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## Inefficient equilibria in lobbying

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### Abstract

Lobbying is often represented as a common agency game. Common agency games typically have multiple equilibria. One class of equilibria, called *truthful*, has been identified by Bernheim and Whinston [Quarterly Journal of Economics 1986;101(1):1–31]. In this paper, we identify another class of equilibria, which we call *natural*, in which each principal offers a positive contribution on at most one alternative. We run an experiment on a common agency game for which the two equilibria predict a different equilibrium alternative. The alternative predicted by the natural equilibrium is chosen in 65% of the matches, while the one predicted by the truthful equilibrium is chosen in less than 5% of the matches. © 2001 Elsevier Science B.V. All rights reserved.

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### 1. Introduction

Common agency games model a situation where several principals simultaneously try to influence the behavior of one agent. The agent must choose one alternative among a set of alternatives. Each of the principals cares about which

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alternative the agent chooses and can promise monetary contributions to the agent conditional on the agent's choice. Namely, each principal can promise a vector of monetary contributions, one for each possible alternative. Only the contribution on the alternative that is chosen will actually be paid. The agent observes all the monetary contributions offered by the principals and makes his choice.

Common agency is often used to represent the lobbying process. The agent is a politician who faces a set of policy alternatives. The politician cares both about monetary contributions, which he can spend on his electoral campaign, and directly about the policy alternative he chooses (either because he is genuinely concerned or because he wants to please voters). Each principal is a lobby who represents a special interest. Each lobby can offer — maybe implicitly — campaign contributions to the politician conditional on his policy stance. A partial list of political economy papers that use common agency includes: Grossman and Helpman (1994, 1996), Dixit et al. (1997), Rama and Tabellini (1998), and Helpman and Persson (1988). All these works rely on theoretical foundations developed by Bernheim and Whinston (1986), who were the first to study common agency. After noting that the typical common agency game has several equilibria, Bernheim and Whinston discuss a particular class of equilibria, which they name truthful. A truthful equilibrium is called 'truthful' because in it the contribution schedule of each principal follows the shape of the payoff function of that principal (the exact definition will be given later). Bernheim and Whinston show several striking properties: (1) a truthful equilibrium always exists (that is, the set of equilibria of a given common agency game always contains a truthful equilibrium); (2) an equilibrium is coalition-proof if and only if it is payoff-equivalent to a truthful equilibrium; and (3) in a truthful equilibrium the agent chooses an alternative which maximizes the sum of the payoffs of the agent and of the principals.<sup>1</sup>

Point (3) is of foremost importance. The alternative selected in a truthful equilibrium is 'efficient' in that it maximizes the sum of payoffs of the players. In the case of lobbying, this means that if all citizens can make unlimited monetary contributions to politicians, the policy alternative chosen will be efficient. If there are inefficiencies connected with lobbying, they must be because some groups of citizens do not have access to politicians, contributions are somehow restricted, or 'corruption' contracts are incomplete.

However, the results of Bernheim and Whinston and the subsequent applications hinge on the assumption that the truthful equilibrium is played. As Besley and Coate (2000) have pointed out in the context of lobbying, if principals do not play

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<sup>1</sup>Bergemann and Välimäki (1998) extend Bernheim and Whinston's analysis to a multi-period common agency game. They define truthful equilibrium and coalition-proof equilibrium in a dynamic setting and show that points (2) and (3) hold in this setting as well.

truthful strategies, there can be coordination failures that give rise to inefficiencies. Hence, it is important to understand to what extent we can rely on the truthful equilibrium.

In this paper, we ask whether the truthful equilibrium is the only reasonable equilibrium. One conjecture is that principals may behave in a simpler way. Instead of making positive offers on all, or most, possible alternatives, as the truthful equilibrium requires, each principal makes only one strictly positive offer, on the alternative that she hopes to get. We call such strategy natural and — if it exists — we call the corresponding equilibrium natural as well.

We prove that a natural equilibrium always exists. It can actually be constructed in a simple way by identifying two candidate alternatives that split the set of principals in two camps. These alternatives have the property that they can mobilize the greatest amount of resources between principals. Once the two candidate alternatives are identified, each principal makes a strictly positive offer only on the alternative preferred by her side.

A natural equilibrium need not be coalition-proof. That occurs when the two opposite camps fail to contribute to a compromise alternative that would give higher net payoffs to both sides. Thus, the natural equilibrium does not have the efficiency property discussed in point (3) above. The alternative chosen may not be the one that maximizes the sum of payoffs of the players involved. Hence, if we believe that people play natural rather than truthful, we should be more pessimistic on the possibility that the lobbying process generates an efficient outcome.

To ascertain which class of equilibrium is a better predictor of behavior we run an experiment. We use a two-principal three-alternative common agency game, in which the truthful equilibrium and the natural equilibrium predict different alternatives. At every round, a principal is matched with a different principal. Each principal announces a schedule of contributions. The experimenter acts as agent by choosing the alternative that maximizes the sum of contributions.

The results of the experiment are mixed. On the one hand, the natural equilibrium is a much better predictor of the outcome of the game. The ‘natural’ alternative is chosen in 65% of the matches, while the ‘truthful’ alternative is chosen in 3.6% of the matches. On the other hand, most subjects do not use natural contribution schedules, as 81% of the chosen schedules include positive offers on more than one alternative. However, the truthful equilibrium too is a poor predictor of the subjects’ strategies because truthful contributions are observed in only 17% of the cases.

The plan of the paper is as follows. Section 2 contains the theory part. After summarizing the main results by Bernheim and Whinston on truthful equilibria, we define natural equilibrium and prove its properties. Section 3 describes the design of the experiment. Section 4 reports the results. Section 5 concludes with an informal discussion of possible reasons why people do not seem to play truthful strategies. The Appendix contains the proofs and the instructions for the experiment.

## 2. Theory

### 2.1. The model

In a common agency game, the players are one agent and  $m$  principals. The set of principals is denoted with  $M = \{1, \dots, m\}$ . The agent chooses an alternative out of a finite set of alternatives  $S$ . Each principal tries to induce the agent to take a particular alternative rather than another by offering him a monetary payment which we call a *contribution*. Let  $t_s^j$  denote the contribution that principal  $j$  promises to make to the agent if the agent chooses alternative  $s \in S$ . The strategy of principal  $j$  is a contribution schedule  $t^j$ , namely a vector of contributions, one for each alternative in  $S$ . Contributions are restricted to be nonnegative. If the agent selects alternative  $s$  he receives a total monetary contribution  $\sum_{j \in M} t_s^j$ . Contributions promised on alternatives other than the chosen alternative are not paid (this is the difference between common agency and an all-pay auction).

The agent cares only about how much money he receives. He is indifferent about what alternative he chooses.<sup>2</sup> Hence, the agent chooses  $s$  to maximize  $\sum_{j \in M} t_s^j$ .

Each principal cares about how much money she pays to the agent and which alternative the agent chooses. Let  $G_s^j$  denote the utility (gross payoff) principal  $j$  derives from  $s$ . The net payoff of principal  $j$  if alternative  $s$  is chosen is  $G_s^j - t_s^j$ .<sup>3</sup>

The game is played in two stages. First, all principals simultaneously and noncooperatively choose their contribution schedules. Second, the agent observes the principals' contribution schedules and selects an alternative.

### 2.2. Truthful equilibria

Bernheim and Whinston (1986) note that a typical common agency game has many equilibria.<sup>4</sup> They propose to focus on one type of equilibrium, which they call truthful, and they prove a number of important properties of truthful equilibria. This subsection reviews Bernheim and Whinston's results.

**Definition 1.** The contribution schedule  $t^j$  of principal  $j \in M$  is said to be truthful if it can be written as  $t_s^j = \max(0, G_s^j - u^j)$  for all  $s \in S$ , where  $u^j$  is a constant. A

<sup>2</sup>The results presented here can easily be extended to accommodate an agent with direct preferences on the alternative he chooses, as in Bernheim and Whinston (1986).

<sup>3</sup>Dixit et al. (1997) have shown that the main results of Bernheim and Whinston are still valid if the principals or the agent have nonseparable preferences.

