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On the possibility of efficient private provision of public goods through government subsidies

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Abstract

This paper investigates the possibility of implementing an efficient provision of a public good through distortionary tax-subsidy policies in a simple one-shot game of voluntary contributions. Within the class of all linear tax-subsidy policies two cases are distinguished. The first is where individual taxes only depend on the sum of all other individuals' contributions. Although such policies may increase total supply of the public good, it is shown that the implementation of an efficient amount is not possible unless the government has complete information about individual characteristics. In the second case, where taxes depend on the distribution of contributions, the equilibrium supply of the public good is no longer unique. For any efficient interior solution there might also exist inefficient boundary solutions. Moreover, unlike the boundary solutions, the efficient interior solution is in general not stable. © 1997 Elsevier Science S.A.

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1. Introduction

Voluntary contributions to a public good typically entail underprovision of that good. Many authors have therefore considered models in which a 'government'

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subsidizes private contributions so as to increase the total supply of the public good. Usually, in these models, government's subsidy payments are financed by appropriate taxation. While lump-sum transfers typically leave the equilibrium amount of the public good unchanged, distortionary tax-subsidy policies may indeed increase total equilibrium supply of the public good.[1] Given this possibility to influence aggregate supply of a public good, the question arises whether by choosing an appropriately designed tax-subsidy policy a government can implement an efficient amount of the public good. This is the problem addressed by the present paper. The analysis is restricted to the most natural case of linear tax-subsidy policies. Unfortunately, in this case our results are rather negative. Indeed, it is shown that either (i) the government needs to know individual preferences in order to implement an efficient allocation, or (ii) the contribution game admits a multiplicity of equilibria with not all of them corresponding to efficient allocations. Moreover, in the latter case, an efficient equilibrium is in general unstable.

The framework for our analysis is the following general tax-subsidy scheme. Each agent's own contribution is subsidized at some fixed individual subsidy rate. At the same time, each agent faces a tax that is a linear function of all other agents' contributions to the public good. Subsidy payments and taxes are linked in such a way that the government's budget is balanced for any possible distribution of individual contributions. In determining the level of her own contribution to the public good, each agent optimizes against all other agents, taking their decisions as given. Aggregate supply of the public good then results from the equilibrium level of individual contributions in this simultaneous one-shot game. Given this general model, two cases have to be distinguished.

Firstly, it can be shown that within our framework the model suggested by Andreoni and Bergstrom (1996) corresponds to the case where each agent's tax only depends on the sum of all other individuals' contributions (and not on their distribution). In the following, we refer to this case as the case of individually uniform tax rates. In this case, each choice of subsidy rates induces a unique aggregate equilibrium supply of the public good which is increasing with the subsidy rates (see Andreoni and Bergstrom, 1996). Moreover, it is easy to determine subsidy rates that induce an efficient allocation provided that all individual contributions are positive in equilibrium. However, in this paper we prove that given such subsidy rates all individual contributions remain positive in

[1] Warr (1983) has shown that lump-sum transfers do not alter the equilibrium amount of the public good provided that the set of contributors does not change. Bergstrom et al. (1986) provide a general analysis of income redistributions. Warr's neutrality result is confirmed in Bernheim (1986) who considers distortionary income taxes. For the possibility to influence aggregate supply of a public good through subsidies to voluntary contributions in a framework with 'naive' individuals who ignore the government's budget constraint, see Roberts (1987), (1992) and Boadway et al. (1989). Non-neutrality of tax financed subsidies with rational individuals who take into account the government budget constraint has been established in the models of Andreoni and Bergstrom (1996) and Falkinger (1996).

equilibrium only if the resulting effective prices for the individuals are the Lindahl prices. This implies that in order to implement an efficient allocation the government must have complete information about individual preferences (or at least about the individuals' demand for the public good). However, if a government does have complete information, there is of course no point in designing a complicated tax-subsidy policy. Indeed, in that case the government has complete control over the state of the economy via much simpler tax policies, e.g. the government could take care of the supply of the public good and impose appropriate lump-sum taxes to cover its expenditures.

In order to overcome this difficulty, one has to allow agents' tax payments to depend on the distribution of the other agents' contributions to the public good. Indeed, an example of a subsidy policy where the existence of an efficient interior solution is not the exception is the model recently suggested by Falkinger (1996). In this model, the population is divided into subgroups and each agent's tax only depends on the contributions made by individuals belonging to the same subgroup. If, however, the agents' tax payments depend on the distribution of the other individuals' contributions other problems arise. Firstly, in that case the equilibrium amount of the public good is no longer uniquely determined. Specifically, we prove the following result. For any subsidy scheme where tax rates are not individually uniform there exist individual preferences and distributions of incomes such that besides an efficient interior solution there is also a non-efficient boundary solution where at least one individual contributes zero. Moreover, in contrast to the boundary solution the efficient interior solution is not stable in an appropriately defined sense. This result casts some serious doubt on the possibility to implement efficient allocations by linear tax-subsidy policies involving different tax rates for the other agents' contributions.

