Learning and Market Clearing: Theory and Experiments*

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Abstract

This paper investigates theoretically and experimentally whether traders learn to use market-clearing trading institutions or whether other (inefficient) market institutions can survive in the long run. Using a framework with boundedly rational traders, we find that market clearing institutions are always stable under a general class of learning dynamics. However, we show that there exist other, non-market clearing institutions that are also stable. Therefore, in the long run traders may fail to coordinate exclusively on market clearing institutions. Using a replica-economies approach, we find the results to be robust to large market size. The theoretical predictions were confirmed in a series of platform choice experiments. Traders coordinated on platforms predicted to be stable, including market-clearing as well as non-market clearing ones, while platforms predicted to be unstable were avoided in the long run. (JEL C72, D4, D83)

Keywords: Market Institutions, Market Clearing, Coordination, Learning

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1 Introduction

The formation of a market requires a group of agents, some of them willing to buy and some of them willing to sell. Preferences and cost functions are sufficient to develop a theory if market clearing is taken as granted. Actual markets, though, are not merely characterized by demand and supply. Market exchange requires an institutional framework in which action and message sets are specified, and in which a process of matching and price formation can take place.

An enormous variety of market institutions can be observed in the field, even for the same good. Both call markets and continuous double auction markets are used to exchange financial assets. Real estate is sold both at auctions and by means of direct negotiations. In some countries' rental markets, established but *informal* institutions (e.g. group tenant visits) bias the market in favor of the owners. In other countries, middlemen act as platforms which compete actively for tenants. In addition there is always an alternative "word-of-mouth" market tenants might resort to.

These details of the market institution are consequential. In addition to theoretical and empirical evidence, there is a large body of experimental evidence in this direction.[1] Trading rules affect the efficiency of the market outcome, the convergence towards equilibrium, the volatility of the prices, and the distribution of surplus over the market participants. Given that "institutions matter" and given the competition between different market institutions, we might ask which institutions are used in the long run. What are the properties of successful institutions? Do the surviving market institutions support market clearing and efficient outcomes? Are there circumstances under which inefficient trading rules can persist, or are forces and mechanisms present that drive a market towards efficient organization?

Internet auction platforms like eBay, Yahoo, and Amazon provide a good example of competition between different market institutions. The trading rules of these auctions platforms differ e.g. in their ending rules and in the type of the Buy-Now option sellers may use. Experimental (Ariely, Ockenfels, and Roth, 2005) and theoretical analysis (Reynolds and Wooders, 2009) reveals that the level of realized prices as well as efficiency are influenced by these differences. A similar conclusion can be drawn for the possibility of secret reserve prices (Bajari and Hortaçsu, 2003) and for buy prices (Budish and Takeyama, 2001). In the context of multi-unit auctions, Ausubel and Cramton (2002) show that uniform and pay-as-bid auctions lead to different realized prices. Hence, competing institutions differ not only in their institutional setup, but the different institutional setups lead to systematic differences in the realized prices.

The survival of a specific market institution depends on whether traders employ this institution or avoid it. The decision about the use of a particular market institution gives rise to a game that combines aspects of a coordination and a minority game. On the one hand, poten-

[1] An overview of the classical experimental evidence on the importance of market institutions is provided by Plott (1982) (see also Holt, 1995).

tial buyers and sellers have to coordinate on a particular institution in order to make mutual beneficial trade possible. On the other hand, a trader is better off the fewer competing traders opt for the same trading platform. Due to the coordination aspect such a game exhibits a multiplicity of Nash equilibria. All the traders might coordinate on an institution that does not lead to market clearing outcomes. They might even coordinate on an institution that leads to a Pareto-inefficient outcome. Hence, we ask under what circumstances traders will indeed *learn* to coordinate on an efficient, market-clearing institution.

To provide an answer to this question, we conducted a theoretical and experimental study of a market for a homogeneous good. Potential traders have to choose simultaneously at which institution they want to trade. They choose between a market clearing institution and other institutions that do not lead to market clearing, but realize other prices. Traders who have chosen such an institution might obtain more favorable prices but necessarily face rationing. The theoretical part of the analysis is based on a learning model where each trader has a tendency to switch from one institution to a different one next period if another institution exhibits better current-period results. Traders evaluate the results according to evaluation functions that satisfy a number of weak behavioral assumptions, compatible with standard microeconomic models but allowing also for boundedly rational behavior. The learning model is related to stochastic models of learning in games (see Fudenberg and Levine, 1998, for an overview). In particular, traders are not assumed to anticipate future prices, market-clearing or otherwise. They tend to switch to strategies (institutions) which are better in the current period, without anticipating the effects of their strategy change. Within this framework the market clearing institution is *always* stochastically stable independently of the characteristics and the number of the other available institutions. This strong prediction, however, does not imply that only market clearing will be observed in the long run. On the contrary, we find that certain non-market clearing institutions are also stochastically stable. Hence, the theoretical analysis suggests that in the long run market-clearing institutions will be used, but in general not exclusively.

The experimental test of this result concentrates on the learning aspect of the theoretical model. More specifically, groups of 14 subjects each had to choose between two or three institutions. In the first treatment the payoffs were designed in such a way that subjects had to choose between a market-clearing and another stochastically stable institution. In the second treatment the choice was between the market clearing and a non-stable institution, while in the third treatment the choice was between all three institutions. The results show that whenever the market clearing as well as the other stochastically stable institution were available, there was no tendency to coordinate on a single institution. Both institutions remained active in the long run, i.e. after 90 repetitions of the game. Subjects, though, learned to avoid the non-stable institution when available. We also found strong evidence that individual traders' choice behavior was in accordance with our learning model. Overall, the experimental results confirmed the theoretical predictions.

The possibility that traders might choose between different trading institutions plays a role in several existing models (e.g. Ishibuchi, Oh, and Nakashima, 2002; Kugler, Neeman, and Vulkan, 2006; Gerber and Bettzüge, 2007). Those, however, do not investigate whether traders learn to coordinate on efficient institutions guaranteeing market clearing prices and quantities. There also exists a large experimental literature on learning in games, but to the best of our knowledge this literature does not examine the question of on which trading platforms traders coordinate.

The theoretical analysis in the paper at hand is related to our own work on competition among simultaneously available trading institutions. Alós-Ferrer, Kirchsteiger, and Walzl (2010) consider a game among two market designers confronted with boundedly rational buyers and sellers, where all sellers are endowed with a constant-return-to-scale technology. For any given characteristics of the institutions chosen by the market designers, the game played between the buyers and the sellers is a particular case of the model considered here. Alós-Ferrer and Kirchsteiger (2010) considers a related model where boundedly rational traders choose among different, possibly non-market-clearing institutions within a general equilibrium framework. This approach, however, is conceptually different from the model considered here. First, since the focus of Alós-Ferrer and Kirchsteiger (2010) is on rationing, each institutions is characterized directly by a parameter determining the amount of rationing. Second, traders' behavior is modeled through probabilistic behavioral rules rather than evaluation functions. Last, neither Alós-Ferrer and Kirchsteiger (2010) nor Alós-Ferrer, Kirchsteiger, and Walzl (2010) provide an experimental test of the underlying learning approach.

The paper proceeds as follows. Next we describe the model and its basic assumptions. In Section 3 we describe the learning process. Sections 4 and 5 present the stability results for market-clearing and non-market clearing institutions, respectively. Section 6 investigates the robustness of the theoretical results with respect to market size. Section 7 presents the experimental test of our model. Section 8 concludes. Proofs are relegated to Appendix A, and the experimental instructions are given in Appendix B.

2 The Model

There is a homogeneous good to be traded by a finite set I of n of buyers and a finite set J of m of sellers. We denote the price of the good by p.

A typical buyer will be modeled through a demand function, a typical seller through a supply function satisfying the following assumptions.

M1. The demand function $d : \mathbb{R}_+ \to \mathbb{R}_+ \cup \{+\infty\}$ is continuous and strictly decreasing in p in the range where $d(p) > 0$. Further, $d(0) > 0$, $d(p) \geq 0$ for all $p \geq 0$, and $\lim_{p\to\infty} d(p) = 0$.

M2. The supply function $s : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and (weakly) increasing. Further, $s(0) = 0$.

M3. There exists a price $p > 0$ with $d(p) > 0$ and $s(p) > 0$.

We allow for instance for linear demand functions of the form $d(p) = \max(a - bp, 0)$, but also for everywhere-positive functions as $d(p) = p^{-a}$, which are extended-real because $d(0) = +\infty$. Notice, though, that assumption **M1** implies $d(p) < +\infty$ for all $p > 0$.

For an individual trader the market outcome is given by the price at which he trades, and by the quantity he can trade. In order to model the learning process, we describe how buyers and sellers evaluate the market outcome. Denote by q_S the quantity sold by a typical seller, and by q_B the quantity bought by a typical buyer. The evaluations of the market outcomes, $v_B(q_B, p)$ and $v_S(q_S, p)$, depend on the quantity the traders buy and sell, respectively, and on the price p at which they trade. Hence, the evaluations (payoffs) are given by functions $v_B : \mathbb{R}^2_+ \to \mathbb{R}$ and $v_S : \mathbb{R}^2_+ \to \mathbb{R}$.

The primitives in our model are the demand, supply, and the payoff (evaluation) functions. We want to emphasize that this framework is more general than the usual microeconomic approach, where demand and supply are derived from maximization of the payoffs (i.e. from utility- and profit maximization). We have deliberately chosen this general framework in order to allow for the possibility that demand and supply are not based on rational choices of the agents. Furthermore, in our framework the evaluation of the market outcome, which—as explained later in detail—drives the learning process, need not be identical with consumers' utility and producers' profits. In other words, we allow for more general (even boundedly rational) modes of behavior. For example our framework allows for producers whose supply is derived from profit maximization, but who evaluate the market outcome by the revenue raised (without taking production costs into account). Such an inconsistency between the supply behavior and the learning process (which might e.g. be due to the different divisions within a firm deciding about quantity supplied and the market chosen) can be modeled within our approach, since such a model fulfills our core assumptions (explained below). It is worth emphasizing, however, that the usual microeconomic model of utility-maximizing consumers and profit-maximizing producers is also covered by our framework, as we will show later.

Demand and supply are given meaning by the following assumptions which relate them to the evaluation of the market outcome.