<sup>4</sup>We will focus on subgame perfect Nash-equilibria, which for simplicity will be referred to as equilibria.

truthful equilibrium is an equilibrium of the common agency game in which all principals offer truthful contribution schedules.<sup>5</sup>

A truthful contribution schedule follows the shape of the payoff function of the principal plus or minus a constant, except that, when the contribution would be negative, the nonnegativity constraint requires a zero contribution instead. The main feature of a truthful contribution schedule is that (but for the nonnegativity constraint) a principal who plays truthful is indifferent with regards to the alternative that the agent ends up choosing.

The properties of truthful equilibria that are relevant to our analysis can be summarized as follows:<sup>6</sup>

**Theorem 1 (Bernheim and Whinston).** *For any common agency game,*

- (i) *For any  $j \in M$ , given  $\{t^i\}_{i \neq j}$ , the set of best responses of principal  $j$  contains a truthful contribution schedule.*
- (ii) *There exists a truthful equilibrium.*
- (iii) *Every truthful equilibria is coalition-proof and every coalition-proof equilibrium is payoff-equivalent to a truthful equilibrium.*
- (iv) *In a truthful equilibrium, the agent chooses an efficient alternative, that is  $s^* \in \operatorname{argmax}_{s \in S} \sum_{j \in M} G_s^j$ .*

Part (i) of Theorem 1 says that, given the contribution schedules of the other principals, a principal can restrict her attention without loss to truthful contribution schedules.

Note that (i) does not imply that a truthful equilibrium actually exists. Bernheim and Whinston do, however, show the existence of a truthful equilibrium (Part (ii)), that is, they prove that the set of equilibria of a given common agency game contains an equilibrium which is truthful.

Part (iii) links truthful equilibria to coalition-proofness. The definition of coalition-proofness for common agency can be found in Bernheim and Whinston's article. For the goal of the present paper, an informal definition will suffice. An equilibrium of a common agency game is coalition-proof if there exists no coalition of principals that can benefit by agreeing on a 'self-enforcing' joint deviation from the equilibrium. The definition of self-enforcing deviation is

<sup>5</sup>We will sometimes say that a 'principal plays truthful' meaning that she offers a truthful contribution schedule. An analogous expression will be used for natural contribution schedules.

<sup>6</sup>Theorem 1 is not stated directly in that form in Bernheim and Whinston (1986). Part (i) corresponds to Bernheim and Whinston's Theorem 1. Part (ii) is an immediate consequence of Bernheim and Whinston's Theorem 2. Part (iii) is Bernheim and Whinston's Theorem 3. Part (iv) is Bernheim and Whinston's Theorem 2.

recursive. A joint deviation for a given coalition is self-enforcing if there exists no coalition within the given coalition that can benefit from a (self-enforcing) deviation from the proposed joint deviation. Part (iii) of Theorem 1 says that there is an essential equivalence between the set of truthful equilibria and the set of coalition-proof equilibria. All truthful equilibria satisfy coalition-proofness and an equilibrium which is not truthful, or payoff-equivalent to a truthful equilibrium, does not satisfy coalition-proofness.<sup>7</sup>

Part (iv) guarantees efficiency. In a truthful equilibrium the agent chooses an alternative that maximizes the Utilitarian welfare function of the principals. The implications of this result have already been discussed at length in the Introduction.

### 2.3. Natural equilibria

This subsection contains the original theoretical contribution of this paper. We introduce the concept of natural equilibrium and compare it to Bernheim and Whinston's truthful equilibrium. The idea is simple. In a truthful equilibrium, each principal usually makes strictly positive offers on more than one alternative. This may look rather complicated and one can ask if there are other equilibria. In particular, one may wonder if there is an equilibrium in which each principal makes only one strictly positive offer. In view of its simple structure, we may call it a natural equilibrium. Formally,

**Definition 2.** The contribution schedule  $t^j$  of principal  $j \in M$  is said to be natural if  $t_s^j = 0$  for all  $s \in S$  except, at most, one. A natural equilibrium is an equilibrium of the common agency game in which all principals offer natural contribution schedules.

We stress that a natural equilibrium is an equilibrium of the game (and not an equilibrium of the game with a restricted strategy space). Hence, in a natural equilibrium each principal has no incentive to use a more complicated strategy than the equilibrium natural contribution schedule.

How many of the points of Theorem 1 apply also to natural equilibria? As we shall see, (i) and (ii) do and (iii) and (iv) do not:

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<sup>7</sup>The fact that a truthful equilibrium is coalition-proof among the  $m$  principals does not imply that it is Pareto-efficient among the  $m$  principals if there are more than two principals. Indeed, Konishi et al. (1999) provide a simple three-principal example of common agency game in which there exists a non coalition-proof equilibrium which gives each principal a strictly higher net payoff than every coalition-proof equilibrium of the same game.

**Theorem 2.** For any common agency game,

- (i) For any  $j \in M$ , given  $\{t^i\}_{i \neq j}$ , the set of best responses of principal  $j$  contains a natural contribution schedule.
- (ii) There exists a natural equilibrium.
- (iii) A natural equilibrium need not be coalition-proof.
- (iv) In a natural equilibrium, the agent may choose an inefficient alternative.

The proof (in the Appendix) constructs a particular type of natural equilibrium, which exists in every common agency game. We may call this equilibrium the *maximum-conflict equilibrium*. Principals split in two sides. One side supports one alternative, the other side another. Each principal contributes only to the alternative supported by her side. The two candidate alternatives have the property that they maximize the sum of the payoffs that each principal gets from the alternative they prefer between the two:

$$(s^*, \bar{s}) = \arg \max_{s' \in S, s'' \in S} \sum_{j \in M} \max\{G_{s'}^j, G_{s''}^j\} \tag{1}$$

This maximization problem has two equivalent formulations, which shed light on the nature of the equilibrium (the equivalences are shown in the proof of Theorem 2). First, suppose we split principals in two subsets and let each subset choose the alternative that maximizes its surplus. The value of the problem of finding the split that maximizes the sum of surpluses of the two subsets is the same as the value of the problem above:

$$\sum_{j \in M} \max \left\{ G_{s^*}^j, G_{\bar{s}}^j = \max_{\substack{M' \cup M'' = M \\ M' \cap M'' = \emptyset}} \right\} \left( \max_{s \in S} \sum_{j \in M'} G_s^j + \max_{s \in S} \sum_{j \in M''} G_s^j \right) \tag{2}$$

Second, the candidate alternatives  $(s^*, \bar{s})$  satisfy:

$$\sum_{j \in M} \max\{0, G_{s^*}^j - G_{\bar{s}}^j\} = \max_s \sum_{j \in M} \max\{0, G_s^j - G_{\bar{s}}^j\} \tag{3}$$

$$\sum_{j \in M} \max\{0, G_{\bar{s}}^j - G_{s^*}^j\} = \max_s \sum_{j \in M} \max\{0, G_s^j - G_{s^*}^j\} \tag{4}$$

That is, given one of the two candidate alternatives (say  $s^*$ ), the other candidate alternative ( $\bar{s}$ ) is the alternative toward which a deviation would be most beneficial.

Now, it is easy to find equilibrium transfers. Assume without loss of generality that  $\sum_{j \in M} G_{s^*}^j \geq \sum_{j \in M} G_{\bar{s}}^j$ . Let  $M^*$  be the set of principals that prefer  $s^*$  to  $\bar{s}$  and  $\bar{M}$  the ones who prefer  $\bar{s}$  to  $s^*$ . If transfers satisfy the following conditions then we have a maximum-conflict equilibrium:

- (a) Principals in  $\bar{M}$  (the losing side) set  $t_{\bar{s}}^j \geq G_{\bar{s}}^j - G_{s^*}^j$ , that is they offer on their alternative at least as much as the additional payoff if the agent chose it instead of the equilibrium alternative.
- (b) Principals in  $M^*$  (the winning side) set  $t_{s^*}^j$  low enough that none of them would like to avoid paying  $t_{s^*}^j$  by deviating to the alternative preferred by the losing side.
- (c) The total transfers from the winning side equal the total transfers from losers:  $\sum_{i \in M^*} \hat{t}_{s^*}^i = \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i$ .
- (d) All other transfers are zero.