The plan of the paper is as follows. In Section 2 we present the general framework of our analysis and discuss its relation to the literature. Section 3 is devoted to the case of individually uniform taxation. In Section 4, we consider linear tax rules that are not individually uniform. Concluding remarks are offered in Section 5.

2. The model

Consider an economy with n individuals, indexed by $i=1, \ldots, n$. Each individual's utility is given by a strictly quasi-concave utility function $u^i(c_i,G)$, where c_i denotes i's consumption of a private good and G the consumption of a purely public good. Throughout, private consumption and the public good are assumed to be strictly normal goods at every level of wealth. Furthermore, we assume that each individual's utility function is continuously differentiable. Each individual has an initial endowment of m_i units of the private good. For simplicity, let the price of the private good be equal to 1. Hence, one may think of m_i as consumer i's income. The public good is produced from private goods at a cost of one unit private good per unit of public good.

The public good is supplied by voluntary contributions of the consumers. For each i, denote by g_i consumer i's contribution to the public good. Furthermore, let $G_{-i} := \Sigma_{j \neq i}\, g_j$ denote the sum of the contributions of all agents different from i. A common assumption in a model of private provision of a public good is that each individual takes the activities of all other agents as given for her own decision. Consequently, consumer i's decision problem is

$$\max_{c_i, g_i} u^i(c_i, g_i + G_{-i}) \quad \text{s.t.} \\ c_i + g_i = m_i \textit{ and } g_i \geq 0. \tag{2.1}$$

A pair of n-tuples $(c_1^*, \ldots, c_n^*)$ and $(g_1^*, \ldots, g_n^*)$ that solves (2.1) for all i is hence a Nash equilibrium of the corresponding contribution game played by the n individuals. It is well known that without any intervention an equilibrium of the game described so far entails underprovision of the public good. Many authors have therefore considered extensions of this model allowing for the possibility that a government subsidizes private contributions (see, among others, Andreoni (1988); Andreoni and Bergstrom (1996); Boadway et al. (1989); Brunner and Falkinger (1995); Falkinger (1996) and Roberts (1987), (1992)). In its most general form, such a government intervention may be described as follows. Each individual's private contribution is subsidized at a rate σ_i, where $0 \leq \sigma_i < 1$. Hence, if i contributes g_i she receives a payment of $\sigma_i g_i$. Government expenditure, in turn, is financed by taxes where each agent's tax payment depends on all other individuals' contributions. In the present paper, we confine ourselves to the most natural case where each agent's tax is a linear function of all other agents' contributions. Denoting by $t_{ij} \geq 0$ agent i's tax rate with respect to agent j's contribution, consumer i's budget constraint may thus be written as[2]

$$c_i + (1 - \sigma_i)g_i = m_i - \sum_{j \neq i} t_{ij} g_j, \tag{2.2}$$

with the additional constraint that $g_i \geq 0$.

Remark: the subsidy scheme described by (2.2) is the most general form of government subsidies through a change of relative prices when taxes are linear. Firstly, observe that there is no rationale to let individual i's tax depend on other individuals' private activities, i.e. their private consumption c_j. Clearly, individual i's tax may depend on her own private consumption c_i. However, any reasonable form of such a dependence must be linear in c_i. Since all that matters are relative prices, such a dependence is already incorporated in the σ_is. By a similar argument

[2] Our analysis is completely general with respect to the distribution of income. Without loss of generality, we therefore neglect lump-sum transfers in our model.

one may also assume without loss of generality that agent i's tax does not depend on her own contribution g_i to the public good.

Note that if agents optimize against each other according to (2.2) there is a problem of bankruptcy, since some configurations of strategies (i.e. choices of (c_i, g_i)) may entail negative net-income for some consumers. In this case there would be no well-defined payoff. Consequently, in analyzing the corresponding contribution 'game', we will assume throughout that in equilibrium individual contributions $(g_1^*, \ldots, g_n^*)$ satisfy the following condition.[3]

NB (No bankruptcy). For all $i \in \{1, \ldots, n\}$,

$$m_i - \sum_{j \neq i} t_{ij} g_j^* \geq 0.$$

As in most of the recent literature, we will assume that subsidies and taxes are linked so as to satisfy the government's budget constraint which is given by:

$$\sum_i \sigma_i g_i = \sum_i \sum_{j \neq i} t_{ij} g_j \quad \textit{for all } g_1, \ldots, g_n. \tag{2.3}$$