A1. In the absence of rationing, a lower price is better for buyers and worse for sellers. That is, for all p, p' with $p < p'$,

$$v_B(d(p), p) > v_B(d(p'), p') \quad \text{whenever } d(p) > 0,$$
$$\text{and } v_S(s(p), p) < v_S(s(p'), p') \quad \text{whenever } s(p) > 0.$$

A2. Given the price, traders prefer not to be rationed. That is, for all $p > 0$ and all $0 < q_B < d(p)$, $0 < q_S < s(p)$,

$$v_B(d(p), p) > v_B(q_B, p) \text{ and } v_S(s(p), p) > v_S(q_S, p).$$

A3. Given the price, traders prefer being rationed to not being able to trade. That is, for all $p > 0$ and all $0 < q_B < d(p)$, $0 < q_S < s(p)$,

$$v_B(q_B, p) > v_B(0) \text{ and } v_S(q_S, p) > v_S(0)$$

where $v_B(0) = v_B(0, p')$ and $v_S(0) = v_S(0, p')$ for all $p' \geq 0$ are the payoffs of not being able to trade, which we explicitly assume not to depend on (hypothetical) prices.

Essentially, these assumptions are fulfilled as long as traders focus on getting more favorable prices and dislike rationing. Of course, as the next examples show, standard models fulfill **A1-A3**, but we want to emphasize that our results depend only on these minimal properties.

Example 1. **Utility and Profit Maximization.** A first example fulfilling all assumptions above is obtained as follows. Consider identical consumers endowed with a strictly quasiconcave, continuous and strictly monotone utility function. Fix the prices of all goods except good 1, and denote by p_{-1} the vector of (fixed) prices of goods other than 1. Assume the (reduced) demand function for good 1, $d(p_1) = x_1(p_1, p_{-1})$, to be strictly decreasing in p_1 (ruling out that it is a Giffen good). The consumers' evaluation of the market outcome is simply given by the utility derived from this outcome. That is,

$$v_B(q_B, p_1) = u(q_B, x_{-1}(W - p_1 q_B, p_{-1}))$$

where W denotes the consumer's wealth, q_B is the quantity of good 1 actually bought by a buyer at the chosen institution (that is, taking into account possible rationing) and $x_{-1}(W - p_1 q_B, p_{-1})$ is the optimal demand for goods other than 1 given the remaining wealth and the prices p_{-1}.

Sellers are identical firms that produce good 1 with a strictly convex technology without fixed costs, leading to an increasing supply function $s_1(p_1)$. The evaluation of the market outcome is given by profits, i.e.

$$v_S(q_S, p_1) = p_1 q_S - C(q_S)$$

where C is the cost function and q_S is the quantity of good 1 actually sold by a seller at the chosen institution.

It is easy to show that valuation and demand and supply functions constructed in this way fulfill assumptions **M1-M3** and **A1-A3**.

Example 2. **Consumer and Producer Surplus.** Another specific way to derive valuation functions for the current model is to arbitrarily specify demand and supply functions satisfying **M1-M3** and let the evaluation of the market outcome be the corresponding consumers' and producers' surplus. It is easy to see that valuation functions constructed in this way also fulfill assumptions **A1-A3**.

2.1 Trading Institutions

The good can be traded at different market institutions. For any institution z, denote by n_z, m_z the number of buyers and sellers active at z. Let $p^*(n_z, m_z)$ be the market clearing price at z, i.e. $p^*(n_z, m_z)$ is the solution to

$$n_z d(p) = m_z s(p). \qquad \text{(MC)}$$

Under **M1-M3**, for every $n_z, m_z > 0$ there exists a unique $p^*(n_z, m_z)$ solving equation (MC), and it is strictly larger than zero. Note also that the equilibrium quantity is strictly positive.[2] Moreover, the market clearing price $p^*(n_z, m_z)$ depends only on the ratio

$$r = \frac{n_z}{m_z}$$

through the implicit equation $rd(p) = s(p)$, and hence we can write $p^* = p(r)$. It is important to note that the function $p(r)$ is strictly increasing in r (because $d(p)$ is decreasing and $s(p)$ is increasing in p).

Institutional biases come in many flavors and it is frequently hard to formally pin down the bias. We adopt a hands-on approach which nevertheless allows us to tackle a wide range of examples. Because differences in the institutional setup lead to systematic differences in the realized prices, we characterize the institutions directly by the trading price generated by the institution, but abstract from the specific rules generating this price. In order to accommodate different kinds of institutions, we give here a general definition and proceed to illustrate it presenting some families of examples. Let

$$S(n, m) = \left\{(n_z, m_z) \in \mathbb{N}^2 \,|1 \leq n_z \leq n,\ 1 \leq m_z \leq m\right\}$$

be the set of all feasible combinations of traders and sellers which can potentially show up at the same institution.

Definition 1. An institution is characterized by a *bias function*, $\beta_z : S(n, m) \to \mathbb{R}_{++}$ which measures the ratio between the actual price realized under that market institution, p_z, and the market clearing price. More specifically,

$$p_z(n_z, m_z) = \beta_z(n_z, m_z) p^*(n_z, m_z).$$

We say that the institution z is *market clearing* if $\beta_z(n_z, m_z) = 1$ for all $(n_z, m_z) \in S(n, m)$. We say that it is biased in favor of the sellers, or simply that it is a *seller institution*, if $\beta_z(n_z, m_z) > 1$

[2]**M1-M3** imply that there exists an equilibrium, and that any equilibrium price is strictly larger than zero. Because of **M3** and monotonicity of supply and demand any equilibrium quantity is strictly positive. Then, by **M1** demand at any equilibrium is strictly decreasing, implying uniqueness.

for all $(n_z, m_z) \in S(n, m)$. Analogously, we say that it is biased in favor of the buyers, or simply that it is a *buyer institution*, if $\beta_z(n_z, m_z) < 1$ for all $(n_z, m_z) \in S(n, m)$.

According to this definition, trade at each institution occurs at only one particular, deterministic price. One might want to give up these assumptions. It can be shown that allowing for institutions that violate this intra-institutional "law of one price" or for institutions with stochastic prices would not change our main results.[3]

We also remark that we do not assume that institutions are systematically biased in favor of the sellers or the buyers. A given institution might yield $\beta_z(n_z, m_z) < 1$ for certain pairs (n_z, m_z), and $\beta_z(n_z, m_z) > 1$ for others. Examples are given below.

If the price is not at the market-clearing level, we assume that the quantity traded is determined by the "shorter" market side and that the other market side cannot trade as much as it wishes according to its demand or supply function. This rationing is assumed to be the same for every trader of the same market side. More specifically, denote by $Q_z(n_z, m_z)$ the overall quantity traded at z. We can now distinguish between three cases:

Case 1: $\beta_z(n_z, m_z) = 1$. In this case the market-clearing prices and quantities are realized, and no trader is rationed. The institution is market clearing. The quantities are given by $Q_z(n_z, m_z) = m_z s(p^*(n_z, m_z)) = n_z d(p^*(n_z, m_z))$; $q_B^z = d(p^*(n_z, m_z))$; $q_S^z = s(p^*(n_z, m_z))$

Case 2: $\beta_z(n_z, m_z) < 1$. In this case the price is below the market-clearing price, and hence the quantity is determined by supply and buyers are rationed: $Q_z(n_z, m_z) = m_z s(p_z(n_z, m_z))$; $q_S^z = s(p_z(n_z, m_z))$; $q_B^z = \frac{m_z}{n_z} s(p_z(n_z, m_z)) < d(p_z(n_z, m_z))$.

Case 3: $\beta_z(n_z, m_z) > 1$. In this case the price is above the market-clearing price, and hence the quantity is determined by demand and sellers are rationed: $Q_z(n_z, m_z) = n_z d(p_z(n_z, m_z))$; $q_B^z = d(p_z(n_z, m_z))$; $q_S^z = \frac{n_z}{m_z} d(p_z(n_z, m_z)) < s(p_z(n_z, m_z))$.

In summary, given an institution z characterized by a function $\beta_z(\cdot, \cdot)$, and given $r = \frac{n_z}{m_z} > 0$ and $\beta = \beta_z(n_z, m_z)$ we can compute the seller and buyer quantities as

$$q_S^z(\beta, r) = \begin{cases} s\left(\beta \cdot p(r)\right) & \text{if} \quad \beta \leq 1 \\ r \cdot d\left(\beta \cdot p(r)\right) & \text{if} \quad \beta \geq 1 \end{cases}$$

and

$$q_B^z(\beta, r) = \begin{cases} \frac{1}{r} \cdot s\left(\beta \cdot p(r)\right) & \text{if} \quad \beta \leq 1 \\ d\left(\beta \cdot p(r)\right) & \text{if} \quad \beta \geq 1 \end{cases}$$

At this point we have to emphasize that we do not aim to analyze how a deviation from market clearing prices comes about. Rather, we just assume that market clearing institutions as well as institutions preventing markets from clearing are in principle feasible. And the purpose

[3] A proof of this claim is available upon request. We have also implicitly assumed that institutions are anonymous, i.e. the bias depends only on the number of sellers and buyers operating at the institution, and not on their identities. Our results remain valid if this assumption is relaxed.

of this paper is to investigate whether a non-market clearing institution can survive vis-a-vis a market clearing one.

The formulation above is general enough to encompass many familiar examples.

Example 3. **Limit price institutions.** An institution exhibits a *price cap* if there exist $p^H > 0 \in \mathbb{R}_+$ such that, for all $(n_z, m_z) \in S(n, m)$,

$$\beta_z(n_z, m_z) \leq \frac{p^H}{p^*(n_z, m_z)}.$$

Analogously, an institution exhibits a *price floor* if there exist $p^L > 0 \in \mathbb{R}_+$ such that, for all $(n_z, m_z) \in S(n, m)$,

$$\beta_z(n_z, m_z) \geq \frac{p^L}{p^*(n_z, m_z)}.$$

Price caps are often observed in housing markets, whereas price floors are prominent in labor markets - minimum wages.

Further, an institution exhibits a *fixed price* if there exist $p^F > 0$ which is simultaneously a price floor and a price cap. Such extreme public price regulation has been often observed for basic goods like food in wartime.

Other institutions do not exhibit a direct, public price regulation. Rather, market institutions like the posted offer or the posted bid institution enhance trade at prices systematically above or below the market clearing price. The most simple type of such institutions is the following.

Example 4. **Constant-bias institutions.** A constant-bias institution z is characterized by a bias parameter $\beta_z > 0$, i.e. $\beta_z(n_z, m_z) = \beta_z$ for all $(n_z, m_z) \in S(n, m)$. Thus we can write

$$p_z(n_z, m_z, \beta_z) = \beta_z p^*(n_z, m_z).$$

Constant-bias institutions are a simple, parametric example which will actually be enough for some of our purposes.

Example 5. **Oligopolistic institutions.** We say that a seller institution z is *oligopolistic* if $\beta_z(n_z, m_z)$ is strictly larger than one and strictly decreasing in m_z, for any given n_z. Such institutions arise e.g. if the price is the result of a Nash equilibrium where sellers internalize buyers' demand and compete among themselves in quantities. The intuition is simply that as more and more sellers compete (larger m_z), they lose market power and the oligopolistic price approaches the competitive one (hence the bias approaches one).