Points (b) and (c) are compatible with (a) because the winning side has more to gain from  $s^*$  than the losing side has to gain from  $\bar{s}$ . Moreover, (d) is an equilibrium strategy because the maximum-conflict conditions (3) and (4) guarantee that enough money is put on  $s^*$  and  $\bar{s}$  to guard against deviations to other alternatives. Hence — as is proven formally in the Appendix — it is always possible to find transfers that satisfy (a) through (d), and there always exists a maximum-conflict equilibrium.

However, a natural equilibrium need not be coalition-proof. There could be an alternative  $s'$  different from the two candidate alternatives that would be a good compromise in the sense that  $\sum_{j \in M} G_{s'}^j \geq \sum_{j \in M} G_{s^*}^j \geq \sum_{j \in M} G_{\bar{s}}^j$ . In order to get it, the two sides should agree to lower their contributions on  $s^*$  and  $\bar{s}$  and offer something on  $s'$ . As such agreement would be self-enforcing, the natural equilibrium need not be coalition-proof.

This brings us to point (iv). The compromise alternative may be efficient and yet the natural equilibrium does not select it, as illustrated by the following examples.

#### 2.4. Examples

In the two examples we are going to discuss there are two principals (*A* and *B*) and three alternatives (*I*, *II*, *III*). Here are the payoffs and the equilibrium transfers (for truthful and natural equilibria):

Game 1 the efficient $s$ is <i>I</i>				Game 2 the efficient $s$ is <i>II</i>					
		<i>I</i>	<i>II</i>	<i>III</i>		<i>I</i>	<i>II</i>	<i>III</i>	
Payoffs	$G^A$	17	10	0	Payoffs	$G^A$	17	11	0
	$G^B$	0	6	12		$G^B$	0	7	12
Truthful eq. $s^* = I$	$t^A$	12	5	0	Truthful eq. $s^* = II$	$t^A$	11	5	0
	$t^B$	0	6	12		$t^B$	0	6	11
Natural eq. $s^* = I$	$t^A$	12	0	0	Natural eq. $s^* = I$	$t^A$	12	0	0
	$t^B$	0	0	12		$t^B$	0	0	12

In both examples, *I* is Principal *A*'s preferred alternative, *III* is Principal *B*'s preferred alternative, and *II* is a 'compromise' alternative. The difference between Game 1 and Game 2 is only in the payoffs for alternative *II*, which are slightly higher in Game 2. In Game 1, the efficient alternative is *I* (because  $G_I^A + G_I^B$  is higher than both  $G_{II}^A + G_{II}^B$  and  $G_{III}^A + G_{III}^B$ ). Instead, in Game *II* the efficient alternative is *II*.

By Theorem 1 part (iv), the truthful equilibrium selects the efficient alternative. Indeed, the equilibrium alternative of the truthful equilibrium is *I* in Game 1 and *II* in Game 2 (the details on how to compute a truthful equilibrium can be found in Bernheim and Whinston (1986)).

In Game 1, the difference between the natural and the truthful equilibrium is not payoff-relevant. Instead, in Game 2 it is. The net payoff of *A* is 6 in the truthful and 5 in the natural. The net payoff of *B* is 1 in the truthful and 0 in the natural. The natural equilibrium is not coalition-proof. Both principals would be better off if they both made a positive offer on the compromise alternative (and this would be a self-enforcing deviation). Indeed, in this example, both principals strictly benefit from moving from the natural to the truthful equilibrium.

2.5. Remarks and extensions

Neither the natural nor the truthful equilibrium are in general unique. However, for any common agency game the alternative selected by a truthful equilibrium is unique, except in the nongeneric case in which  $\arg \max_{s \in S} \sum_{j \in M} G_s^j$  has more than one element. The same cannot be said of the natural equilibrium. For instance, it is easy to see that in the following game there are natural equilibria with outcome *I* and *III* (according to whether *C* becomes active on *III*):

	<i>I</i>	<i>II</i>	<i>III</i>
$G^A$	7	0	0
$G^B$	0	6	4
$G^C$	0	0	4

However, uniqueness is achieved in the maximum-conflict equilibrium, except in the nongeneric case in which the maximization problem in (1) has multiple solutions. In the example above, the maximum-conflict equilibrium is unique and supports *III* (as does the truthful equilibrium).

Given this uniqueness, it is easier to identify properties of maximum-conflict equilibria than of the larger set of natural equilibria. The rest of this section focuses mostly on the comparison between maximum-conflict equilibria and truthful equilibria. However, references are made to natural equilibria whenever something is known.

We have seen that the allocation (alternative chosen and transfers paid) of a truthful equilibrium maximizes a Utilitarian objective while that of a maximum-

conflict equilibrium need not. One might also wonder whether it is possible that a truthful equilibrium Pareto-dominates a maximum-conflict equilibrium. This question can be asked with respect to gross payoffs or net payoffs.

In the gross sense, the alternative chosen in a maximum-conflict equilibrium is undominated, in that there exists no alternative  $s \neq s^*$  such that  $G_s^j \geq G_{s^*}^j$  for all  $j \in M$  with at least one strict inequality. This can be seen from (2) in which  $s^*$  maximizes the surplus of the subset  $M^*$  and hence it cannot be the case that  $s^*$  is dominated by some other alternative for all principals in  $M^*$ . While the maximum-conflict equilibrium is undominated, one may construct other natural equilibria that select dominated alternatives.

In the net sense, the question is whether there is a truthful equilibrium and a maximum-conflict equilibrium such that the former gives a higher net payoff than the latter to each principal. A general answer is hard to give because of the nonuniqueness issue discussed above. However, if there are only two principals, we can exclude domination by showing that the agent always benefits from maximum conflict (the proof is in the Appendix):

**Proposition 1.** *If  $m = 2$ , the payoff of the agent (of either principal) is greater or equal (smaller or equal) in any maximum-conflict equilibrium than in any truthful equilibrium.*<sup>8</sup>

The core of the analysis of this paper extends to the continuous case. Theorem 2 holds as stated also when  $S$  is an infinite set, provided that some regularity conditions are satisfied.<sup>9</sup>

In addition to continuity, many economic applications, such as Grossman and Helpman (1994), also assume differentiability and concavity. In this setup, the alternative supported in a truthful equilibrium is conveniently found through first-order conditions. Instead, the alternative supported in a maximum-conflict equilibrium cannot be found by first-order conditions alone because (1) requires identifying the subsets  $M^*$  and  $\bar{M}$ , which is a combinatorial problem.

To see this point, suppose that  $s \in \mathfrak{R}$  and the payoff function of principal  $j \in M$  is  $G_s^j = -(s - \hat{s}_j)^2$ , with  $\hat{s}_1 \leq \dots \leq \hat{s}_m$ . The truthful alternative is found through the first-order condition:

$$\hat{s} = \frac{\sum_{j \in M} \hat{s}_j}{m}$$

<sup>8</sup>One might also use a definition of Pareto efficiency that excludes the payoff of the agent. See, however, footnote 7.

<sup>9</sup>With an infinite  $S$  and provided that, for all  $J \in M$ ,  $\max_s \sum_{j \in J} G_s^j$  has a solution, the proof of part (ii) of Theorem goes through as it is, because the only operations that are performed across alternatives are maximizations. The proof of part (i) requires the usual conditions to ensure that the best-response set is nonempty.

Instead, from (2), finding the alternative supported by the maximum-conflict equilibrium requires solving:

$$\min_{k \subseteq M} \left( \sum_{j=1}^k (s^* - \hat{s}_j)^2 + \sum_{j=k+1}^m (\bar{s} - \hat{s}_j)^2 \right)$$

$$\text{subject to } s^* = \frac{\sum_{j=1}^k \hat{s}_j}{k} \quad \text{and} \quad \bar{s} = \frac{\sum_{j=k+1}^m \hat{s}_j}{m - k}$$

The maximum-conflict equilibrium takes the form of a left-right split of principals. There will be a principal  $k$  such that all principals to the left of  $k$  support the leftist alternative, while all the other principals support the rightist alternative. Given  $k$ , the two alternatives are found through first-order conditions. However, finding  $k$  is a discrete problem, and involves comparing  $m$  values.

