $w = 5$.

   (b) Calculate the certainty equivalent and the probability premium for a gamble $(16, 4; 1/2, 1/2)$.

   (c) Calculate the certainty equivalent and the probability premium for a gamble $(36, 16; 1/2, 1/2)$. Compare this results with the one in b) and interpret.

5. A wage earner, Mister T, with initial wealth $w = 1000$, has a Bernoulli utility function $u(x) = \sqrt{x}$. His monthly wage amounts to 3000 euros. He can be sacked with probability 0.05. An insurance company designs an insurance contract against unemployment. Mister T would pay an insurance premium of 200 euros to receive 2000 euros if he were sacked. There are only two periods: a first one at which decisions are taken and a second one at which all uncertainty is resolved.

(a) Does Mr. T decide to insure himself against unemployment?

(b) What is the maximum insurance premium the insurance company can ask such that Mr. T still wants to buy an insurance contract?

6. Verify that if a distribution $G(\cdot)$ is an elementary increase in risk from a distribution $F(\cdot)$, then $F(\cdot)$ second-order stochastically dominates $G(\cdot)$.

7. The purpose of this exercise is to verify the equivalence of the three statements of proposition 6.D.2 of MWG in a two-dimensional probability simplex. Suppose there are three monetary outcomes: 1, 2 and 3 euros. Consider the probability simplex in Figure 6.B.1(b) in MWG.

   (a) If two lotteries have the same mean, what are their positions relative to each other in the probability simplex?

   (b) Given a lottery L, determine the region of the simplex in which lie the lotteries L' whose distributions are second-order stochastically dominated by L.

   (c) Given a lottery L, determine the region of the simplex in which lie the lotteries L' whose distributions are mean-reserving spreads of L.

Extra exercises students are encouraged to do:

8. Suppose that an individual has a Bernoulli utility function $u(x) = -e^{-ax}$, where $a > 0$. His (nonstochastic) initial wealth is given by w. There is one riskless asset and there are N risky assets. The return per unit invested in the riskless asset is r. The returns of the risky assets are jointly normally distributed random variables with means $\mu = (\mu_1, ..., \mu_N)$ and variance-covariance matrix V. Assume there is no redundacy in the risky assets, so that V is of full rank. Derive the demand function for these $N + 1$ assets.

9. Assume that in a world with uncertainty there are two possible states of nature ($S = 1, 2$) and a single consumption good. There is a single decision maker whose preferences over lotteries satisfy the axioms of the expected utility theory and who is risk-averter. For simplicity, we assume that utility is state-independent. Two contingent commodities are available to the decision maker. The first (second) pays

one unit of the consumption good in state $S = 1$ ($S = 2$) and zero otherwise. Denote the vector of quantities of the two contingent commodities by (x_1, x_2).

(a) Show that the preference relation of the decision maker on (x_1, x_2) is convex.

(b) Argue that the decision maker is also risk averter when choosing between lotteries whose outcome are vectors (x_1, x_2).

(c) Show that the Walrasian demand functions for x_1 and x_2 are normal.