Notice that, in this formulation, sellers' market power is embodied by the institution. The market price p is higher than the market-clearing price. Still, at that market price, sellers are rationed, i.e. sell less than $s(p)$. For instance, if the market price is the Cournot-Nash one, it is only after rationing takes place that the sellers exactly supply the Cournot-Nash quantity. The institution, hence, can be seen as a coordination or commitment device.

3 The Learning Process

3.1 The Stage Game

If more than one institution is available, traders themselves can choose the institution at which they want to be active. For example, if the price for a certain good is fixed by the state, traders might choose between the official market with the fixed price, and a black market where trade is conducted at market-clearing prices. Labor might be hired at the official market where a minimum wage legislation applies, and at a black market without a price floor. Goods might be traded at a posted offer market, where the price tends to be above the market clearing level, and at a double auction, where the market outcome tends to coincide with the competitive equilibrium.

In this section, we explicitly model the choice between trading institutions. Our aim is to be able to predict which institution(s) will be observed to be active, and whether the outstanding importance of market clearing institutions in economics can be justified by this choice process.

A generic trader is denoted by k, while i always denotes a buyer and j always denotes a seller. There are $Z+1$ institutions available, $z = 0, 1, ..., Z$. Institution 0 is a market clearing institution ($\beta_0 = 1$). We make no assumption over the remaining others. In particular, there might be some other, competing, market-clearing institution.

We proceed now by formulating the choice process as a game. At first all traders choose, simultaneously and independently, the institutions at which they want to trade the good.[4] Then, for each trading institution z, the number of buyers and sellers who have opted for this institution, n_z and m_z, and the bias function β_z determine—as described in Section 2.1—the price and the quantity exchanged at z. This in turn determines the payoffs (evaluations) of the traders having opted for z.

It is easy to see that this choice process constitutes a coordination game. If all traders coordinate on a particular institution, every individual trader would be worse off if he deviated to another institution, since by deviating he would lose all trading partners (see **A3**). Hence, nothing guarantees coordination on the market clearing institution; further, full coordination on any institution constitutes a strict Nash equilibrium.

3.2 The Basic Learning Process

We proceed now to model the learning process. First we define the state space. For any point in time t, the state of the process is given by

$$\omega(t) = (\omega_B(t), \omega_S(t)) \in \{0, 1, ..., Z\}^n \times \{0, 1, ..., Z\}^m$$

i.e. $\omega(t)(k) \in \{0, 1, ..., Z\}$ denotes the institution chosen by trader k at t.

[4] We abstract from multihoming considerations here.

Since interactions are anonymous and traders are symmetric, the following notation will turn out to be convenient:

$$\begin{aligned} n_z(\omega) &= |\{i \in I \,|\omega(i) = z\}| \\ m_z(\omega) &= |\{j \in J \,|\omega(j) = z\}| \end{aligned}$$

i.e. $n_z(\omega) \in \{0, 1, ..., n\}$ is the number of buyers and $m_z(\omega) \in \{0, 1, ..., m\}$ the number of sellers choosing institution z, and $n_0(\omega) + ... + n_Z(\omega) = n$, $m_0(\omega) + ... + m_Z(\omega) = m$ hold. Let Ω denote the state space.

The learning process is based on the implicit assumption that traders understand the strategic nature of the coordination problem. Therefore, they do not regard the situation as an individual decision problem (as they would in a reinforcement learning model). Furthermore, we assume that traders only know the prices and the quantities of currently active institutions, and hence do not have enough information to accurately predict the outcomes in all trading institutions which are in principle feasible. Thus, they lack the information necessary to compute a best reply to the current choices of all other traders.

Suppose that a trader has the possibility to revise his choice of institution (we will specify in which form revision opportunities arrive below). What can a trader do in such a situation? From his individual (myopic) standpoint, if he considers himself to be small relative to market size, the best thing he can do is to observe the outcomes (i.e. prices and quantities) of the currently active institutions and to evaluate these outcomes through his own evaluation function. That is, he will switch to that institution whose current prices and quantities he perceives as best according to his payoff function. A trader can perceive this behavior as approximately rational, since when he chooses a new institution, the implied changes in prices and traded quantities will most of the time be small, and hence this behavior is close to best reply. Of course, in the current (symmetric) model, this behavior could also be interpreted as imitation of successful traders of the own market type. We want to stress, though, that the described behavior does not require the observation of payoffs achieved by other traders, but merely prices and traded quantities.

Fix a state ω. Call an institution z *active* if $m_z(\omega) > 0$ and $n_z(\omega) > 0$, and inactive if $m_z(\omega) = 0$ or $n_z(\omega) = 0$. With this notation, the considerations above are captured by the following assumption.

D0. Traders who receive the opportunity to revise observe prices and traded quantities at all active institutions. Then they choose the institution which yields the best outcome as evaluated by their own payoff functions, and go there next period (ties broken randomly).[5]

[5] Inactive institutions are not even observed, since no price is even posted. Hence, in the extreme case in which all institutions are inactive, traders simply stay at their respective institutions. We find this implicit assumption sensible; however, it could be changed without affecting the results.

That is, provided that trader k receives revision opportunity at period t, in period $t+1$ he will choose an institution among those that in period t were yielding the highest observed payoffs for traders of his own type. Note that an agent takes his decision for period $t+1$ given the state $\omega(t)$ and the associated payoffs. This decision determines the institution chosen for period $t+1$. Combining all such decisions of the individual traders determines $\omega(t+1)$, and hence the basic dynamics.

3.3 Revision Opportunities

When can agents revise their choices? It is common in learning models to explicitly introduce some inertia allowing for the possibility that not all agents are able to revise strategies simultaneously. Different specifications are possible. One prominent example is *independent inertia* (e.g. Samuelson, 1994; Kandori and Rob, 1995)), where each agent has an independent, strictly positive probability of not being able to switch. A different example is *asynchronous learning* (e.g. Binmore and Samuelson, 1997; Benaïm and Weibull, 2003; Blume, 2003), where each period one and only one agent is able to revise, all agents having strictly positive probability of receiving the revision draw. In our case, a natural variant of this dynamics would be *asynchronous learning within types*, where in every period, only one buyer *and* one seller are selected (randomly and independently) and given the opportunity to revise.

Different specifications of how revision opportunities arrive give rise to different dynamics and often affect the results (see e.g. Alós-Ferrer and Netzer, 2010). Rather than adopting a specific formulation, we postulate a general class of dynamics encompassing the standard examples mentioned above, and many others (see Alós-Ferrer, 2003 and Alós-Ferrer and Netzer, 2010 for a discussion).

Let $E(k, \omega)$ denote the event that agent k receives revision opportunity when the current state is ω, and let $E^*(k, \omega) \subseteq E(k, \omega)$ denote the event that agent k is the only agent of his type (i.e. the only buyer or the only seller) receiving revision opportunity in ω. With this notation, the general class of dynamics we consider is given by the following assumptions.

D1. $\Pr(E^*(k, \omega)) > 0$ for every agent k and state ω.

Notice that **D1** implies that $\Pr(E(k, \omega)) > 0$, i.e. every agent has strictly positive probability of being able to revise at any given state. Further, since we have two clearly differentiated populations, we introduce a weak form of independence between the revision opportunities in those populations (it can be thought of as an anonymity requirement).

D2. For every agent k and state ω, either $\Pr(E^*(k, \omega) \cap E^*(k', \omega)) > 0$ for any agent k' of the other type, or $\Pr(E^*(k, \omega) \cap E(k', \omega)) = 0$ for any such k'.

Assumptions **D1** and **D2** are rather general. It is easy to see that they are fulfilled by the standard types of revision opportunities mentioned above. The reason we explicitly choose

Assumptions **D1-D2** is that, in the literature of learning in games, predictions are not always robust to minute changes in the assumptions on the dynamics. We want to make explicit that our model is not so sensitive to the details of the dynamics.

In our context, it is plausible that traders are more likely to revise when the perceived gains from revision are higher. For instance, one might postulate that the probability of revision increases with the difference between the payoff at the institution currently chosen by the trader and the largest payoff generated at any other institution. For the case of two institutions, this would be equivalent to the *proportional imitation rule* of Schlag (1998). Such a sensitivity of revision opportunities to payoff differences is allowed by the specification above, since the revision probability $\Pr(E(k,\omega))$ is a function of the state ω.

3.4 Stochastic Stability

The dynamics described till now is a Markov chain on the (finite) state space Ω, to which standard treatment applies (see e.g. Karlin and Taylor, 1975). We refer to this dynamics as the *unperturbed process*.

Given two states ω, ω', denote by $P(\omega, \omega')$ the probability of transition from ω to ω' in one period. An *absorbing set* of the unperturbed dynamics is a minimal subset of states which, once entered, is never abandoned. An *absorbing state* is an element which forms a singleton absorbing set, i.e. ω is absorbing if and only $P(\omega, \omega) = 1$.

In general, the unperturbed process presents a multiplicity of absorbing sets. In order to select among them, and following the literature, the dynamics is enriched with a perturbation in the form of mistakes or experiments as follows. With an independent probability $\varepsilon > 0$, each agent, in each period, might make a mistake ("mutate"), and simply pick an institution at random,[6] independently of other considerations. This can be interpreted literally as a decision mistake or, alternatively, as an *experiment* on the side of the agent. For instance, such an experiment might correspond to an agent being replaced by a new, unexperienced one which simply builds some arbitrary theory, or to an agent discarding past information and being attracted to a new institution after observing an institutional (unmodeled) marketing campaign.

The dynamics with mistakes (experimentation) is called *perturbed* learning process. Since experiments make transitions between any two states possible, the perturbed process has a single absorbing set formed by the whole state space. Such processes are called *irreducible*. An irreducible process has a unique *invariant distribution*, i.e. a distribution over states $\mu \in \Delta(\Omega)$ which, if taken as initial condition, would be reproduced in probabilistic terms after updating (more precisely, $\mu \cdot P = \mu$ where P is the matrix of transition probabilities).

For a given ε, the corresponding invariant distribution is denoted by $\mu(\varepsilon)$. The *limit invariant*

[6] We mean that an institution is picked up according to a pre-specified probability distribution having full support, for instance uniformly. It is well-known that the exact distribution does not affect the stochastic stability results, as long as it has full support.

distribution (as the rate of experimentation tends to zero) $\mu^* = \lim_{\varepsilon \to 0} \mu(\varepsilon)$ exists and is an invariant distribution of the unperturbed process (Kandori, Mailath, and Rob, 1993; Young, 1993; Ellison, 2000). The limit invariant distribution singles out a stable prediction of the unperturbed dynamics, in the sense that, for any $\varepsilon > 0$ small enough, the play approximates that described by μ^* in the long run. The states in the support of μ^*, i.e. $\{\omega \in \Omega \mid \mu^*(\omega) > 0\}$ are called *stochastically stable states* or long-run equilibria. The set of stochastically stable states is a union of some absorbing sets of the original, unperturbed chain ($\varepsilon = 0$).