### 3. Experimental design

To see which equilibrium concept is a better predictor of game behavior, we carry out an experiment. The common agency game we use is Game 2 discussed in the previous section. The payoff matrix is rewritten here for the reader’s convenience. Payoffs are in Dutch guilders (1 US dollar is approximately worth two guilders) (Table 1).

In the first stage of the game, both principals choose simultaneously a contribution schedule. All contributions must be nonnegative. To exclude the possibility of losses, the contribution to an alternative cannot be above the gross payoffs the principal received for that alternative.<sup>10</sup>

In the second stage of the game, since the agent has no intrinsic interest in the alternatives, he should choose the alternative with the highest sum of contributions. Truthful equilibria and natural equilibria differ in the contribution schedules of the principals, not in the behavior of the agent. Principals’ strategy choices are the focus of interest of our experiments, not agent’s behavior. Therefore, we substitute the agent by a rule stating that the winning alternative (i.e. the alternative with the highest sum of contributions) is chosen automatically.

Table 1

	<i>I</i>	<i>II</i>	<i>III</i>
A	17	11	0
B	0	7	12

<sup>10</sup>Since losses are difficult to enforce it is common in experimental economics to restrict the strategy set such that losses are excluded. This only removes some dominated strategies that are not part of neither the truthful nor the natural equilibrium.

As payments of infinitesimal amounts are impossible in practice, we use a discretized version of common agency. The minimum increment is set at 0.05 (5 cents is the smallest Dutch coin).<sup>11</sup> Another practical problem is the tie-breaking rule. In the theory part, the rule is endogenous. This is not possible here and we assume that if the agent is indifferent between two or three alternatives, then he randomizes between them with equal probabilities. In the experiment, ties are broken by rolling a die.

These changes in the game (substitution of the agent by a rule, probabilistic tie-breaking rule, finite strategy sets) lead to inessential changes in the equilibrium predictions. Specifically, the truthful and the natural equilibrium contribution schedules are given by (the Appendix contains a formal check that these two are Nash-equilibria):<sup>12</sup>

The equilibrium net payoffs are 5 for *A* and 0 for *B* (for the natural equilibrium), and 6 for *A* and 1 for *B* (for the truthful equilibrium). Thus, the natural equilibrium is not coalition-proof. In fact, the sum of net payoffs for both principals is 40% higher in the truthful equilibrium than in the natural equilibrium.

The experiments are conducted in a classroom, in two sessions with 16 subjects each. Each subject plays the game six times, three times in the role of principal *A* and three times in the role of *B*.<sup>13</sup> Each subject knows beforehand whether she is principal *A* or *B* in a certain round. Since it is common knowledge that each pair consists of one principal *A* and one principal *B*, everyone also knows whether her partner is principal *A* or *B*. However, nobody knows the identity of her partner.

At the beginning of each round the principals choose simultaneously their contribution schedule.<sup>14</sup> Then the experimenter calculates for each pair of

Table 2

	<i>I</i>	<i>II</i>	<i>III</i>
(a) The natural equilibrium contribution schedules			
A	12	0	0
B	0	0	11.95
(b) The truthful equilibrium contribution schedules			
A	10.95	5	0
B	0	6	10.95

<sup>11</sup>Simon and Zame (1990) consider a class of infinite games which comprise common agency games and show that in this class the limit of the equilibrium of a discretized game as the discretization becomes finer is an equilibrium of the continuous game.

<sup>12</sup>There could be other natural or truthful equilibria but they differ from the ones in Table 2 by at most 5 cents. The equilibria in Table 2 are the ones with lowest contributions.

<sup>13</sup>This guarantees that looking at the whole experiment all subjects are in a similar position. By that, the impact of distributional concerns (fairness, envy, altruism), which very often shape experimental results, is minimized.

<sup>14</sup>For the details of the experimental procedure, see Appendix B.

principals which alternative is chosen (with ties being broken with a fair coin). Each subject is then informed about the alternative chosen as well as the strategy of the other principal. Players are not informed about the alternatives chosen and the contributions offered in the other matches. After the last round, the net payoffs a subject made in all rounds are summed up and paid to her in cash.

The subjects are matched so that nobody plays twice with the same partner. This is common knowledge. Furthermore, we use a matching protocol that maximizes the number of independent observations in the later rounds under the constraint that nobody is matched twice with the same partner. In the first round the 16 subjects form eight pairs in each session. At the beginning of the second round, two first round pairs are merged to form a group consisting of four subjects. Since this grouping remains the same in rounds 2 and 3 we refer to these groups as ' $r_{2/3}$ -groups'. In rounds 2 and 3 each subject is matched with those members of her  $r_{2/3}$ -group with whom he has not been matched in the first round. This matching protocol guarantees that every member of a  $r_{2/3}$ -group does not experience any (previous or contemporary) decision of a non-member — any influence from a decision of a non-member on the behavior of a member can be excluded. Therefore, the decisions made within a  $r_{2/3}$ -group forms an independent observation<sup>15</sup>.

At the beginning of the fourth round, two  $r_{2/3}$ -groups are merged into one group of eight participants. Since this grouping remains the same in rounds 4, 5, and 6 we refer to these groups as ' $r_{4-6}$ -groups'. In rounds 4 to 6, each subject is matched with three of those members of her  $r_{4-6}$ -group with whom she has not been matched in a previous round. This matching procedure guarantees that the decisions of every member of a  $r_{4-6}$ -group are not influenced by any (previous or contemporary) decision of a non-member — the decisions made within a  $r_{4-6}$ -group form an independent observation. On the whole we conduct two sessions. Therefore, we observe 16 first round pairs, eight  $r_{2/3}$ -groups, and four  $r_{4-6}$ -groups.

The experiments took place at the Center for Economic Research, Tilburg University, The Netherlands. The participants were students of different fields, mainly of business administration and law. None of them was a student of ours and none had knowledge of game theory or common agency theory.

A session lasted about 25 minutes net of going through the instructions. The average earnings of a participant was 15.12 Hfl, which was about 8.13 US\$ at the time the experiments were conducted (October 1998). Principal A earned on average 4.26 Hfl per round, whereas B earned on average 0.78 Hfl per round.

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<sup>15</sup>Cooper et al. (1995) introduced, and Kamecke (1997) analyzed, a different matching protocol that preserves the best-reply structure of a one-shot game while maximizing the number of rounds. However, as also Kamecke explains, this does not imply that other, nonstrategic influences between the players (such as learning) are excluded. Hence, such a protocol does not maximize the number of independent observations, and it is, therefore, not helpful to increase the significance of statistical tests.

## 4. Results

The predictions of truthful and natural equilibria differ in two aspects: the alternative chosen and the contribution schedules used.

### 4.1. Chosen alternative

On the whole, we observe 96 choices of alternatives. In four matches, a tie between two alternatives occurs, which is broken by using a die. In what follows, a tie between two alternatives is counted as 0.5 for each alternative. In total, we observe 62.5 cases where *I* is the winning alternative (65% of all cases), 3.5 cases with *II* winning (3.6% of all cases), and 30 cases with *III* winning (31.4% of all cases). This already indicates:

#### Result 1.

- (a) Alternative *II* is hardly ever chosen.
- (b) Alternative *I* is chosen in most of the cases.

To establish this result, one can use a binomial test.<sup>16</sup> The hypothesis that the winning probability of *II* is larger than or equal to 10% is rejected at a 5% level. The hypothesis that the winning probability of *I* is 50% or less is rejected even at a 1% level.

Result 1 can also be inferred from Fig. 1 which depicts the evolution of the relative frequencies of the chosen alternatives during the course of the experiments.

