Graduate Microeconomics I
Problem Set 8

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1. Consider the following game, where player 2 can be of type A or B, and where player 1 cannot distinguish between the two types. It is common knowledge that the probability for player 2 to be type A is 1/2. The matrixes of payoffs are the following: if player 2 is of type A,

		Player 2	
		Left	Right
Player 1	Top	1,1	4,3
	Down	4,0	1,4

and if player 2 is of type B,

		Player 2	
		Left	Right
Player 1	Top	1,5	4,0
	Down	4,2	1,0

What is the set of Bayesian Nash Equilibria?

2. Consider the linear Cournot model, where the price is given by $P(q_1, q_2) = a - b(q_1 + q_2)$. Suppose that each firm has a probability μ of having unit cost c_L and $(1 - \mu)$ of having cost c_H, $c_H > c_L > 0$. Solve the Bayesian Nash Equilbria.

3. Two firms must decide whether to enter a market or to stay outside. If only on firm enters it will be a monopolist and have profits π^m. If both firms enter, there will be a duopoly and each firm will have profits π^d. Each firm faces a cost $\theta_i \in [0, \bar{\theta}]$ (where $\bar{\theta}$ is finite), drawn independently from a uniform distribution. The entry cost is private information to the firm. The payoff of firm i if it is the only one to enter the market is $\pi^m - \theta_i$; if both firms enter the market, each firm gets $\pi^d - \theta_i$. Suppose that $\pi^m > \pi^d > 0$.

(a) Draw the strategic form representation of this game and write its pure strategies.

(b) Compute the Bayesian Nash Equilibrium.

4. Consider now two players deciding whether to contribute or not to a public good. The public good is supplied if at least only one of them contributes. If none of them contributes the public good is not supplied. The costs for each player i is c_i. This cost is private knowledge to the player but it is common knowledge that c_i is drawn independently from a uniform distribution on $[\underline{c}, \bar{c}]$. The benefit of the public good is 1 if it is provided and 0 otherwise. Draw the matrix of payoff of this game and compute the Bayesian Nash equilibria.

5. Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either attack or not attack. In addition, each army is either strong or weak with equal probability (the draws for each army are independent), and an army's type is only known by its general. Payoffs, are as follows: the island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a cost of fighting, which is s if it is strong and w if it is weak, where $s < w$. There is no cost of attacking it is rival does not. Identify all pure strategy Bayesian Nash equilibria of this game.