We will rely on the characterization of the set of stochastically stable states introduced by Kandori, Mailath, and Rob (1993) and Young (1993) and further developed by Ellison (2000). Detailed overviews can be found e.g. in Fudenberg and Levine (1998) or Samuelson (1997).

4 Stochastic Stability of Market-Clearing Institutions

We proceed now to analyze the complete model. A first intuition for our main results is obtained when we compare the payoffs sellers and buyers receive at simultaneously active market-clearing and non-market-clearing institutions.

Lemma 1. *Assume* **A1** *and* **A2**. *Consider any distribution of traders on any number of institutions, where both a market clearing institution* 0 *and another institution* z *are active. Let* $p_z = p_z(n_z, m_z)$. *Then the following holds:*

For $\beta_z(n_z, m_z) \neq 1$: *If* $v_S(q_S^0, p_0) \leq v_S(q_S^z, p_z)$, *then* $v_B(q_B^0, p_0) > v_B(q_B^z, p_z)$. *Hence, if* $v_B(q_B^0, p_0) \leq v_B(q_B^z, p_z)$, *then* $v_S(q_S^0, p_0) > v_S(q_S^z, p_z)$.

For $\beta_z(n_z, m_z) = 1$: *Either* $v_S(q_S^0, p_0) \leq v_S(q_S^z, p_z)$ *and* $v_B(q_B^0, p_0) \geq v_B(q_B^z, p_z)$, *or the reverse (weak) inequalities hold.*

Lemma 1 shows that, whenever traders of a given market side obtain larger payoffs in a biased institution than their counterparts in the market clearing one, traders of the other market side which are active in the market clearing institution must obtain larger payoffs than those active in the biased one. This result is crucial for the analysis of the learning model. Intuitively, it points out a reason for (some) traders to move towards the market-clearing institution in the presence of another one.

We are interested in the stability of institutions. Clearly, every *monomorphic state*, where all traders coordinate in one and the same institution, constitutes an absorbing state. These are actually the only relevant absorbing states. In principle (and particularly for dynamics with asynchronous learning), there might be non-singleton absorbing sets. However, it can be shown (see Lemma 4(ii) in the Appendix) that those would be made up of states where the market clearing institution $z = 0$ is never active.

Since monomorphic states correspond to full coordination on a particular market institution, we aim to identify which of those states are stochastically stable.

Definition 2. We say that an institution $z \in \{0, ...Z\}$ is stochastically stable if the corresponding monomorphic state ω_z characterized by

$$n_z(\omega_z) = n \text{ and } m_z(\omega_z) = m$$

is stochastically stable.

Intuitively, a stochastically stable institution is one such that, in the long run, traders frequently coordinate on it. In principle, several institutions could be stochastically stable, but if a particular institution is not, we can assert that, in the long run, this institution will be simply not be used by traders.

Our first main result establishes that market-clearing institutions are always active in the long run.

Theorem 1. *Assume* **M1-M3**, **A1-A3**, *and consider any dynamics satisfying* **D0-D2**. *Any market clearing institution is stochastically stable.*

This result implies that, independently of which other institutions are available, coordination on the market clearing one will always be observed at least (a non-negligible) part of the time in the long run. It is striking that this result is completely independent of what the characteristics of other institutions are. A market-clearing institution remains stochastically stable independently of how many other institutions are available and what their characteristics are, from limit pricing to oligopolistic institutions or any conceivable alternatives.[7]

Remark 1. A common criticism on the literature of learning in games is that the speed of convergence to the predicted outcomes might depend inversely (and exponentially) on population size and hence the predictions might be irrelevant for large population sizes. This criticism does not affect our results. The technical reason (see Ellison (2000) for details) is that the number of mutations involved in the stability analysis is small (two) and independent of population size. Intuitively, the transitions that destabilize non-market clearing institutions in favor of market-clearing ones only require a few experiments, followed by high-probability revisions where traders imitate successful behavior.

5 Stable Non-Market Clearing Institutions

In the previous section we have shown that market clearing institutions are always stochastically stable. However, it turns out that there exists also stochastically stable biased institutions.

[7]Due to the efficiency properties of the equilibrium, we view this result as "good news". In certain contexts, however, the interpretation might be different. A black labor market might be considered as a market-clearing institution which competes with regulated labor markets. Our result might thus provide an insight into the stability of moonlighting.

Strikingly, it is possible to show that even some constant-bias institutions are stochastically stable, independently of which other institutions are available.

In general the effects of a bias on the payoffs of the traders are ambiguous. Take as an example an active institution z where prices are higher than the equilibrium price ($\beta_z(n_z, m_z) > 1$). Recall the notation $r = n_z/m_z$ for the buyers-sellers ratio at z ($0 < r < \infty$). Compared to a market clearing institution having exactly the same r, prices as well as quantities are unfavorable for buyers, and a further increase in β_z would lead to a further decrease in buyers' payoffs. For sellers the situation is different. For them, prices at z are more favorable than at a market clearing institution. This comes at the price of a decrease in the quantity sellers can sell. Therefore the impact of a further increase of β_z on sellers' payoffs is unclear.

To build an intuition, consider the standard case with demand and supply derived from utility and profit maximization. Under standard assumptions, the price set by a cartel formed by all the sellers is strictly larger than the market-clearing price. Hence, for a given number of buyers and sellers, a small increase of β above one should be beneficial for the sellers (and, of course, detrimental for the buyers).

Similar considerations can be made for the impact of the bias on the buyers. For prices close to the equilibrium price, the positive direct effect of a price decrease on the consumers is larger than the negative effect due to the decrease in consumed quantity.

These considerations lead to Assumption **A4** below. Given a realized bias $\beta_z = \beta_z(n_z, m_z) > 0$, and given $r = \frac{n_z}{m_z} > 0$, the payoffs for buyers and sellers at institution z can be rewritten as

$$V_B(\beta_z, r) = v_B\left(q_B^z(\beta_z, r), \beta_z \cdot p(r)\right) \quad \text{and} \quad V_S(\beta_z, r) = v_S\left(q_S^z(\beta_z, r), \beta_z \cdot p(r)\right).$$

The payoffs of, say, the buyers are given by $V_B(\beta_z, r)$; from the buyers' point of view, though, they depend just on the actually experienced bias and buyers-sellers ratio. The following assumption spells out the effects of small deviations of the equilibrium price from the realized one for a given ratio of buyers and sellers.

A4. For any fixed ratio of buyers and sellers r with $0 < r < \infty$, there exist $\underline{\beta}(r) < 1 < \overline{\beta}(r)$ such that $V_B(\beta, r) > V_B(1, r)$ for all $\underline{\beta}(r) < \beta < 1$, and $V_S(\beta, r) > V_S(1, r)$ for all $1 < \beta < \overline{\beta}(r)$.

This condition is immediately fulfilled if the buyer's payoff $V_B(\beta, r)$ is strictly decreasing in β at $\beta = 1$,[8] and the seller's payoff $V_S(\beta, r)$ is strictly increasing in β at $\beta = 1$.

Note that the comparison of payoffs spelled out in this assumption is fundamentally different from the results of Lemma 1. There, the comparison was among payoffs yielded by two simultaneously active institutions with different traders, while in **A4**, the comparison is implicitly

[8]Neither $V_B(\beta, r)$ nor $V_S(\beta, r)$ are in general differentiable at $\beta = 1$, because at this point there is a transition from rationing of the demand side to rationing of the supply side. Hence, the traded quantity as a function of β has a "kink" at $\beta = 1$.

among payoffs yielded by two different institutions, provided the buyers-sellers ratio is the same in both of them.

Assumption **A4**, though, is instrumental for showing that there are some non-market clearing institutions fulfilling a property analogous to the one spelled out in Lemma 1, i.e. there is always a reason for some traders to move towards them even in the presence of a market-clearing institution.

Definition 3. Fix the number of buyers and sellers operating on the whole market. An institution $F \neq 0$ is *favored* if, given any distribution of these traders on (only) F and the market clearing institution 0 such that both of them are active, then the following holds:

If $v_S\left(q_S^0, p_0\right) \geq v_S\left(q_S^F, p_F\right)$, then $v_B\left(q_B^0, p_0\right) < v_B\left(q_B^F, p_F\right)$ (or, equivalently, if $v_B\left(q_B^0, p_0\right) \geq v_B\left(q_B^F, p_F\right)$, then $v_S\left(q_S^0, p_0\right) < v_S\left(q_S^F, p_F\right)$).

Favored institutions are those such that an statement analogous to Lemma 1 holds for them versus the market clearing one. This is actually enough to show that favored institutions are stochastically stable.

Theorem 2. *Assume* **M1-M3**, **A1-A4**, *and consider any dynamics satisfying* **D0-D2**. *Let* $z \in \{1, ..., Z\}$ *be any favored institution. Then, independently of which other institutions are available,* z *is stochastically stable.*

This result shows that, potentially, there might exist stochastically stable, non-market clearing institutions. In order to actually establish their existence, it is enough to investigate under which circumstances do such favored institutions exist.

Obviously, one can just take the maximum $\underline{\beta}(r)$ and minimum $\overline{\beta}(r)$ among all (finitely many) buyers-sellers ratio which are actually possible. The intuition would be that institutions which always yield biases between those bounds should be favored. This intuition fails, though. The problem is the following. Imagine a biased institution z with, say, constant bias $\beta_z < 1$ and a market-clearing one are simultaneously active. In principle, since $\beta_z < 1$, prices at z are lower than at the market clearing institution, for a given proportion of buyers and sellers. The actual proportions at z and the market clearing institution, though, might be so different as to offset the effect of the bias. For, since the market clearing price is an increasing function of the buyers-sellers ratio $r_z = \frac{n_z}{m_z}$, if the ratio at the market-clearing institution, r_0, is much smaller than the one at z, r_z then the price at the former, $p(r_0)$ might be so much smaller than the (theoretical) market-clearing price at z, $p(r_z)$, that the actual price there, $\beta_z \cdot p(r_z)$, might still be larger than $p(r_0)$ even though $\beta_z < 1$.

This problem, though, might be overcome by taking tighter bounds, taking full advantage of the fact that m and n are finite. Then one obtains the following result.

Theorem 3. *Assume* **M1-M3**, **A1-A4**, *and* **D0-D2**. *Fix the number of buyers* n *and sellers* m *operating on the whole market. Then, there exist* $\underline{\beta}^*(n, m)$ *and* $\overline{\beta}^*(n, m)$ *with* $\underline{\beta}^*(n, m) < 1 <$

$\overline{\beta}^*(n,m)$ such that any institution F satisfying $\underline{\beta}^*(n,m) < \beta_F(n_z,m_z) < \overline{\beta}^*(n,m)$, $\beta_F(n_z,m_z) \neq 1$ for all $(n_z,m_z) \in S(n,m)$ is favored, hence stochastically stable.