In all rounds the frequency of alternative *II* is less than 10%, and in three rounds

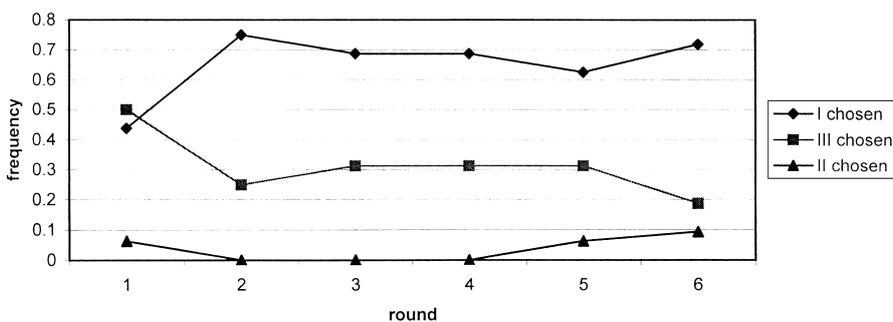


Fig. 1. The relative frequency of chosen alternatives.

<sup>16</sup>See, Siegel and Castellan (1988) for a description of the binomial test.

we do not observe any case of *II* winning. In all rounds, alternative *I* as well as alternative *III* occurs more often than *II*. In the last two rounds, however, the gap between *II* and *III* narrows. On the other hand, alternative *I* wins in more than half of the cases in all rounds except round 1, and there is no tendency of the frequency of *I* to decline.

Due to spillovers between partners and due to change of partner from round to round, the individual observations are of course not independent. Tests based on individual observations like the binomial tests used above might be not appropriate. Hence, we also analyze the frequencies by which the eight  $r_{2/3}$ - and the four  $r_{4-6}$ -groups choose the different alternatives. In each of the 12 groups alternative *I* is chosen more often than *II*. Therefore, *II* is clearly dominated by *I*.<sup>17</sup> Furthermore, the frequency of *III* is always lower than that of *I*, except for two  $r_{2/3}$ -the groups where both alternatives are chosen equally often.<sup>18</sup> Therefore, we have to conclude that the ‘natural’ alternative *I* was ‘dominating’, and the ‘truthful’ alternative *II* hardly ever won.

#### 4.2. Contribution schedules

We now turn to the contribution schedules chosen by the principals. The average contributions over the six rounds are displayed in Table 3.

This implies that actual contributions for all alternatives are lower than the contributions of the truthful equilibrium. Compared with the natural equilibrium strategy, *A*’s (*B*’s) contributions to *I* (*III*) are too low, whereas their contributions to *II* are too high (compare Table 3 with Table 2a and b). If we look at the development of the average contributions of the rounds, we find that *A*’s contribution to *I* increases, whereas there is no clear trend for *A*’s contributions to *II*. *B* tends to increase her contributions to *II* as well as to *III* (see Fig. 2).

This already suggests that we hardly observe equilibrium play in the experiment. To see this point more clearly, let us say that a contribution schedule is a

Table 3  
Average contribution schedules

	<i>I</i>	<i>II</i>	<i>III</i>
Principal <i>A</i>	9.96	2.92	0
Principal <i>B</i>	0	3.53	9.08
Sum of contributions	9.96	6.45	9.08

<sup>17</sup>Using a Wilcoxon signed-ranks test for the hypothesis that the frequencies of *I* and *II* are equal, we have to reject this hypothesis at 1% level for the  $r_{2/3}$ -groups. For the  $r_{4-6}$  groups the corresponding *P* value is 6.25%, which is the lowest value attainable with four observations.

<sup>18</sup>Applying the Wilcoxon signed-ranks test for the  $r_{2/3}$ -groups the difference in frequencies turns out to be significant at a 5% level. For the  $r_{4-6}$ -groups the corresponding *P* value is 6.25%, which is the lowest possible value with four observations.

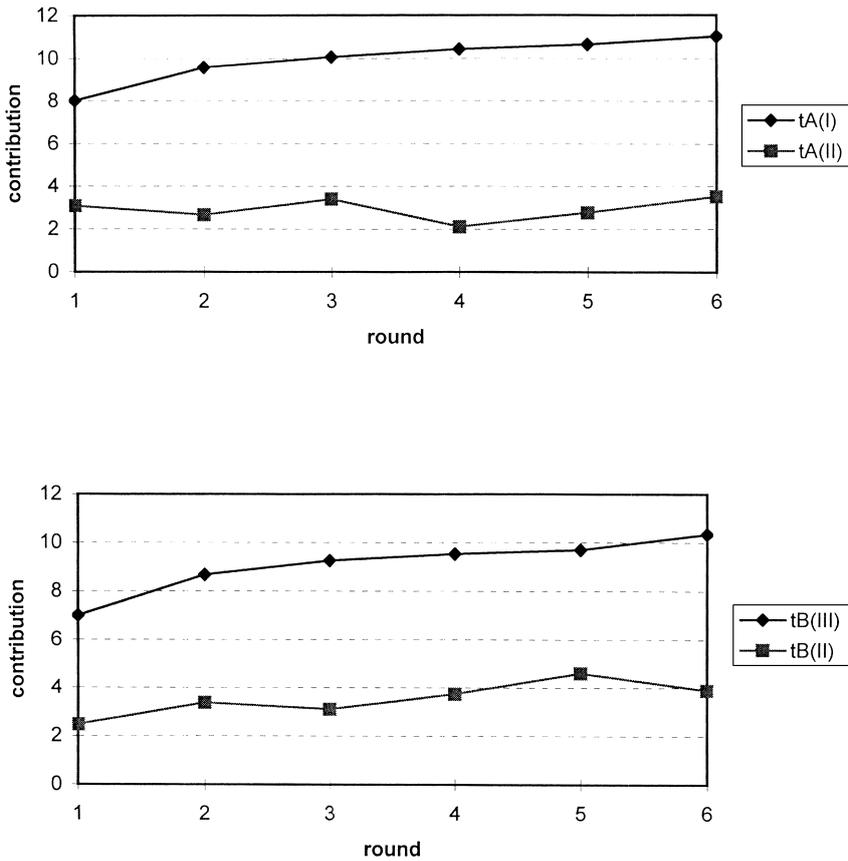


Fig. 2. The average contribution schedule of player A and player B.

*near equilibrium schedule* if the contribution to any alternative differs no more than 5 cents from the equilibrium strategy of either the truthful or the natural equilibrium. Given this definition, we find 10 cases (out of 96) where principal A chooses a near equilibrium schedule of the natural equilibrium. We never observe that A's strategy is a near equilibrium schedule of the truthful equilibrium. Furthermore, B's schedule is never a near equilibrium schedule of either the natural or the truthful equilibrium.<sup>19</sup>

The next question is whether the contribution schedules, despite being different from equilibrium schedules, are still natural or truthful. For instance, the schedule

<sup>19</sup>There are other equilibria besides the truthful and the natural (who are basically modifications of these two types with higher contributions). However, strategies belonging to these other equilibria are very rarely observed either.

$(t_I^A = 10, t_{II}^A = 4, t_{III}^A = 0)$  is not the equilibrium truthful contribution schedule but is still a truthful contribution schedule. Similarly,  $(t_I^A = 10, t_{II}^A = 0, t_{III}^A = 0)$  is a non-equilibrium natural schedule. In the case of truthful equilibrium, given the 5-cent discretization, we call truthful also schedules that differ by no more than 5 cents from a truthful schedule, like  $(t_I^A = 10.05, t_{II}^A = 4, t_{III}^A = 0)$ . Table 4 shows how often natural and truthful schedules are chosen.

Principal *A* plays natural more often than truthful, and this pattern is stronger in the last three periods. *B* chooses both types of strategies quite rarely, and there seems to be no change over time.