The class of subsidy schemes described by (2.2) and (2.3) contains as a special case the model recently suggested by Andreoni and Bergstrom (1996). In their specification, one has for each i, $\sigma_i = \beta(1 - s_i)$ for some parameter $\beta \in [0,1]$ and $0 < s_i < 1$. Furthermore, for all i and all j, $t_{ij} = s_i \beta$, hence consumer i's total tax burden is $s_i \beta G_{-i}$. Rearranging individual i's budget constraint yields

$$c_i + (1 - \beta) g_i = m_i - s_i \beta G. \tag{2.4}$$

Hence, each consumer's contribution g_i is subsidized at a rate of β, and at the same time the consumer is taxed for a fixed share s_i of total government expenditure on subsidies. As a consequence, consumer i's effective price for the public good is $1 - \beta + s_i \beta$. Clearly, the government's budget is balanced if and only if $\Sigma_i\ s_i = 1$. Notice that each individual is taxed for each unit of the public good provided by any other agent at a constant rate, i.e. each individual's tax payment only depends on the sum of all other agents' contributions. Hence, the Andreoni/Bergstrom model belongs to the class of subsidy schemes with individually uniform taxation. In this model, Andreoni and Bergstrom (1996) prove that for each $\beta \in [0,1]$ and any family $s_1, \ldots, s_n$ with $0 < s_i < 1$ and $\Sigma_i [0,1)\ s_i = 1$, there exists a unique Nash equilibrium $g_1^*(\beta), \ldots, g_n^*(\beta)$ of the corresponding contribution game. Furthermore, total supply in equilibrium $G^*(\beta) := \Sigma_i\ g_i^*(\beta)$ is increasing in the parameter β.

The first order conditions for private contribution behaviour are given by

$$MRS^i = 1 - \beta + s_i \beta,$$

[3] Note that this condition is always (sometimes implicitly) assumed in related models of the literature.

where $MRS^i(c_i,G) := \frac{\partial u^i(c_i,G)/\partial G}{\partial u^i(c_i,G)/\partial c}$ is the absolute value of the marginal rate of substitution between the public good and private consumption. Summing up over all individuals thus gives

$$\sum_{i=1}^{n} MRS^i = n - \beta(n-1).$$

Consequently, if all individual contributions are positive in equilibrium — an assumption which will be shown to be extremely restrictive — an efficient provision of the public good would require $\beta = 1$. Indeed, with $\beta = 1$ one obtains Σ_i MRS$^i = 1$, the condition characterizing efficient allocations.[4] Although also for $\beta = 1$ there is a unique aggregate equilibrium supply of the public good, individual equilibrium contributions are no longer unique for that particular value of β (see Section 3).[5] Nevertheless, one may ask if by choosing β sufficiently close to 1 one can implement an amount of the public good arbitrarily close to an efficient amount as the result of a unique Nash equilibrium. This question is addressed in Section 3.

Another special case of the subsidy scheme described by (2.2) and (2.3) is the model recently proposed by Falkinger (1996). In his model, the population is partitioned into subgroups and each individual is rewarded or penalized on the basis of the average contribution of the subgroup to which the individual belongs (cf. Section 4). Note that, since individual taxes only depend on the contributions made within the same subgroup, taxation is not individually uniform in that model.

3. Efficient allocations with individually uniform tax rates

In this section, we investigate existence of efficient allocations for the case of individually uniform tax rates. Hence, assume that in (2.2) each individual i is taxed at a constant rate for each contribution made by another individual, i.e. assume that for all i and all j, $t_{ij} = t_i$ for some $t_i \geq 0$. The analysis is substantially simplified by the observation that any subsidy scheme of the form of (2.2) with that property can be rewritten as in (2.4). Hence, the Andreoni/Bergstrom model described by (2.4) exactly corresponds to the case of individually uniform taxation. Indeed, it can be shown that given the governments budget constraint (2.3) the assumption of individually uniform taxation implies that for all i, j, $t_i + \sigma_i = t_j + \sigma_j$. Defining $\beta := t_i + \sigma_i$ and $s_i := t_i/\beta$, the budget constraint (2.2) then

[4] By strict normality, private consumption and total supply of the public good are always positive in equilibrium. Hence, efficiency in equilibrium is always characterized by the standard Samuelson rule.

[5] This seems to be the reason why Andreoni and Bergstrom (1996) explicitly exclude the case $\beta = 1$ in their analysis.

easily transforms into (2.4). Also, it can be checked that $\Sigma_i\ s_i = 1$.[6] The case $\beta \leq 1$ corresponds to the case where $\Sigma_{j=1}^{n}\ \sigma_j \leq n-1$. Of course, this is the only case of interest, since with $\Sigma\sigma_j > n-1$ one would obtain 'overprovision' of the public good.