In particular, any constant-bias institution F with $\underline{\beta}^*(n,m) < \beta_F < \overline{\beta}^*(n,m)$ is stochastically stable.

This result shows that potential favored institutions do exist[9] for any n, m, and that the vicinity of the market clearing institution consists of such favored institutions. Those non-market clearing institutions for which $\underline{\beta}^*(n,m) < \beta_z(n_z,m_z) < \overline{\beta}^*(n,m)$ are such that they improve one market side relative to the market clearing institution for distribution of buyers and sellers. In other words *for any given distribution of buyers and sellers* such a non-market clearing institution is favored by one market side over the market clearing one.

The last result shows that, in general, there exist non-market clearing institutions which do not disappear in the long run. Strikingly this includes even some very simple institutions, characterized by a constant (if small) bias.

6 Stable Non-Market Clearing Institutions and the Market Size

Theorem 2 gives us sufficient conditions for the existence of stochastically stable institutions other than the market clearing one. By Theorem 3, favored institutions always exist for given market size, even if institutions are simply characterized by a constant bias parameter. However, one might ask whether it is possible that only the market-clearing institution is stable if the market becomes very large. It is indeed possible to construct examples (for particular combinations of demand and supply functions) where the set of favored institutions degenerates as market size grows; however, being favored is just a sufficient condition for stochastic stability, and hence focusing on this property would not allow us to obtain a satisfactory answer. In this section, we investigate this question by letting the size of the market grow and by analyzing stochastic stability directly.

Specifically, we adopt a "replica economy" approach as follows. We fix an economy with n buyers and m sellers, and consider the K-replicated economy formed by K copies of the initial economy, i.e. with $K \cdot n$ buyers and $K \cdot m$ sellers. By Theorem 1, the market-clearing institution remains stochastically stable for all K. We aim to show that certain non-market clearing institutions are also stochastically stable for arbitrarily large K.

We consider slightly stronger versions of our assumptions **M1-M3.** The following assumptions exclude that demand and supply functions have trivial parts. Note that, if e.g. demand might be zero at a positive price, it would still be zero for any replicated economy, and hence the sense in which the economy becomes larger would be unclear.

[9]That is, there are bias functions such that, if an institution is characterized precisely by that function, it will be favored. This does not mean that we assume a favored institution always to be actually available in the market.

M1'. The demand function $d : \mathbb{R}_+ \to \mathbb{R}_+ \cup \{+\infty\}$ is continuous and strictly decreasing, with $d(p) > 0$ for all $p \geq 0$, and $\lim_{p\to\infty} d(p) = 0$.

M2'. The supply function $s : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous and (weakly) increasing. Further, $s(0) = 0$ and $s(p) > 0$ for all $p > 0$.

Note that **M1'-M2'** imply **M1-M3**. The key additional implications of these assumptions are that $\lim_{r\to+\infty} p(r) = +\infty$ and $\lim_{r\to 0} p(r) = 0$.[10]

In order to study large economies, we need to specify an additional assumption on the dynamics. The reason is that assumptions **D1** and **D2** are not tailored to the case of large economies. In particular, consider a dynamics where only one agent revises every period (which is allowed by assumptions **D1** and **D2**). As K increases, the speed of learning in this dynamics effectively converges to zero. A more reasonable dynamics would be e.g. one ensuring that at least one agent *in each replica* receives the opportunity to revise, ensuring that the speed of learning remains constant (or at least does not vanish) as K increases.[11] The following assumption fulfills this role.

D3$_K$. For every state ω, the probability that any given set of K buyers i revise (and nobody else) is strictly positive. Analogously, the probability that any given set of K sellers revise (and nobody else) is strictly positive.

The following theorem proves existence of (constant-bias) stochastically stable non-market clearing institutions even for those cases where the set of favored institutions degenerates.

Theorem 4. *Assume* **M1'-M2'**, **A1-A4**, **D0-D2** *and* **D3$_K$** *for the dynamics of each K-replicated economy. Suppose z with constant β_z is a favored institution for the economy with $K = 1$. The following hold.*

(i) If $m \leq n$ (more buyers than sellers) and $\underline{\beta}(1) < \beta_z < 1$, then there exists a K^* such that z is stochastically stable for all $K \geq K^*$.

(ii) If $m \geq n$ (more sellers than buyers) and $1 < \beta_z < \overline{\beta}(1)$, then there exists a K^* such that z is stochastically stable for all $K \geq K^*$.

[10] Recall that $p(\cdot)$ is strictly increasing in r. If $\lim_{r\to\infty} p(r) \neq +\infty$, it follows that $p(r)$ is bounded above by some $L > 0$. Since $s(\cdot)$ is increasing and $d(\cdot)$ is decreasing, it follows from $rd(p(r)) = s(p(r))$ that r is bounded above by $s(L)/d(L)$, a contradiction. Analogously, if $\lim_{r\to 0} p(r) \neq 0$, we would obtain that r is bounded below by some strictly positive $s(\varepsilon)/d(\varepsilon)$, a contradiction.

[11] This problem is well-known in the stochastic approximation literature. For instance, Benaïm and Weibull (2003) assume a fixed relationship between population size and the length of a time interval to ensure that the expected time between two revision opportunities of a given individual does not grow as the population size increases.

Note that the bounds $\underline{\beta}(1) < 1 < \overline{\beta}(1)$, are independent of market size (recall **A4**). Further, by Theorem 3, the set of constant-bias favored institutions for $K = 1$ include a non-negligible interval around $\beta = 1$. Therefore, the set of stochastically stable institutions does not in general shrink to the market clearing institution when the market size increases, even if the set of favored institutions degenerates. Theorem 4 shows that, under general conditions, there will be biased stochastically stable institutions even for large market size. That is, there is no "core convergence" result in this setting. Even though the set of stochastically stable institutions will always contain the market-clearing institution, other institutions will remain active as the size of the economy grows.

The intuition is the following. Suppose one market side (buyers or sellers) is overrepresented in the population. Then, this market side has less market power than the other side. If, for some reason, an institution biased in the favor of this market side attracts a few sellers and buyers in similar numbers, the overrepresented side will necessarily prefer the latter institution. Once the new institution becomes active, the fact that it is favored for $K = 1$ implies that, if the appropriate proportions of agents are present in it (for instance if the numbers of buyers and sellers are multiples of K), in practice it will behave as a favored institution in the replicated economy. This creates positive-probability paths destabilizing the market-clearing institution.

7 Experimental Analysis

In this section, we test the theoretical predictions derived from our model. In particular, we investigate whether traders use stochastically stable institutions independently of whether they are market clearing or not, i.e. independently of whether they maximize the sum of the gains from trade. On the other hand, we also check whether institutions that are not stochastically stable are abandoned in the long run.

Technically, stochastic stability entails a double limit, as time goes to infinity and as the experimentation rate vanishes. None of these limits can be reproduced in reality. Hence, it becomes especially important to test whether theoretical predictions based on stochastic stability are also relevant within reasonable time horizons and in the presence of naturally noisy human decisions.

7.1 The Experimental Design

In order to test which market institutions survive in the long run, we ran experiments where buyers and sellers had to choose between three different market institutions. The focus of the experiment was the choice of the trading platform, and not the trading behavior at a given platform. Therefore, subjects did not actually conduct trading interactions on the platforms. Rather, each subject only had to choose between the feasible platforms, and his payoff was directly determined by this choice, by his type (buyer or seller), and by the number of other buyers and sellers that opted for the same market institution. That is, the subjects played a simultaneous move game.

The buyers' demand functions and payoffs were derived from a quasilinear utility function for two goods, $u(q_0, q) = q_0 + v(q)$ with v a strictly increasing function. Specifically, for the derivation of the numerical payoffs used in the experiment, we used $v(q) = 5q - \frac{1}{2}q^2$ (for $q \in [0, 5]$). We then replaced $q_0 = w - pq$ and used $w = 1$ to obtain the valuation $v_B(q, p) = 1 - pq + 5q - \frac{1}{2}q^2$. In order to obtain payoffs in a reasonable range for the experiment, we then applied a monotonic transformation to these values.[12] The sellers' supply and payoffs were derived from the profit function $\pi(p, q) = pq - \frac{1}{8}q^2$, i.e. sellers were producers with quadratic costs.

One platform (platform A) was market clearing ($\beta = 1$), the second one (platform B) was biased with $\beta = 0.8$, and the third one biased with $\beta = 0.4$. The resulting payoff matrices are shown in Appendix B. As it can be easily checked from the payoff matrices, the sum of payoffs was maximized when all traders opted for A. But whenever the distribution of traders over the platforms was such that B was active, traders of one market side were better off at B than the traders of the same market side at any of the other platforms. So B was favored and hence stochastically stable. Examination of the payoff matrices in Appendix B shows that two mutations suffice for a successful transition away from platform C, while a significantly larger number of mutations is necessary in order to reach platform C from the states where full coordination in either of the other platforms obtains. Following standard arguments, this suffices to establish that platform C is not stochastically stable (and hence not favored).

We conducted three different treatments. In Treatment 1 (T1) subjects had to choose between platforms A and B. In Treatment 2 (T2) traders chose between A and C, and in Treatment 3 (T3) they chose between all three platforms. The theoretical model predicts that in the long run subjects will opt for both platforms in T1 and only for platform A in T2. In T3 A and B should stay active while nobody should opt for C in the long run.

Each treatment was run with 6 groups of 7 buyers and 7 sellers each. Each subject played the game for 90 times ("periods"), during which the group composition did not change. Each subject was member of only one group. In each period, subjects had to choose between the available platforms within 30 seconds. At the end of each period traders were informed about their own payoffs as well as about the distribution of the group members over the feasible platforms. The instructions (see Appendix B), avoided terms like market platform, buyer/seller, etc. Instead, it used terms like decision, type I(II), etc. The experiments were conducted at the University of Konstanz (Germany). The subjects were undergraduates of all fields except economics and psychology. A subject's overall payoff was the sum of the payoffs earned in all the 90 periods. The exchange rate between the ECU of the payoff matrices and Euro was 0.7 Euro cent. Overall, the average subject received 11.55 Euros. A session lasted about 70 minutes.

[12] The transformation was $v' = 10 + 8(\arctan(1.1(v - 9.2) - \arctan(-9.02))$. Payoffs were then rounded.

Figure 1: Evolution of the number of traders in institution A (market clearing) in T1 (averaged across six sessions). The remaining traders are at institution B (not market clearing, but also stochastically stable).