In the majority of cases, principals choose strategies that are neither truthful nor natural. However, one might ask whether the chosen schedules display some of the characteristics of either truthful or natural strategies. Let us start with truthful. A truthful schedule is characterized by the feature that (but for the nonnegativity constraint) the principal is indifferent with regard to the chosen alternative. This implies for the game at hand that *A*'s schedule should make him indifferent between *I* and *II*, whereas *B*'s schedule should make him indifferent between *III* and *II*. Hence, the difference between *A*'s contribution to *I* and *II* should be 6, whereas *B*'s contribution to *III* and *II* should differ by 5. We use the individual schedules to run a *t*-test for the hypothesis that  $t_I^A - t_{II}^A = 6$  ( $t_{III}^B - t_{II}^B = 5$ ). This hypothesis has to be rejected at the 1% (5%) level in favor of the counter-hypothesis that the difference is larger than 6 (5).<sup>20</sup> This leads to:

**Result 2.** *Player A's contribution schedules are not designed to make her indifferent between alternative I and alternative II.*

**Result 3.** *Player B's contribution schedules are not designed to make her indifferent between alternative III and alternative II.*

Do the observed schedules exhibit the main feature of natural strategies, namely that the principals focus on one alternative and bid aggressively on it? To answer this question, notice first that, as long as the difference between *A*'s contribution to

Table 4

Number of cases of nearly natural and nearly truthful contribution schedules chosen by principal *A* and *B* in all and last three rounds (percentages in parentheses)

	A: all rounds	A: last three rounds	B: all rounds	B: last three rounds
Natural	24 (25%)	17 (35%)	13 (14%)	6 (13%)
Truthful	16 (17%)	6 (13%)	17 (18%)	9 (19%)

<sup>20</sup>Since the individual observations are not statistically independent, we also looked at the average schedules of the eight  $r_{2/3}$ -groups and the four  $r_{4-6}$ -groups. We found in six of the  $r_{2/3}$ -groups and in all  $r_{4-6}$ -groups that  $t_I^A - t_{II}^A > 6$  and  $t_{III}^B - t_{II}^B > 5$ .

*I* and to *II* is larger than or equal to 7, *II* could not win *irrespective* of *B*'s contributions. Indeed the average difference between *A*'s contribution to *I* and *II* is slightly above 7. If we look at the individual strategies, we observe 38 (out of 96) cases where the difference is larger than or equal to 7 (see Table 5). In the last three rounds the difference is not below 7 in 26 (out of 48) cases.

Therefore, we observe that especially in the last rounds most of *A*'s strategies exclude *II* from winning for sure.

Up to now the discussion concentrated on whether *A*'s strategy excludes the choice of *II*, under the assumption that *B* chooses the most extreme contribution to *II*, namely 7. However, *B*'s actual contributions are never that high (no case of  $t_{II}^B = 7$  is observed), and we can plausibly assume that *A*'s expectations of how *B* is playing are influenced by this experience. Hence, we assume that *A* expects *B*'s contribution to *II* to be at the highest level *A* previously experienced. Then we calculate whether for these expectations *A*'s contribution schedule excludes *II* from winning. Notice that this approach is rather unfavorable to *I*, because it specifies the expectations such that *II* is most likely to defeat *I*. Nonetheless, *A*'s actual contribution schedules jointly with these expectations about *B*'s contributions imply that on average the sum of contributions for *I* exceeds the sum of contributions for *II* by 4.135. Furthermore, we find that in 60 individual cases (out of 64 cases) *A*'s schedule is such that alternative *I* would defeat *II*. In two cases the sum of contributions would be equal and only in two cases *II* would defeat *I*.<sup>21</sup>

Let us now turn the attention to player *B*, who excludes the choice of *II* whenever the difference between the contributions to *III* and *II* is not less than 11. This happens 19 times, 15 times in the last three rounds (see Table 5). Hence, we observe quite some cases that *B* contributes so aggressively to *III* that *II* is excluded. This tendency, however, is less pronounced for *B* than for *A*. If we take the highest contribution of *A* to *II* as *B*'s expectation about *A*'s behavior, the average expected sum of contributions for *III* exceeds that for *II* by 2.16. Furthermore, *B*'s schedule is in 53 (out of 64) cases designed such that *III* would defeat *II*, and only in seven cases *II* would defeat *III*; in four cases a tie would occur.

All this evidence can be summarized by

Table 5  
Number of aggressive bids of *A* ( $t_I^A - t_{II}^A > 7$ ) and *B* ( $t_{III}^B - t_{II}^B > 11$ ) (percentages in parentheses)

	$t_I^A - t_{II}^A > 7$	$t_{III}^B - t_{II}^B > 11$
All rounds	38 (40%)	19 (20%)
Last three rounds	26 (54%)	15 (31%)

<sup>21</sup>In the first two rounds, no subject has played the role of *A* previously. Hence, no subject has previous experience about *B*'s contributions before the third round.

**Result 4.** *Player A's contribution schedules are designed to get alternative I and to exclude alternative II.*

**Result 5.** *Player B's contribution schedules are designed to get alternative III and to exclude alternative II.*

The strategies employed by the players resemble neither truthful nor natural strategies. However, Results 2 and 3 show that players do not choose schedules which made them — as required by the concept of truthful contributions — indifferent between the alternatives. Results 4 and 5 indicate that players *A* as well as players *B* rather want to enforce their most preferred alternative which is in line with the spirit of the concept of natural contributions. Since players *A* are in the better position, they succeed to do so most of the time. Therefore, the natural alternative is chosen in most cases. If not, player *B*'s most preferred alternative is chosen, whereas the truthful (and efficient) alternative is hardly ever observed.

## 5. Conclusions

We have introduced a new class of equilibria for common agency games — natural equilibria — and we have compared it with the class that is commonly used in the literature — truthful equilibria. In the experimental evidence that we gather, the alternative chosen is almost never the one predicted by the truthful equilibrium.

We see two possible explanations of the fact that people do not seem to play truthful. First, the truthful equilibrium may be quite complex. The complexity is not in terms of strategy implementation (how much space it takes to write down the optimal strategy). Rather, it is difficult for players to arrive at the truthful strategy equilibrium.<sup>22</sup> Unfortunately, we do not know of any standard way of evaluating this type of complexity.<sup>23</sup>

A second explanation has to do with risk dominance as discussed by Harsanyi and Selten (1988). Intuitively, if a principal is not sure what the other principals are doing, playing truthful may be riskier than playing natural. Suppose for instance that in our game principal *A* does not know whether *B* will play truthful or natural. If *B* is playing natural and *A* plays truthful, *A* gets a payoff of zero — a

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<sup>22</sup>See, Rubinstein (1998) for a discussion on possible notions of bounded rationality and complexity.

<sup>23</sup>An earlier version of this paper analyzes the computational complexity (as defined in computer science) of finding a truthful/natural equilibrium. For the truthful case, the complexity grows exponentially with the number of principals. For the natural case, it grows only polynomially. Hence, as the number of principals increases, the time needed to find a truthful equilibrium becomes infinitely longer than the time needed to find a natural equilibrium.

rather negative outcome. Instead, if  $B$  plays truthful and  $A$  plays natural,  $A$  only overbids one unit. To make this argument in a formal way, we should use the tracing procedure proposed by Harsanyi and Selten, which unfortunately is not defined for games with continuous strategies like ours.

The two explanations may reinforce each other. If playing the truthful equilibrium is so complex that some principals are not able to do it, then even those principals who would know how to play truthful might decide to play natural out of risk-dominance considerations.

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**Appendix A. Proof of Theorem 2 and Proposition 1**

*Part (i)*

Given  $\{t^i\}_{i \neq j}$ , let  $\tilde{t}^j$  denote a best response contribution schedule for  $j$  (it is obvious that the best-response set is nonempty). Let  $\hat{s}$  be the alternative chosen by the agent. Consider the contribution schedule  $\hat{t}^j$  such that  $\hat{t}_s^j = \tilde{t}_s^j$  if  $s = \hat{s}$  and  $\hat{t}_s^j = 0$  otherwise. As  $\hat{t}^j$  leaves  $j$ 's net payoff unchanged, it belongs to the set of best responses of  $j$  given  $\{t^i\}_{i \neq j}$ .