Firstly, we analyze the case of interior equilibrium of the contribution game corresponding to (2.4). It has already been observed in the previous section that efficiency in an interior equilibrium requires $\beta = 1$. In order to characterize the equilibrium of contribution game for $\beta = 1$, consider the following closely related problem for individual i.

$$\begin{aligned} &\max_{c_i, G} u^i(c_i, G) \quad \text{s.t.} \\ &c_i + s_i G = m_i \quad \textit{and} \quad G \geq 0. \end{aligned} \tag{3.1}$$

The solution $\tilde{G}_i(m_i,\ s_i)$ to this problem is individual i's demand for the public good provided that its price is s_i and that no other individual contributes to the public good. Consequently, we refer to $\tilde{G}_i(m_i,\ s_i)$ as individual i's stand-alone contribution. Now, compare (3.1) to individual i's maximization problem given the subsidy scheme (2.4) for the value $\beta = 1$.

$$\begin{aligned} &\max_{c_i, g_i} u^i(c_i, g_i + G_{-i}) \quad \text{s.t.} \\ &c_i + s_i G = m_i \quad \textit{and} \quad g_i \geq 0. \end{aligned} \tag{3.2}$$

Obviously, the only relevant difference to the problem (3.1) is the non-negativity constraint. Denote by M the set of those individuals with maximal stand-alone contribution, i.e.

$$M := \{j \in \{1, \ldots, n\} : \tilde{G}_j(m_j, s_j) \geq \tilde{G}_i(m_i, s_i) \textit{ for all } i = 1, \ldots, n\}.$$

Furthermore, let $G^*(1)$ denote aggregate equilibrium supply of the public good resulting from (3.2).

Fact 1: For any $j \in$ M, $G^*(1) = \tilde{G}_j(m_j,\ s_j)$.

Indeed, suppose that $G^*(1) < \tilde{G}_j(m_j,\ s_j)$. Then, by definition of $\tilde{G}_j(m_j,\ s_j)$, individual j would have an incentive to increase her own contribution, in which case $G^*(1)$ could not be an equilibrium value. Next, suppose that $G^*(1) > \tilde{G}_j(m_j,\ s_j)$ for some $j \in$ M. Then, in fact, $G^*(1) > \tilde{G}_i(m_i,\ s_i)$ for all i. However, by strict normality, $G^*(1) > 0$, hence there must exist at least one individual who contributes a positive amount. By definition of $\tilde{G}_i(m_i,\ s_i)$, any such individual would have an incentive to lower her contribution. Hence again, $G^*(1)$ cannot be an

[6] For a detailed proof of these assertions, see Brunner and Falkinger (1995) (Lemma 5.1).

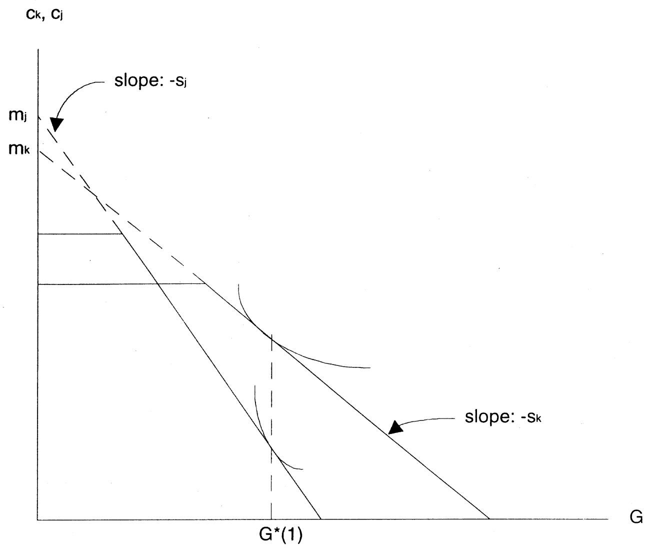


Fig. 1. Equilibrium supply of G for $\beta=1$.

equilibrium value. Consequently, $G^*(1)=\tilde{G}_j(m_j, s_j)$ for all $j \in$ M (see Fig. 1 which depicts equilibrium supply for two individuals belonging to M).

Note, in particular, that by Fact 1 total equilibrium supply $G^*(1)$ is uniquely determined.

Fact 2: If $\tilde{G}_i(m_i, s_i)<G^*(1)$, then i's unique individual equilibrium contribution $g_i^*(1)$ equals zero.

Indeed, $\tilde{G}_i(m_i, s_i)<G^*(1)$ implies that i has an incentive to lower her contribution as long as it is positive.