7.2 Experimental Results

First, we investigate which platforms are opted for in the long run. Then we investigate the individual decision behavior and, in particular, whether the model's assumption on the learning process are supported by the data.

Figures 1 and 2 present the results of T1 and T2, respectively. The figures plot the time evolution of the number of traders in the market-clearing institution A, averaged across the six sessions of each experiment. The remaining traders are in institution B in the case of T1, and in institution C in the case of T2. The figures show a remarkable compliance with the theoretical predictions. In T1, both institutions are stochastically stable, and in the experiment both remain active over time, with traders allocating themselves among both. In T2, only the market-clearing institution is stochastically stable, and indeed traders quickly learn to coordinate on it and avoid the other institution.

Figure 3 presents the results of T3, where all three institutions were available. Since there is no significant difference between buyers and sellers in their choice of platform, we do not present the results for buyers and seller separately, but rather plot the average total number of traders in each of the three institutions. The results are again in agreement with the theoretical

Figure 2: Evolution of the number of traders in institution A (market clearing) in T2 (averaged across six sessions). The remaining traders are at institution C (not stochastically stable).

predictions. Institution C, which is not stochastically stable, is quickly abandoned in favor of the other two, stochastically stable institutions

In each of the three treatments, at least half of the traders opt for platform A. When feasible, however (T1 and T3) platform B also remains active in the long run. But when available (T2 and T3), platform C becomes inactive during the first 15 rounds and stays empty or almost empty until the end. In summary, these observations yield:

Result 1: In the long run, traders opt for the stochastically stable platforms A and B, while platform C is avoided.

This result is not an artifact of taking the average over all groups. Rather, it can also be observed for each individual group. Table 1 presents, for all individual groups, the percentage of traders opting for the different feasible platforms during the last 30 periods. For example, in group 2 of T3 57.6% of the traders opted for platform A during the last 30 rounds, 42.4% for platform B, and 0% for platform C.

In all groups at least 50% of the traders opted for platform A during the last 30 periods. If available, at least 30% opted for platform B. But less than 5% opted for platform C, and in 10 out of 12 cases less than 1 percent of the traders opted for C when available.

Overall, this result gives a strong support for the main predictions of the theoretical model, namely that market clearing as well as other stochastically stable institutions will be used in

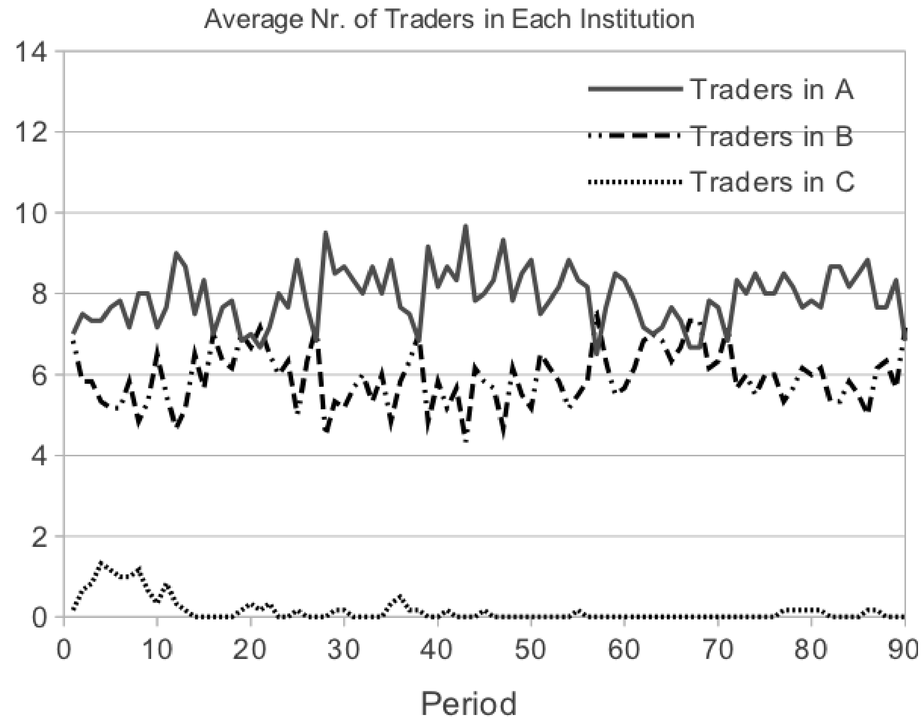

Figure 3: Evolution of the number of traders in each institution in T3 (averaged across six sessions).

the long run, while other institutions will be avoided. To further investigate the reasons for this result, we take a closer look at the individual behavior. In particular, we investigate the behavior of traders who change the platform from one period to the next ("platform switching").

In Table 2 we provide the number of switches observed for the different treatments as absolute numbers and as percentage of the number of total decisions. We also look at the number of cases where subjects switch to an institution which gave the traders of their own type the highest possible payoff in the last period, i.e. switches consistent with our model (fourth column). Since in these cases the subjects imitate the most successful last period choice, we call them *imitation switches*. Table 2 also provides the percentage of imitation switches over all switches (fifth column).

Table 2 shows that in T1 and T3 subjects switch in about 30-35% of all possible cases (the maximum possible number of switches per treatment is 7476, 89 periods times 14 subjects times 6 groups). In T2, however, subjects switch in only 6.8% of all cases. In T1 and T3 60-65% of all switches were consistent with our learning models, while in T2 the respective percentage is 52%.

Taking the group percentages of imitation switches we can test the null hypothesis that these percentages are equally likely to be strictly above 50% as weakly below 50%, i.e. that there is no

Group	1	2	3	4	5	6	Average all groups
T1-A	61.7	68.1	57.6	58.6	59.2	62.6	61.3
T1-B	38.3	31.9	42.4	41.4	40.8	37.4	38.7
T2-A	100.0	100.0	95.3	100.0	99.8	100.0	99.2
T2-C	0	0	4.7	0	0.2	0	0.8
T3-A	60.7	57.6	54.5	55.3	51.9	54	55.6
T3-B	38.1	42.4	45.5	44.5	48.1	46	44.1
T3-C	1.2	0	0	0.2	0	0	0.3

Table 1: Percentage of choices made during the last 30 periods.

Treatment	Switches	%	Imitation switches	%
T1	2315	31.0	1529	65.9
T2	512	6.8	267	52.1
T3	2587	34.6	1624	62.8

Table 2: Switches and Imitation Switches. The percentage of switches is over the total number of decisions from period 2 onwards. The percentage of imitation switches is over the total number of switches.

tendency of imitation switching. In all 6 groups of T1 the percentage of imitation switching was between 62% and 70%. In T3 these these percentages were between 59% and 65%. Therefore, a binomial test shows that for T1 and T3 the null hypothesis has to be rejected at any significance level. For T2, we find one group with 50% imitation learning, and 5 groups with percentages between 51% and 55%. In this case the null hypothesis also has to be rejected at the 5% level.

This provides evidence for imitation learning. Overall, we observe a tendency towards imitation learning in all three treatments, but this tendency is weaker in T2 than in T1 and T3. This difference is not surprising. Given the near perfect coordination on the unique stochastically stable platform A in T2, there were only very few possibilities were an imitation switch was possible at all. This implies that a smaller percentage of switches should be due to imitation, since other reasons for a switch (e.g. experimentation) should be of comparable strength across all treatments. In summary:

Result 2: Individual traders tend to switch to a platform which in the last period gave the highest payoff to traders of their own type. This tendency is stronger in T1 and T3 than in T2.

To further investigate individual behavior, the learning model must be further specified. In particular, as mentioned in Section 3.3, we hypothesized that the likelihood of a revision would depend on the observed payoff differences between the own and other institutions. Hence, we test a learning model where the revision probability is strictly increasing in the difference between the highest last-period-own-type payoff and the last-period-own payoff.

Denote by Δ the difference between the highest last-period-own-type payoff and the last-period-own payoff. For the case of two platforms, s denotes a dummy which simply takes value 1 if a switch to the other platform occurs. For T3, the definition of s is more involved, because many more possibilities exist. We define s as a variable which takes the value 1 if either last period's platform did not deliver maximal payoffs and a switch to the last-period-best among the other two platforms occurred, or last-period's platform did deliver maximal payoffs and a switch to some other platform occurred. This definition is the natural generalization of the dummy variable for the two platform case. The logic is as follows. Consider first the case where $\Delta > 0$, that is, last period's platform did not deliver the highest payoffs. In the two-platform case, the decision consistent with our basic decision rule involves a switch, i.e. $s = 1$. In the three-platform case, $s = 1$ indicates again the choice consistent with the basic decision rule, which corresponds to a switch to the appropriate platform, but not to the third one. In the case $\Delta = 0$, the decision consistent with our basic decision rule in the two-platform case is to stay, i.e. $s = 0$. In the three-platform case, $s = 0$ again indicates the choice consistent with the basic decision rule. The main difference between the dummy variables in the two- and three-platform cases is that, when $\Delta > 0$, with three platforms a value of $s = 0$ might indicate either that the agent did stay in his previous platform (which might correspond to either inertia or a mistake), or also to a switch to a "third platform", which is neither his previous one nor the one which delivered highest payoffs. Switches of the last type can obviously not occur in the two-platform case. But in T3 only 85 decisions (out of 7476) were of this type.

Since each trader has to decide 89 times whether to switch or not, we have a strongly balanced panel data set. We conduct a probit regression with random effects with s as dependent variable. The most important independent variable is Δ, and to allow for nonlinearities, we include Δ^2. We also include the period, a type dummy, and dummies for the groups. Since all group dummies are insignificant except for one group in T3, they are not reported in Table 3.

The regressions deliver the following main result.

Result 3: In all three treatments, the switching probability is strictly increasing in the difference between highest last-period-own-type payoff and the last-period-own payoff.

As can be seen from Table 3, in all three treatments the impact of Δ on s is positive and highly significant (p-values are shown in brackets below the corresponding coefficients). The negative coefficient of Δ^2 shows that the marginal impact of Δ is decreasing, but it remains positive for all feasible levels of Δ. That is, in accordance with our theoretical model, the likelihood of a switch to the last period's best platform is indeed increasing in the difference between the highest and the own payoff of the last period.

The period variable is significantly negative in all three treatments. This implies that the likelihood of a switch decreases. One could interpret this as an indication that that not only the last period experience determines the switching behavior. Rather, previous experiences also

	T1	T2	T3
Δ	.0289	.2601	.0252
	(0.000)	(0.000)	(0.000)
Δ^2	-.0004	-.0087	-.0004
	(0.006)	(0.000)	(0.000)
Period	-.0038	-.0251	-.0050
	(0.000)	(0.000)	(0.000)
Type	-.0484	-.1412	-.0276
	(0.674)	(0.222)	(0.735)
Constant	-.6442	-.9573	-.5140
	(0.000)	(0.000)	(0.001)
N. Obs.	7476	7476	7476
Log likelihood	-4239.4064	-11151.2425	-4496.9168
Chi^2	.0000	.0000	.0000

Table 3: Regression Results. Random-effects probit regressions on switches agreeing with the decision rule. Entries in brackets are p-values.

matter, and therefore in later periods the last period experience has a smaller impact than in earlier periods.