*Part (ii)*

We prove that there exists a maximum-conflict equilibrium. As a preliminary step, we show (2) and (3)–(4). Let  $i^j(J)$  be an indicator function that takes value 1 if  $j \in J \subseteq M$  and 0 if  $j \notin J$ . Then,

$$\begin{aligned} \max_{s' \in S, s'' \in S} \sum_{j \in M} \max\{G_{s'}^j, G_{s''}^j\} &= \max_{s' \in S, s'' \in S} \max_{J \subseteq M} \sum_{j \in M} (i^j(J)G_{s'}^j + (1 - i^j(J))G_{s''}^j) \\ &= \max_{J \subseteq M} \max_{s' \in S, s'' \in S} \sum_{j \in M} (i^j(J)G_{s'}^j + (1 - i^j(J))G_{s''}^j) \\ &= \max_{\substack{M' \cup M'' = M \\ M' \cap M'' = \emptyset}} \left( \max_{s \in S} \sum_{j \in M'} G_s^j + \max_{s \in S} \sum_{j \in M''} G_s^j \right). \end{aligned}$$

To show (3), note that

$$\sum_{j \in M} \max\{G_{s^*}^j, G_{\bar{s}}^j\} = \max_{s' \in S, s'' \in S} \sum_{j \in M} \max\{G_{s'}^j, G_{s''}^j\} = \max_{s' \in S} \sum_{j \in M} \max\{G_{s'}^j, G_{\bar{s}}^j\}$$

Then,

$$\begin{aligned} \sum_{j \in M} \max\{0, G_{s^*}^j - G_{\bar{s}}^j\} &= \sum_{j \in M} \max\{G_{s^*}^j, G_{\bar{s}}^j\} - G_{\bar{s}}^j \\ &= \max_{s' \in S} \sum_{j \in M} \max\{G_{s'}^j, G_{\bar{s}}^j\} - G_{\bar{s}}^j = \max_{s' \in S} \sum_{j \in M} \max\{0, G_{s'}^j - G_{\bar{s}}^j\} \end{aligned}$$

The other equivalence, (4) is shown in an analogous manner.

Now, to construct a maximum-conflict equilibrium, take  $s^*$ ,  $\bar{s}$ ,  $M^*$ , and  $\bar{M}$  as defined in the text, and suppose principals offer nonnegative contribution schedules  $\{\hat{t}^j\}_{j \in M}$  that satisfy the following four conditions:

(a) If  $j \in \bar{M}$ ,

$$\hat{t}_{\bar{s}}^j \geq G_{\bar{s}}^j - G_{s^*}^j \tag{5}$$

(b) If  $j \in M^*$ ,

$$\hat{t}_{s^*}^j \leq \min\left\{G_{s^*}^j - G_{\bar{s}}^j, G_{s^*}^j - \max_s G_s^j + \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i\right\} \tag{6}$$

(c)

$$\sum_{i \in M^*} \hat{t}_{s^*}^i = \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i \tag{7}$$

(d) All other transfers are zero.

We want to show that there exists a matrix  $\hat{t}$  that satisfies (a), (b), (c), and (d) and that such a matrix is an equilibrium of the common agency game.

To prove existence, construct a  $\hat{t}$  such that, if  $j \in M^*$ ,

$$\hat{t}_{s^*}^j = G_{s^*}^j - G_{\bar{s}}^j; \tag{8}$$

Because, by definition,  $\sum_{j \in M} G_{s^*}^j \geq \sum_{j \in M} G_{\bar{s}}^j$ ,

$$\sum_{j \in M^*} (G_{s^*}^j - G_{\bar{s}}^j) \geq \sum_{j \in M} (G_{\bar{s}}^j - G_{s^*}^j)$$

and it is always possible to set  $\{\hat{t}_{\bar{s}}^i\}_{i \in \bar{M}}$  such that both (a) and (c) are satisfied. Satisfying (d) is trivial. Hence, we can always find a  $\hat{t}$  that satisfies (8) (a), (c) and (d). To show that this  $\hat{t}$  also satisfies (b), notice that, given (8), (b) rewrites as:

$$\sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i \geq \max_s G_s^j - G_{\bar{s}}^j$$

and, given (c), it rewrites as:

$$\sum_{i \in M^*} (G_{s^*}^i - G_{\bar{s}}^i) \geq \max_s G_s^j - G_{\bar{s}}^j \tag{9}$$

Let  $s^j \in \arg \max_s G_s^j$ . Then:

$$\begin{aligned} \sum_{i \in M^*} (G_{s^*}^i - G_{\bar{s}}^i) &= \sum_{i \in M} \max\{0, G_{s^*}^i - G_{\bar{s}}^i\} \\ &= \max_s \sum_{i \in M} \max\{0, G_s^i - G_{\bar{s}}^i\} \geq \sum_{i \in M} \max\{0, G_{s^j}^i - G_{\bar{s}}^i\} \geq G_{s^j}^j - G_{\bar{s}}^j \end{aligned}$$

where the first equality is the definition of  $M^*$  and the second equality is (3). This proves (9). Hence, we have shown that a  $\hat{t}$  satisfying (a), (b), (c), and (d) exists.<sup>24</sup>

Now we want to show that, if  $\hat{t}$  satisfies conditions (a) through (d), then  $(s^*, \hat{t})$  is an equilibrium of the common agency game because neither the agent nor any of the principals have a profitable deviation. For the agent, (c) and (d) guarantee he (weakly) prefers  $s^*$  to any other  $s$ . Hence, the agent has no incentive to deviate from  $s^*$ . For principals, suppose that principal  $j$  deviates from  $\hat{t}^j$  and plays  $\tilde{t}^j$  instead. There are six possibilities:

- (i)  $j \in M^*$  and the agent still chooses  $s^*$ . The deviator cannot benefit because, in order for the agent to still choose  $s^*$ , it must be the case that  $\tilde{t}_{s^*}^j \geq \hat{t}_{s^*}^j$ .
- (ii)  $j \in M^*$  and the agent chooses  $\bar{s}$ . By (b),  $\hat{t}_{s^*}^j \leq G_{s^*}^j - G_{\bar{s}}^j$  and hence  $G_{\bar{s}}^j - \tilde{t}_{\bar{s}}^j \leq G_{s^*}^j - \hat{t}_{s^*}^j$ .
- (iii)  $j \in M^*$  and the agent chooses  $s'$ , different from both  $s^*$  and  $\bar{s}$ . In order for the agent to choose  $s'$ ,  $j$  must offer more than the amount the agent gets for  $\bar{s}$ . But (b) guarantees that

$$\hat{t}_{s^*}^j \leq G_{s^*}^j - \max_s G_s^j + \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i \leq G_{s^*}^j - G_{s'}^j + \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i$$

and hence  $G_{s'}^j - \tilde{t}_{s'}^j \leq G_{s^*}^j - \hat{t}_{s^*}^j$ .

- (iv)  $j \in \bar{M}$  and the agent still chooses  $s^*$ . Clearly not profitable.
- (v)  $j \in \bar{M}$  and the agent chooses  $\bar{s}$ . In order to convince the agent to switch to  $\bar{s}$ ,  $j$  must offer more than what she is already offering now. But, by (a), this implies that  $G_{\bar{s}}^j - \tilde{t}_{\bar{s}}^j \leq G_{s^*}^j$ .
- (vi)  $j \in \bar{M}$  and the agent chooses  $s'$ , different from both  $s^*$  and  $\bar{s}$ . To induce the agent to choose  $s'$ ,  $j$  must beat the transfers of the other principals on  $s^*$ , but:

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<sup>24</sup>The  $\hat{t}$  that we have constructed corresponds to the highest amount that the winning side is willing to pay. This choice makes the proof simpler, but in general there are other maximum-conflict equilibria based on a lower  $\hat{t}$ .

$$\begin{aligned} \sum_{i \in M^*} \hat{t}_{s^*}^i &= \sum_{i \in \bar{M}} \hat{t}_{\bar{s}}^i \geq \sum_{i \in \bar{M}} (G_{\bar{s}}^i - G_{s^*}^i) \\ &= \sum_{i \in M} \max\{0, G_{\bar{s}}^i - G_{s^*}^i\} \geq \sum_{i \in M} \max\{0, G_{s'}^i - G_{s^*}^i\} \geq G_{s'}^j - G_{s^*}^j \end{aligned}$$

where the first equality is (c), the first inequality is (a), the second equality is the definition of  $\bar{M}$ , the second inequality is (4), and the third inequality is obvious. This shows that  $G_{s'}^j - \tilde{t}_{s'}^j \leq G_{s^*}^j - \hat{t}_{s^*}^j$ .

Part (iii) and (iv)

By example. See Game 2.