Fact 2 implies that in equilibrium for $\beta=1$ the set of contributors must be a subset of M.[7] But this implies at once that an efficient interior solution can only exist if M equals the set of all individuals, i.e. only if

$$\tilde{G}_1(m_1,s_1)=\tilde{G}_2(m_2,s_2)=\cdots=\tilde{G}_n(m_n,s_n). \tag{3.3}$$

Noting that (3.3) defines the Lindahl prices, it follows that an efficient interior solution can only exist if the effective prices induced by the subsidy rates are the Lindahl prices. However, the government is assumed to have no information about individual demand for the public good, hence it cannot know the Lindahl prices. Consequently, in general, (3.3) will fail to hold. If (3.3) is violated, equilibrium

[7]Note that $j \in M$ does not imply $g_j^*(1)>0$. Indeed, it follows at once from (3.2) that individual contributions are not uniquely determined in equilibrium for $\beta=1$. Any vector $(g_1^*(1),\ldots, g_n^*(1))$ such that $\Sigma_{j\in M}\ g_j^*(1)=G^*(1)$ is an equilibrium of individual contributions.

supply of the public good at $\beta=1$ necessarily entails overprovision of the public good. Indeed, for any individual i with non-maximal stand-alone contribution one must have $\mathrm{MRS}^i < s_i$ in equilibrium. Consequently, in equilibrium for $\beta=1$, $\Sigma_i\, \mathrm{MRS}^i < 1$. In particular, if (3.3) fails, the equilibrium does not approach an efficient allocation as β tends to 1. On the other hand, it is clear that for $\beta=0$, $\Sigma_i\, \mathrm{MRS}^i(c_i^*(0), G^*(0)) > 1$. Hence, by the intermediate value theorem, if (3.3) is not satisfied, there exists some $\beta^e < 1$ such that the equilibrium allocation corresponding to β^e is efficient.

Summarizing, the following result has been established.

Theorem 1: A necessary condition for the existence of an efficient interior equilibrium under individually uniform taxation is that the effective prices induced by subsidies are the Lindahl prices. If effective prices do not coincide with the Lindahl prices there exists $\beta^e < 1$ *such that the corresponding equilibrium allocation is efficient. In any such equilibrium at least one individual will contribute zero.*

Although by Theorem 1 there always exist β such that the corresponding equilibrium supply of the public good is efficient, it is also clear that in any case the corresponding value of β crucially depends on individual preferences. Hence, a subsidy scheme with individually uniform tax rates can in general not induce an efficient supply of the public good without knowledge of preferences or individual demand functions.

Remark 1: Facts 1 and 2 show that, as β tends to 1, individuals with low stand-alone contributions are 'crowded out' by individuals who are richer and/or more inclined to contribute to the public good. In particular, in the generic case where individual stand-alone contributions are different, the public good is supplied by a single individual as β tends to 1. This conclusion resembles a result obtained by Andreoni (1988) in a model without subsidization but with increasing population, where a similar effect causes the fraction of contributors to shrink to zero as the population gets large.

Remark 2: Suppose that all individuals are endowed with quasi-linear preferences, i.e. suppose that $u^i(c_i, G) = c_i + v_i(G)$ for all i. In this case it is immediately clear — and well-known — that not all individuals will contribute unless prices are personalized according to individual preferences. However, with quasi-linear preferences goods are not strictly normal, and there seems to be no obvious way to use this result in order to derive the same conclusion for the case of strictly normal goods.[8] Indeed, the case of quasi-linear preferences is rather special. In our framework, a necessary condition for the existence of an interior equilibrium at $\beta=1$ would be that for all i, $v_i'(G^*(1)) = s_i$. Prima facie this condition seems similar to (3.3). However, unlike (3.3) satisfaction of this condition cannot be controlled for by redistributing income. Furthermore, for some distributions of

[8] Certainly, it is not possible to approximate a preference displaying strictly normal goods by a sequence of quasi-linear preferences.

quasi-linear preferences there might exist no interior equilibrium for any distribution of income even without subsidization. This can never happen with strictly normal goods.

4. General linear tax rules

In this section, we investigate the case where taxation is not individually uniform, i.e. where at least one individual's tax depends on the distribution of all other agents' contributions.[9] Hence, consider the general tax-subsidy scheme described by (2.2) and (2.3) and assume that

$$t_{ij} \neq t_{ik} \text{ for some } i,j,k \in \{1, \ldots, n\}. \tag{4.1}$$

To illustrate the possibility that under (4.1) efficient interior equilibria may exist also when subsidy rates do not imply the Lindahl prices, consider the following example. As in the model suggested by Falkinger (1996), suppose that each individual is subsidized at a constant rate σ. Furthermore, suppose that the population is partitioned into subgroups and individuals are taxed on the basis of the average contribution of the subgroup to which they belong. Specifically, consider four individuals with identical preferences $u^i(c_i, G) = (c_i)^{3/4} G^{1/4}$ for $i = 1$, 2, 3, 4. Individuals 1 and 2 form the first subgroup denoted by I, individuals 3 and 4 make up the second subgroup denoted by II. For simplicity, we assume that individuals belonging to the same group have the same income, i.e. $m_I := m_1 = m_2$ and $m_{II} := m_3 = m_4$. In this example the average contribution of all other agents in the same subgroup is simply given by the single contribution of the other agent in the same group. Balanced government budget thus implies the following individual budget constraint

$$c_i + g_i = m_i + \sigma(g_i - g_j),$$

where j is the other individual in the same subgroup. If individual equilibrium contributions are positive, an inspection of the first-order conditions shows that efficiency requires $\sigma = 3/4$. Given this value of σ, individual i's reaction function is easily calculated as

$$g_i(g_j, g_k, g_l) = \max\left\{m_i - \frac{3}{2}\, g_j - \frac{3}{4}\,(g_k + g_l), 0\right\},$$