In general, T1 and T3 deliver quite similar results. The size of the coefficients of T2 differs substantially from those of T1 and T3. This indicates again clear but unsurprising differences between T2 and the other treatments. In T3, platform C is essentially disregarded by the traders from the very beginning, so the actual choice for the traders is between A and B, as in T1. In T2, of course, the choice is between A and C and convergence to complete coordination on A occurs, which is not observed in the other treatments.

8 Conclusions

We have presented a model where traders can choose among different trading institutions and asked whether they will learn to coordinate on an institution that guarantees market clearing. Under a general class of learning dynamics, we find that the market clearing institution is always stochastically stable. It is, however, not necessarily the *only* one. We also find non-market clearing institutions that are stochastically stable under general conditions, even if the market becomes large. As a result, coordination on market-clearing institutions will be often observed as the result of learning, but other institutions might also survive in the long run.

In the framework of our model, this conclusion has to be qualified. Formally, multiple stochastically stable institutions correspond to multiple absorbing *states* of the unperturbed dynamics, and not to a single absorbing *set*. This means that whenever several institutions are stochastically stable, most of the time only one institution will be observed at a time. After a time, though, experimentation will induce a short transitional period leading to coordination in a different institution. Hence, most of time we will observe only one active institution. In the long run, different institutions with different prices will be observed; this can be interpreted as reflecting endogenous changes in the institutional environment favored by traders.

Stochastic stability, however, involves a double limit as time goes to infinity and the probability of mistakes goes to zero. Neither of these limits corresponds to a realistic situation. In order to test for the relevance of our theoretical results, we conducted a laboratory experiment on platform selection based on the structure of our model. The results are remarkably in agreement with the qualitative content of the theoretical predictions. Traders quickly learn to avoid institutions which are not stochastically stable, while all stochastically stable institutions remain active. Whenever a stochastically stable institution is confronted with another institution which fails this criterion, the outcome is a sharp selection result, as predicted by the model, even though the length of the experiment was relatively short and decision errors were not influenced in any way. Whenever two stochastically stable institutions are available, the literal prediction of stochastic stability cannot be observed in the lab, since it would involve each institution being observed for an infinitely long time (as $\varepsilon \to 0$) before a switch to the other occurs. The qualitative prediction, however, is that both institutions are equally stable and no quick convergence to either one should be observed for a finite time interval and non-negligible noise level. This qualitative prediction is readily observed in the data. Hence, we can conclude that the predictions of our model are also relevant for actual decision-making. Further, the analysis of individual-level data also supports our behavioral assumptions and, in particular, the hypothesis that switching probabilities increase in the payoff difference between other institutions and the one currently used by a trader.

APPENDICES

Appendix A Proofs

Proof of Lemma 1. Suppose $\beta_z(n_z, m_z) < 1$. Then buyers, but not sellers, are rationed at institution z. We have that $q_S^0 = s(p_0)$ and $q_S^z = s(p_z)$. Suppose now that $v_S(q_S^0, p_0) \leq v_S(q_S^z, p_z)$. By **A1**, we must have that $p_0 \leq p_z$.

Then, again by **A1**, $v_B(q_B^0, p_0) \geq v_B(d(p_z), p_z)$. Since $d(p_z) > q_B^z$ (buyers are rationed), **A2** implies that $v_B(d(p_z), p_z) > v_B(q_B^z, p_z)$.

The case $\beta_z(n_z, m_z) > 1$ is analogous.

If $\beta_z(n_z, m_z) = 1$, no traders are rationed, and the analogous arguments follow with weak inequalities (**A2** does not apply since there is no rationing). ■

We now prove some auxiliary lemmata. The first shows that traders in an inactive institution will always prefer any active institution.

Lemma 2. *Suppose institution z is inactive and institution z' is active. If traded quantities are not zero at z', any trader in z or z' strictly prefers the outcome of institution z' to that of z. If traded quantities are zero at z', traders in z or z' are indifferent between the two institutions.*

Proof. Since $m_z = 0$ or $n_z = 0$, traded quantities are zero at z. Since both the bias $\beta_{z'}(n_{z'}, m_{z'})$ and the market-clearing price $p^*(n_{z'}, m_{z'})$ are strictly positive, it follows that $p_{z'}(n_{z'}, m_{z'}) > 0$. If $d\,(p_{z'}(n_{z'}, m_{z'})) > 0$, the claim then follows from assumption **A3**. If $d\,(p_{z'}(n_{z'}, m_{z'})) = 0$, the claim follows from our explicit assumption as part of **A3** that evaluations do not depend on hypothetical prices. ∎

Lemma 3. *Assume* **A1-A3***. Under* **D0-D2***, given any state ω with $n_0\,(\omega) \geq 1$ and $m_0\,(\omega) \geq 1$, there exists a finite, positive probability path of the unperturbed dynamics leading from ω to the state ω_0 with $n_0\,(\omega_0) = n$ and $m_0\,(\omega_0) = m$.*

Proof. Consider any institution $z \neq 0$, which is chosen by some traders in state ω. If $n_z\,(\omega) = 0$ or $m_z\,(\omega) = 0$, by Lemma 2 we can build a positive probability path to a new state where no trader is at institution z. Hence, without loss of generality, suppose $n_z = n_z\,(\omega) > 0$ and $m_z = m_z\,(\omega) > 0$.

If $\beta_z(n_z, m_z) \neq 1$, it follows from Lemma 1 that in state ω at least one of the two types of traders strictly prefers the market clearing institution. Let k be a trader of that type who is at the non-market clearing institution z. It might happen that k prefers a third institution to the market clearing one, but certainly will not stay in z if given revision opportunity. Further, by Assumption **D1**, there is strictly positive probability that k is the only trader of his type obtaining revision opportunity. Consider the paths where this event happens, and let k' denote a trader of the *other* type (i.e. not of the same type as k) who, in state ω, is in the same non-market clearing institution z. Consider now the event that only k and k' get revision opportunity.

If this event has positive probability, then (if it occurs) k' may or may not change institution, but k will, switching to the market clearing or another institution. If the probability of k and k' being the only revising traders is zero, by Assumption **D2** no agent of the same type as k' will revise this period, and hence k will change institution but no other agent will. In any case, the process reaches a state with strictly less traders at institution z than there were in ω, but at least the same traders in the other institutions (and, in particular, the market clearing one). If $\beta_z(n_z, m_z) = 1$, Lemma 1 yields weak preferences. The argument above applies again, because by Assumption **D0** ties are broken randomly, i.e. if a trader weakly prefers another institution to his current one, there is a maybe small but positive probability that he switches away.

Repeating this argument, we will reach a state ω' with either $n_z\,(\omega') = 0$ or $m_z\,(\omega') = 0$. From this state, all remaining traders will leave institution z as above (by Lemma 2). Hence, we reach a state where strictly less institutions are chosen than in ω.

Repeating this procedure, we will reach a state where only two institutions are chosen by traders, and one of them will necessarily be the market clearing one. Applying again the same argument (using Lemma 1) shows that we can construct a positive probability path to ω_0, where 0 is the only active institution. ■

Lemma 4. *Assume* **A1**, **A2**, *and* **A3**. *Under* **D0**, **D1** *and* **D2**,

(i) the absorbing states of the unperturbed dynamics are the "separated states" ω such that there is no active institution at all, and all monomorphic states ω_z characterized by $n_z(\omega_z) = n$ and $m_z(\omega_z) = m$, corresponding to coordination on a particular institution;

(ii) no state ω with $1 \leq n_0(\omega) \leq n-1$ and $1 \leq m_0(\omega) \leq m-1$ (i.e., where the market clearing institution is active but not all traders of any type are in it) is part of any absorbing set of the unperturbed dynamics.

Proof. The states given in (i) are obviously absorbing because, in the absence of experimentation, traders will never switch to unobserved institutions.[13] To see that there are no other absorbing states, suppose there are traders of the same type in at least two different institutions. Since necessarily one of those institutions is yielding (weakly) higher payoffs than the other, and under Assumption **D1** there is positive probability that one of the traders not in that institution is given revision opportunity, there is a positive probability transition to a different state, a contradiction. Finally, if there is only one active institution but the state is not monomorphic, the result follows from Lemma 2. Part (ii) follows immediately from Lemma 3. ■

The remaining proofs rely on the characterization of the set of stochastically stable states introduced by Kandori, Mailath, and Rob (1993) and Young (1993), and in the concept of Radius and Coradius developed by Ellison (2000). Given two absorbing sets A and B, let $c(A, B) > 0$ (referred to as the *transition cost* from A to B) denote the minimal number of mistakes in a positive probability path starting in an element of A and leading to an element in B. The following lemma contains all the results on stochastic stability that we require for the analysis. Its proof is a straightforward application of Ellison (2000, Theorems 1 and 3) and is analogous to the proof of Lemma 2 in Alós-Ferrer and Kirchsteiger (2010); hence we omit it here.

Lemma 5. *Let A be an absorbing set and define the* Radius *of A by*

$$R(A) = \min \{c(A, B) \,|B \text{ is an absorbing set, } B \neq A\}$$

and the Coradius *of A by*

$$CR(A) = \max \{c(B, A) \,|B \text{ is an absorbing set, } B \neq A\}$$

[13]Thus, separated states are absorbing because, if all institutions are inactive, no prices are observed and traders do not switch. This is inconsequential. Separated states are extremely unstable. Specifically, they are destabilized with a single mutation, in which one trader moves to an institution containing at least one trader of the other type. By Lemma 2, the outcome of the now-active institution is better for all traders than that of the inactive institution. Hence, traders at the inactive institution will switch whenever revision opportunities arise.

Then:

(i) If $R(A) \geq CR(A)$, the states in A are stochastically stable.

(ii) If $R(A) > CR(A)$, the only stochastically stable states are those in A.

(iii) If the states in an absorbing set B are stochastically stable and $R(A) = c(B, A)$, the states in A are also stochastically stable.

Proof of Theorem 1. We have to show the stochastic stability of the state ω_0. If there is any other market-clearing institution, the conclusion follows by renaming. First, notice that, by Lemma 2, no monomorphic state can be left with less than two mutations unless the traded quantity is zero. Since traded quantities at a market-clearing institution are never zero, it follows that $R(\{\omega_0\}) \geq 2$.