**Proof of Proposition 1.** Let  $s^* \in \arg \max_s G_s^1$ ,  $\bar{s} \in \arg \max_s G_s^2$ , and  $\hat{s} \in \arg \max_s G_s^1 + G_s^2$ . Without loss of generality, assume that  $G_{s^*}^1 - G_{\bar{s}}^1 \geq G_{\bar{s}}^2 - G_{s^*}^2$ . By corollary 1 of Bernheim and Whinston (1986), the payments in a truthful equilibrium are  $G_{\bar{s}}^2 - G_{s^*}^2$  from principal 1 and  $G_{s^*}^1 - G_{\bar{s}}^1$  from principal 2. In a natural equilibrium, only 1 makes a positive payment, which is equal to  $G_{\bar{s}}^2 - G_{s^*}^2$ . Principal 1 is weakly better off in a truthful equilibrium if  $G_{\bar{s}}^1 + G_{\bar{s}}^2 - G_{s^*}^2 \geq G_{s^*}^1 + G_{s^*}^2 - G_{\bar{s}}^2$ , which is true by the definition of  $\hat{s}$ . Principal 2 is better off because  $G_{\bar{s}}^2 + G_{\bar{s}}^1 - G_{s^*}^1 \geq G_{s^*}^2$ . The agent is weakly better off in a maximum-conflict equilibrium if  $G_{s^*}^1 + G_{\bar{s}}^2 - G_{\bar{s}}^1 - G_{s^*}^2 \leq G_{\bar{s}}^2 - G_{s^*}^2$ , which, again, is true by the definition of  $\hat{s}$ . □

### Appendix B. Experiment details

This section contains: (1) a check of the equilibria in Table 2; and (2) the instructions and the decision form for the experiment.

#### Equilibria in Table 2

We prove that the contribution schedules in Table 2 constitute a Nash-equilibrium of the experiment game.

#### Natural equilibrium (Table 2a)

Hold the contribution schedule of *B* constant and suppose principal *A* deviates. There are seven possible outcomes: (1) *I* is chosen with certainty; (2) *II* is chosen with certainty; (3) *III* is chosen with certainty; (4) randomization between *I* and *II*; (5) randomization between *I* and *III*; (6) randomization between *II* and *III*; and (7) randomization between *I*, *II*, and *III*. It is immediate to check that in none of these outcomes the (expected) payoff of *A* increases.

Now hold the contribution schedule of *A* constant and suppose *B* deviates. As before, there are seven possible outcomes. Cases (2), (4), (6), and (7) are not

possible by assumption because when *II* occurs she has to pay more for *II* than the benefit she gets. The same is true for (3) because she would have to pay at least 12.05 to get 12. Finally, (5) keeps *B*'s payoff unchanged.<sup>25</sup>

#### *Truthful equilibrium (Table 2b)*

Hold the contribution schedule of *B* constant and suppose principal *A* deviates. The seven outcomes listed above apply. Cases (2), (3), (5), (6), and (7) are clearly not profitable. Case (1) keeps the net payoff constant because *A* needs offer at least 11 on *I*. The same applies to (4). The experiment you participate in will consist of six rounds. In each round you can earn money. After the last round the earnings you have made in all rounds will be summed up, and paid to you in cash.

Now hold the contribution schedule of *A* constant and suppose *B* deviates. Cases (1), (2), (4), (5), and (7) are clearly not profitable. Case (3) keeps the net payoff constant because *B* needs offer at least 11 on *III*. The same applies to (6).

#### *Instructions and decision form*

##### *Instructions*

The experiment you participate in will consist of six rounds. In each round you can earn money. After the last round the earnings you have made in all rounds will be summed up, and paid to you in cash.

At the beginning of each round you will be matched with a second participant, your partner. Your partner will change from round to round, and in each round you will have a different partner — you will not be matched with the same person twice. You will not know the identity of your partner, and your partner will not learn your identity. Furthermore, you are not allowed to communicate with any other participant during the whole experiment.

In each round you and your partner together have to make a choice between three alternatives, denoted by *I*, *II*, and *III*. For the chosen alternative you will get a certain amount of money from the experimenter, your gross earnings. The computation of the gross earnings will be described in detail below.

In order to choose between the alternatives you have to make contributions to each of the three alternatives. You have to fill in the contributions you want to make in your 'decision form' in the line 'your contribution'. Your contribution to an alternative may be zero or positive. Each contribution must not be above the gross earnings you can get from that alternative (see below the description of the gross earnings). Since 5 cents is the smallest available coin, each contribution has to be a multiple of 5 cent. Hence, a contribution of, e.g. 1.35 is allowed, but not a contribution of 1.37.

At the same time you make your decision, your partner also decides about

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<sup>25</sup>Obviously, these deviations would still be unprofitable if we eliminate upper limits on contributions.

his/her contributions. After you have made your decisions, the experimenter will transmit your contributions to your partner, and you will learn your partner's contributions (the experimenter will fill in the line 'contributions of your partner' in your decision form). Now you can easily calculate the sum of your and your partner's contributions for the different alternatives. The alternative with the highest sum of contributions is the chosen alternative.

Depending on which alternative has been chosen, you will get some money from the experimenter, your gross earnings. Similarly, your partner's gross earnings depend on the chosen alternative. But your gross earnings as well as that of your partner also depend on which earning schedule applies to you and your partner, respectively. There are two possible earning schedules, denoted by A and B. Both schedules are shown at the end of these instructions and at your decision form. If schedule A applies to you, B applies to your partner, and if B applies to you, A applies to your partner. I. Notice that the gross earnings you get from a certain alternative are different from the gross earnings your partner receives for the same alternative. Furthermore, in different rounds different schedules apply to you. In three rounds schedule A applies to you, in three rounds schedule B. Which schedule applies to you in a specific round can be seen at the bottom of your decision form for that specific round.

The chosen alternative together with the earning schedule that applies to you determines your gross earnings for that round. From these gross earnings you have to deduct your contribution for the chosen alternative. But you do not have to pay your contributions for the alternatives not chosen. Hence, your net earnings in a certain round are calculated according to the formula:

Your net earnings

$$= \text{Gross earnings due to chosen alternative and schedule applying to you} \\ - \text{Contribution to the chosen alternative}$$

Similarly, your partner's net earnings are the difference between his/her gross earnings and his/her contribution to the chosen alternative.

Assume for example that you contribute 0, 2.05, and 7.25 for the alternatives *I*, *II*, and *III*, respectively. Your partner contributes 4.65, 0, and 0 for *I*, *II*, and *III*. Then the chosen alternative is *III* since the sum of the contributions for *III* is 7.25, whereas the sum for *I* and *II* is only 4.65 and 2.05, respectively. Furthermore, schedule B applies for your in that round. Then your net earnings would be calculated according to: net earnings =  $12 - 7.25 = 4.75$ , whereas your partners net earnings would be  $0 - 0 = 0$ .

It is of course possible that the sum of contributions of two or more alternatives is equal. In that case the tie between the 'winning' alternatives is broken by using a die. In each round the die will be thrown after you and your partner have decided about your contribution, and the result of the throw will be publicly announced. If there are two winning alternatives and the die gives 1, 2, or 3, then *I* is chosen out

of *I* and *II*, *I* is chosen out of *I* and *III*, and *II* is chosen out of *II* and *III*. If all three alternatives have equal sums of contributions, 1 or 2 implies the choice of *I*, 3 or 4 implies the choice of *II*, and 5 or 6 implies the choice of *III*.

Do you have any questions?

Earning schedule A

Alternative	<i>I</i>	<i>II</i>	<i>III</i>
Earnings	17	11	0

Earning schedule B

Alternative	<i>I</i>	<i>II</i>	<i>III</i>
Earnings	0	7	12

*Decision form*

Round #                      Participant #

Alternative	<i>I</i>	<i>II</i>	<i>III</i>
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Your contributions

Contributions of your partner

Sum of contributions

Chosen alternative

Earning schedule A

Alternative	<i>I</i>	<i>II</i>	<i>III</i>
Earnings	17	11	0

Earning schedule B

Alternative	<i>I</i>	<i>II</i>	<i>III</i>
Earnings	0	7	12

In this round: earning schedule A applies to you, earning schedule B applies to your partner.

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