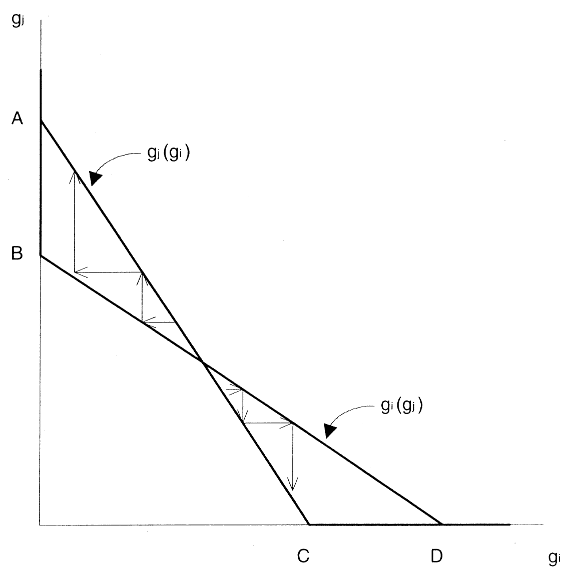
where j belongs to the same subgroup, whereas k and l together form the other subgroup. Assuming that all individuals contribute a positive amount, the

[9] Note that this requires at least three consumers in the economy. Indeed, with only two individuals taxes are automatically individually uniform.

following equilibrium contributions can be derived, $g_1^* = g_2^* = 5m_I/8 - 3m_{II}/8$ and $g_3^* = g_4^* = 5m_{II}/8 - 3m_I/8$. Consequently, there is an efficient interior solution if and only if $3/5 < m_I/m_{II} < 5/3$.

However, in addition to the efficient interior equilibrium there is a variety of inefficient boundary solutions. For instance, if $3/5 < m_I/m_{II} < 4/3$ there are two equilibria where exactly one individual from subgroup *I* contributes zero whereas all other agents make a positive contribution. Similarly, if $3/4 < m_I/m_{II} < 5/3$ there are two equilibria where exactly one individual from subgroup *II* contributes zero. Hence, whenever an efficient interior solution exists there are also inefficient boundary equilibria. In fact, it can be checked that there are further boundary equilibria in addition to those described. In any of these equilibria the no-bankruptcy condition is satisfied, i.e. each agent's private consumption is positive.

Besides the multiplicity of equilibria in this example there is another problem of instability of the interior equilibrium. Indeed, as can be seen from Fig. 2, the reaction curves of two individuals belonging to the same subgroup intersect with



A: $m_j - (3/4)(g_k + g_l)$

B: $(2/3)(m_i - (3/4)(g_k + g_l))$

C: $(2/3)(m_j - (3/4)(g_k + g_l))$

D: $m_i - (3/4)(g_k + g_l)$

Fig. 2. Non-uniqueness and instability of the interior solution.

the 'wrong' slope. This implies instability of the interior equilibrium with respect to any dynamics where individuals adjust their contribution in direction of their best responses.

The difficulties with non-uniform tax rates are by no means specific to the particular example just discussed. Specifically, one has the following result.

Theorem 2: For any linear tax-subsidy scheme with property (4.1) there exist preference and income distributions such that besides the efficient interior equilibrium there is also an inefficient boundary equilibrium for the corresponding contribution game.

Before we proceed to the proof of Theorem 2, we need the following preliminary result. Observe first that an efficient interior solution can only exist if subsidy rates satisfy

$$\sum_{i=1}^{n} \sigma_i = n - 1. \tag{4.2}$$

Let $\tilde{t}_i$ denote individual i's average tax rate, i.e. $\bar{t}_i := \Sigma_{j \neq i}\, t_{ij}/(n-1)$.

Lemma 4.1: Suppose that subsidy rates satisfy (4.2). Either there exists an individual i_0 whose average tax rate t_{i_0} is larger than her effective price $1-\sigma_{i_0}$, or, for all i, $\bar{t}_i = 1 - \sigma_i$.

Proof: Assume by way of contradiction that, for all i, $\bar{t}_i \leq 1 - \sigma_i$ with strict inequality for some i. Summing over i one would obtain

$$\sum_{i=1}^{n} \sum_{j \neq i} t_{ij} < (n-1) \sum_{i=1}^{n} (1 - \sigma_i). \tag{4.3}$$

However, differentiating the government's budget constraint (2.3) with respect to g_i yields $\sigma_i = \Sigma_{j \neq i}\, t_{ji}$ for all i. Using this and interchanging the order of summation in (4.3) one could conclude $\Sigma_{j=1}^{n}\, \sigma_j < n-1$. However, this is in contradiction to (4.2).

