Graduate Microeconomics I
Problem Set 9

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1. Consider the following interaction (sequential game) between a parent and a child. The parent first has to decide whether:

   - to give (G) her child a twinckie,
   - or not (NG),

   and then the child in turn responds in one of three possible ways:

   - hold his breath until he dies (HB),
   - cry for half an hour (C),
   - or do nothing (N).

   The parent does not want to give the child a twinckie because it is less than healthy; however, he also does not like crying and he certainly does not want the child to asphyxiate himself. The parent's utility over this possible outcomes are as follows:

   - give and child holds breath: -100,
   - give and child cries: -3,
   - give and child does nothing: -1,
   - not give and child holds breath: -100,
   - not give and child cries: -2,
   - not give and child does nothing: 10.

   The child loves twinkies, loses energy from crying, and has no wish to kill himself. His utility levels over the possible outcomes are as follows:

   - receive and hold breath: $-\infty$,
   - receive and cry: 3,
   - receive and do nothing: 5,
   - not receive and hold breath: $-\infty$,
   - not receive and cry: -1,
   - not receive and do nothing: 0.

   (a) Write out the extensive form (game tree) for this.

(b) Write out the normal form game, being careful to include every strategy on the child's part and remembering that a strategy lists the player's moves for every possible state.

(c) Find the Nash equilibrium or equilibria.

(d) Find the subgame perfect equilibrium for this game.

(e) Compare the results of (c) and (d) and explain.

2. Consider the following simultaneous move-game

		Player 2		
		Left	Middle	Right
Player 1	Top	4,7	1,5	3,6
	Down	2,1	3,1	5,5

(a) Determine all Nash equilibria of this game.

(b) Suppose the game is played sequentially. Under version 1, player 1 starts and chooses either Top or Down, and then player 2 chooses between Left, Middle and Right; payoffs are specified in the table above. Alternatively, under version 2, player 2 starts and chooses an action from her set of alternatives (already mentioned) and then player 1 chooses between T and D. Imagine there is a social planner that only cares about the sum of the individual payoffs and has to choose between the two alternative sequential-move games. Given the equilibria in pure strategies, what is the optimal order of moves?

3. Consider a two-stage game with two players. Player I moves first and selects between A and B. If player I chooses A, the game ends and each of the players gets a payoff equal to 3. If player I selects B, then the game proceeds to the second stage, where players choose simultaneously; that is, player I chooses between U and D and, player II chooses between L and R. The payoffs achieved in this second stage are as follows:

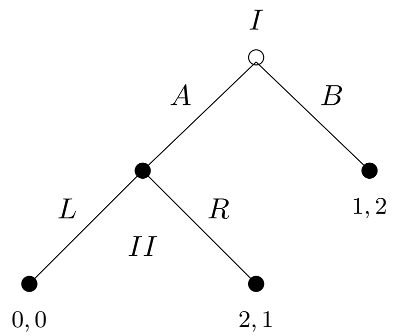
		Player II	
		L	R
Player I	U	1,1	1,5
	D	5,1	0,0

(a) Draw the extensive form of this game.

(b) List all pure strategies for both players.

(c) Determine all pure-Nash equilibria. Which of them are subgame perfect.

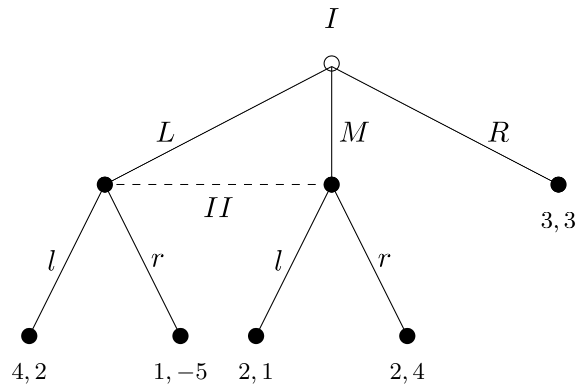
4. Consider the following sequential-move game:



(a) List all pure strategies that players I and II have.

(b) Find the set of Nash equilibria. Which equilibria also belong to the set of subgame perfect Nash equilibria.

5. Consider the following game:



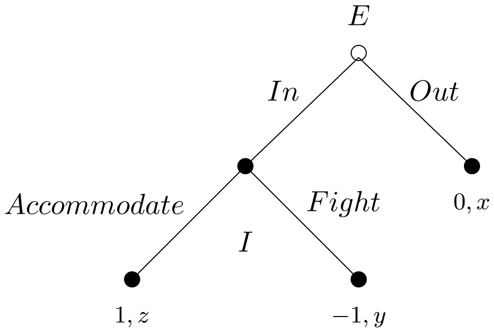
(a) List all pure strategies that players I and II have.

(b) What is the set of Nash equilibria?

(c) What is the set of subgame perfect Nash equilibria.

6. At time 0, an incumbent firm (I) is already in the widget market, and a potential firm (E) is considering entry. In order to enter, firm E must incur a cost $K > 0$. Firm E's only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game illustrate below is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight (the left payoff is for firm E). Once firm E plays "out", it is out of the market forever; firm E earns zero in any period during which it is out of the market, and firm I earns x. The discount factor is δ. Assume that:

- $x > z > y$.
- $y + \delta x > (1 + \delta)z$
- $1 + \delta > K$.



(a) What is the (unique) subgame Nash equilibrium of this game?

(b) Suppose that firm E faces a financial constraint. In particular, if firm I fights once against firm E (in any period), firm E will be forced out of the market from that point on. Now what is the (unique) subgame perfect Nash equilibrium of this game?