The following proof of Theorem 2 is based upon the case distinction described in Lemma 4.1. Firstly, assume that there is an individual i_0 whose average tax rate is larger than her effective price for the public good. Suppose that the valuation for the public good is sufficiently low for all individuals but i_0, and consider an income distribution such that there is nevertheless an interior equilibrium in which all individuals contribute the same amount. Since i_0's average tax is higher than her effective price, and since moreover, i_0's valuation for the public good is high compared to the other agents, there is an additional (inefficient) boundary equilibrium in which only i_0 supplies the public good. Next, consider the case where average tax rate equals effective price for all individuals. Then, since taxation is not individually uniform, there must exist two individuals, i_0 and j_0 such that i_0's tax rate for j_0's contribution is larger than i_0's effective price for the public good. From this, the existence of an inefficient boundary equilibrium in addition to the efficient interior equilibrium can be inferred by a similar argument as in the first case.

Proof of Theorem 2: Assume that all individuals are endowed with Cobb-Douglas type preferences $u^i(c_i, G)=c_i^{\alpha_i}G^{1-\alpha_i}$. In this case, agent i's (unrestricted) reaction function can be calculated as

$$g_i = \frac{1-\alpha_i}{1-\sigma_i}\, m_i - \sum_{j\neq i}\left(\frac{1-\alpha_i}{1-\sigma_i}\, t_{ij} + \alpha_i\right) g_j. \tag{4.4}$$

Consider a family of 'efficient' subsidy rates $\sigma_1, \ldots, \sigma_n$ satisfying (4.2). We distinguish two cases according to Lemma 4.1.

Case 1. There exists i_0 such that $\bar{t}_{i_0} > 1-\sigma_{i_0}$. Without loss of generality, assume that $i_0=1$. Fix $\alpha_1\in(0,1)$. (The exact value of α_1 will be determined later so as to satisfy the no-bankruptcy constraint for individual 1.) For each $i=2, \ldots, n$, consider a strictly increasing sequence $(\alpha_i^l)_{l\in\mathbf{N}}$ converging to 1. For any $l\in\mathbf{N}$ one can find a distribution $(m_1^l, \ldots, m_n^l)$ of incomes such that the contribution vector $(g_1^*, g_2^*, \ldots, g_n^*)=(1, 1, \ldots, 1)$ is an interior equilibrium. Clearly, if α_i tends to 1, m_i is unbounded from above. Therefore, one can choose α_1 sufficiently large so that $c_1^*=m_1-(1-\sigma_1)-\sum_{j\neq i} t_{ij}$ is positive. Similarly, for large enough l, c_i^* is positive for $i=2, \ldots, n$.

Next, we show that for sufficiently large l there is also an inefficient boundary equilibrium where only individual 1 makes a positive contribution. If 1 is the only contributing individual it follows from (4.4) that her optimal contribution is

$$g_1^{**} = \frac{1-\alpha_1}{1-\sigma_1}\, m_1^l.$$

On the other hand, since $(1, 1, \ldots, 1)$ is a solution one obtains, again by (4.4),

$$\frac{1-\alpha_1}{1-\sigma_1}\, m_1^l = 1 + (1-\alpha_1)\sum_{j\neq 1}\frac{t_{1j}}{1-\sigma_1} + \alpha_1(n-1).$$

Since by assumption $\bar{t}_1>1-\sigma_1$, it follows that $g_1^{**}>n$. Now consider all individuals different from 1. Since the vector $(1, 1, \ldots, 1)$ is an interior solution, and since the α_i^l converge to 1 one obtains from (4.4)

$$\frac{1-\alpha_i^l}{1-\sigma_i}\, m_i^l \rightarrow n \textit{ if } l\rightarrow\infty.$$

Hence, since $g_1^{**}>n$, agent i's unrestricted reaction function becomes negative for sufficiently large l provided that all agents $j\neq 1,i$ contribute zero. Consequently, i's best response to $g_1=g_1^{**}$ and $g_j=0$ for $j\neq 1,i$ is $g_i^{**}=0$ for sufficiently large l. This shows that for large l, $(g_1^{**}, 0, \ldots, 0)$ is an additional equilibrium. Observe that this equilibrium cannot be efficient. Indeed, at the equilibrium allocation one has $\mathrm{MRS}^1=1-\sigma_1$. However, all other individuals' unrestricted reaction function becomes strictly negative in equilibrium. Hence,

$\mathrm{MRS}^j < 1 - \sigma_j$ for $j = 2, \ldots, n$. Together with (4.2) this immediately implies inefficiency. Also observe that the no-bankruptcy constraint is satisfied in the boundary equilibrium.

Case 2. Suppose now that for all i, $\bar{t}_i = 1 - \sigma_i$. Since the tax-subsidy scheme satisfies (4.1), there must exist i_0 and $j_0 \neq i_0$ such that $t_{i_0 j_0} > 1 - \sigma_{i_0}$. Without loss of generality, assume that $i_0 = 1$ and $j_0 = 2$. The proof in this case is similar to the proof in Case 1. Again, fix α_1 so that in the end the no-bankruptcy condition is satisfied for individual 1, and consider for each $i = 2, \ldots, n$ a strictly increasing sequence $(\alpha_i^l)_{l \in \mathbf{N}}$ converging to 1. For each l, choose the distribution $(m_1^l, \ldots, m_n^l)$ so that the contribution vector $(g_1^*, g_2^*, \ldots, g_n^*) = \left(\frac{1}{n-1}, 1, \frac{1}{n-1}, \ldots, \frac{1}{n-1}\right)$ is an interior equilibrium. By (4.4) this implies that for all $l \in \mathbf{N}$,

$$\frac{1 - \alpha_1}{1 - \sigma_1} m_1^l > 2 \quad \textit{and} \quad \frac{1 - \alpha_2^l}{1 - \sigma_2} m_2^l = 2.$$

Furthermore, for each $i = 3, \ldots, n$,

$$\frac{1 - \alpha_i^l}{1 - \sigma_i} m_i^l \to 2 \quad \textit{if} \quad l \to \infty.$$

This implies by the same arguments as in Case 1 that, for sufficiently large l, an additional equilibrium is given by

$$(g_1^{**}, g_2^{**}, \ldots, g_n^{**}) = \left(\frac{1 - \alpha_1}{1 - \sigma_1} m_1^l, 0, \ldots, 0\right).$$

Again, this equilibrium is inefficient and satisfies the no-bankruptcy constraint if l is large enough.

The preferences constructed in the proof of Theorem 2 might seem rather extreme. Note, however, that this is due to the great generality of Theorem 2 since it applies to arbitrary linear tax-subsidy schemes satisfying (4.1). In specific examples — such as the one considered above — much less extreme preference distributions yield similar conclusions.

Also note that the instability of the interior equilibrium uncovered in the example is a general phenomenon. Suppose, for instance, that individual preferences are of Cobb-Douglas type. By Lemma 4.1, if a tax-subsidy scheme is not individually uniform, there exist individuals i_0 and $j_0 \neq i_0$ such that $t_{i_0 j_0} > 1 - \sigma_{i_0}$. If α_{j_0} is sufficiently close to 1 this implies that the reaction curves of individuals i_0 and j_0 intersect qualitatively as shown in Fig. 2.

5. Conclusion

In this paper, we have argued that linear tax-subsidy policies in a simple one-shot, simultaneous move game of voluntary contributions to a public good are not an appropriate tool for implementing efficient allocations. In designing such a policy, the central planner (the 'government') faces a dilemma. Either the government chooses a policy where each individual's tax only depends on the sum of all other individuals' contributions, i.e. an individually uniform tax-subsidy scheme, or an incentive scheme where some individuals' tax depends on the distribution of contributions.

In the first case, an efficient interior equilibrium only exists if the government can implement the Lindahl prices through subsidy rates. However, this requires knowledge that the government is assumed not to have. It is worth noting that this problem can be solved in a different framework which has been suggested in the literature. Consider, for instance, the following two-stage game proposed by Danziger and Schnytzer (1991) (see also Althammer and Buchholz (1993); Varian (1994)). In the first stage individuals announce appropriate subsidy rates by which they will subsidize other agents' contributions to a public good. Given these subsidy rates, individual contributions are then simultaneously determined in a second stage. In this two-stage game, it can be shown that the Lindahl subsidies indeed form the unique subgame perfect equilibrium. Given the difference in the informational structure of the two games, the difference in the results is of course not surprising. In both models, individual preferences are common knowledge to any potential contributor. Consequently, in the Danziger/Schnytzer game all players have complete information. On the other hand, in the model considered in the present paper subsidy rates are set by a central planner who has no information about individual preferences. Clearly, either of the two models implies extremely restrictive informational assumptions on the part of potential contributors. Taking these assumptions for granted, we believe that the model considered here has a priori much more practical appeal, in particular, if the number of agents is large.

In the second case, when a subsidy policy is chosen where individuals tax payments depend on the distribution of the other agents' contributions, it has been shown that uniqueness of the equilibrium is no longer guaranteed. Moreover, even if an efficient interior equilibrium exists, it is in general not stable, and typically there exist additional stable and inefficient boundary equilibria.

Our overall conclusion, that linear tax-subsidy policies are not appropriate for implementing efficient allocations, bears some resemblance to negative results obtained in the very different framework of mechanism design models (see e.g. Green and Laffont (1979)). In our context, an interesting open question is whether the multiplicity of equilibria can be avoided by designing suitable non-linear taxation rules. On the other hand, it seems to us that the problem of instability would persist also under more complicated tax policies.

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