Consider any state in any absorbing set other than $\{\omega_0\}$. Notice that two mutations (to the market clearing institution) suffice to reach a state ω with $n_0(\omega) \geq 1$ and $m_0(\omega) \geq 1$. By Lemma 3, there is a positive probability path of the unperturbed dynamics (i.e. requiring no further mutations) leading to ω_0. This shows that $CR(\{\omega_0\}) = 2$ (the equality follows because two mutations are required to leave any other monomorphic state). The result follows from Lemma 5(i). ■

Proof of Theorem 2. Let ω_z denote the monomorphic state corresponding to coordination on institution z. We know from Theorem 1 that ω_0 is stochastically stable. By definition of a favored institution, we see that if exactly two mutations to institution z occur at state ω_0, we reach a state where at least one type of traders strictly prefer that institution. Analogously to the proof of Lemma 3 (through repeated application of Definition 3), from this state there exists a positive probability path involving no further mutations which leads to state ω_z. From the proof of Theorem 1, we already know that it is possible to make the opposite transition with exactly two mutations (but no less). Thus, we obtain that $c(\{\omega_0\}, \{\omega_z\}) = 2 = R(\{\omega_z\})$, and the result follows from Lemma 5(iii). ■

Proof of Theorem 3. For any given n, m, let

$$R(n,m) = \left\{ \frac{a}{b} \, | a = 1, ..., n, \text{ and } b = 1, ..., m \right\}$$

be the set of feasible buyers-sellers ratios and define

$$\underline{\beta}(n,m) = \max_{r \in R(n,m)} \underline{\beta}(r) \qquad \text{and} \qquad \overline{\beta}(n,m) = \min_{r \in R(n,m)} \overline{\beta}(r).$$

Note that for any given number of buyers and sellers, there exists only a finite number of values r can take. Hence, by **A4** $\underline{\beta}(n,m) < 1 < \overline{\beta}(n,m)$. For any given n, m, define $T(n,m)$ as the set of all pairs (r_0, r_z) such that $r_0 = \frac{n_0}{m_0}$ and $r_z = \frac{n_z}{m_z}$ with $n_0, n_z \in \{1, ..., n\}$ and $m_0, m_z \in \{1, ..., m\}$ such that $n_0 + n_z = n$ and $m_0 + m_z = m$. In other words, $T(n,m)$ is the set of all pairs of buyer-seller ratios which are feasible when exactly two institutions are simultaneously active. Finally, define

$$\begin{aligned} \underline{\beta}^*(n,m) &= \max\left\{\underline{\beta}(n,m), \max\left\{\frac{p(r_0)}{p(r_z)}\,|(r_0,r_z)\in T(n,m) \text{ and } r_0 < r_z\right\}\right\} \\ \overline{\beta}^*(n,m) &= \min\left\{\overline{\beta}(n,m), \min\left\{\frac{p(r_0)}{p(r_z)}\,|(r_0,r_z)\in T(n,m) \text{ and } r_0 > r_z\right\}\right\} \end{aligned}$$

Notice that $\underline{\beta}^*(n,m)$ (and analogously $\overline{\beta}^*(n,m)$) is well-defined because $T(n,m)$ is finite and $r_0 < r_z$ implies $\frac{p(r_0)}{p(r_z)} < 1$. Clearly, $\underline{\beta}(n,m) \leq \underline{\beta}^*(n,m) < 1 < \overline{\beta}^*(n,m) \leq \overline{\beta}(n,m)$.

Consider an institution F such that $\underline{\beta}^*(n,m) < \beta_F(n_z, m_z) < \overline{\beta}^*(n,m)$ for all feasible n_z, m_z. We prove that F is favored.

We want to show that, whenever $v_S\left(q_S^0, p_0\right) \geq v_S\left(q_S^F, p_F\right)$, then $v_B\left(q_B^0, p_0\right) < v_B\left(q_B^F, p_F\right)$. Let $\beta_F = \beta_F(n_z, m_z)$ be the realized bias at institution F. Suppose $\beta_F < 1$. Then buyers, but not sellers, are rationed at F. We have that $q_S^0 = s(p_0)$ and $q_S^F = s(p_F)$. Suppose now that $v_S(q_S^0, p_0) \geq v_S(q_S^F, p_F)$. By **A1**, we must have that $p_0 \geq p_F$.

Suppose $r_0 \geq r_F$. Then, $p_0 = p(r_0) \geq p(r_F)$ and, by **A1**, $v_B(q_B^0, p_0) = V_B(1, r_0) \leq V_B(1, r_F)$. By **A4**, $V_B(1, r_F) < V_B(\beta_F, r_F) = v_B\left(q_B^F, p_F\right)$ and the claim follows.

Suppose now $r_0 < r_F$. Then, $p_0 = p(r_0) < p(r_F)$. If, as assumed, $v_S\left(q_S^0, p_0\right) \geq v_S\left(q_S^F, p_F\right)$, then $p_0 \geq p_F = \beta_F \cdot p(r_F)$ by **A1**. It follows that $\beta_F \leq \frac{p(r_0)}{p(r_F)}$, a contradiction with $\beta_F > \underline{\beta}^*(n,m)$. Hence, $v_S\left(q_S^0, p_0\right) < v_S\left(q_S^F, p_F\right)$.

The case $\beta_F > 1$ is analogous. ■

The following Lemma is used in the proof of Theorem 4.

Lemma 6. *Assume* **A1** *and* **A4**. *If* $\underline{\beta}(1) < \beta_B < 1 < \beta_S < \overline{\beta}(1)$, *then*

(i) if $m \leq n$ (more buyers than sellers), in a state where an equal, strictly positive number of sellers and buyers are at an institution z with constant $\beta_z = \beta_B$ and the remaining traders are at a market-clearing institution (and there are traders of both types in the latter), buyers strictly prefer z;

(ii) if $m \geq n$ (more sellers than buyers), in a state where an equal, strictly positive number of sellers and buyers are at an institution z with constant $\beta_z = \beta_S$ and the remaining traders are at a market-clearing institution (and there are traders of both types in the latter), sellers strictly prefer z.

Proof. We will show part (i). Part (ii) is analogous. Let $0 < \ell < \max(m,n)$ be the number of sellers and buyers at the alternative institution z. Since $m \leq n$, we have that $m - \ell \leq n - \ell$ and hence

$$r = \frac{n-\ell}{m-\ell} \geq 1$$

i.e. there are (weakly) more buyers than sellers at the market clearing institution. By **A4**, since $\underline{\beta}(1) < \beta_B < 1$,

$$V_B(\beta_B, 1) > V_B(1, 1)$$

and, by **A1**

$$V_B(1,1) = v_B\left(q_B^z(1,1), p(1)\right) \geq v_B\left(d(p(r)), p(r)\right)$$

because $q_B^z(1,1) = d(p(1))$ and $p(1) \leq p(r)$ since $r \geq 1$ and p is increasing in r. Hence,

$$V_B(\beta_B, 1) > v_B\left(d(p(r)), p(r)\right)$$

which proves the claim, because $v_B\left(d(p(r)), p(r)\right)$ is the buyers' payoff at the market clearing institution, and $V_B(\beta_B, 1)$ is the payoff of the buyers at the non-market clearing institution with $\beta_z = \beta_B$. ■

Proof of Theorem 4. We will show part (i). Part (ii) is analogous. Let ω_1 denote the monomorphic state corresponding to coordination on the buyers' institution z. By hypothesis, z is a favored institution for the economy with $K = 1$. Further, we know from Theorem 1 that ω_0 is stochastically stable. In order to show stochastic stability of ω_1 for large K, by Lemma 5(iii) it is enough to show that two mutations at ω_0 from the market clearing institution 0 to z suffice for a transition.

Let a buyer and a seller mutate from 0 to z. Then $r_z = 1$ and $r_0 = \frac{Kn-1}{Km-1} \geq 1$. By Lemma 6(i), the mutant buyer is strictly better off. By $\mathbf{D3}_K$, let K buyers revise, including the mutant, and follow him to z (so exactly $K-1$ buyers switch). Now $r_z = \frac{K}{1}$ and $r_0 = \frac{(n-1)K}{Km-1}$.

Since $\lim_{r\to\infty} p(r) = +\infty$ under **M1'-M2'**, $r_z \to \infty$ as $K \to \infty$, and $r_0 \to \frac{n-1}{m}$ (finite), even though $\beta_z < 1$ it follows that there exists K^* such that, for all $K \geq K^*$, $p_z = \beta_z p^*(r_z) > p^*(r_0) = p_0$.

Sellers are not rationed at 0 (by definition) and they are not rationed at z either since $\beta_z < 1$. Hence by **A1** sellers are strictly better off at z since they face a higher price. By $\mathbf{D3}_K$, there is positive probability that K sellers, including the lone seller already in z, revise and move to z from 0, while no other trader receives revision opportunity. In the new state, we have $r_z = \frac{K}{K} = 1$ and $r_0 = \frac{(n-1)K}{(m-1)K} = \frac{(n-1)}{(m-1)}$. By Lemma 6(i), $V_B(\beta_z, r_z) > V_B(1, r_0)$ and buyers at z are strictly better off. By $\mathbf{D3}_K$, there is positive probability that K buyers from 0 revise and follow them to z.

We know that the market institution z is favored for the economy with $K = 1$. That means that one market side is better off at z for all prices resulting from population proportions *which are feasible in the economy with* $K = 1$ (recall the construction of the set $T(m, n)$ in the proof of

Theorem 3). In the state we have just reached, we have $r_z = \frac{2K}{K} = 2$ and $r_0 = \frac{(n-2)K}{(m-1)K} = \frac{n-2}{m-1}$, which are feasible population proportions in the economy with $K = 1$.

We conclude that one market side is better off at z. Let K traders of the appropriate market side switch (using $\mathbf{D3}_K$). The new population distribution is always a multiple of K for each trader type and each institution, hence we can apply the fact that z is favored in the economy with $K = 1$ again. Proceeding iteratively, eventually we reach a state where a complete market side is at institution z. By **A3**, we can complete the transition by moving groups of K traders of the other market side to z until the market-clearing institution becomes empty. ■

Appendix B Experimental Instructions and Payoff Matrices

The instructions and the control questionnaire below are translated from German into English as literally as possible. These instructions were distributed to type I traders (i.e. sellers) of treatment T3 (choice between 3 platforms). The instructions for type II traders (buyers) were symmetric (of course with the appropriate payoffs in the examples). The instructions for T1 and T2 were similar, with the only difference that all the references to choice C were deleted.

Each participant was provided with the appropriate payoff tables for the institutions used in the experiment. Table 4 displays the payoffs obtained by buyers and sellers at each of the three institutions used in the experiments. Within each table, each row corresponds to the number n of buyers present at the institution, each column to the number m of sellers.

Instructions - Type I

The experiment you are about to participate in is part of a research project on decision behavior. The instructions are simple, and if you read them carefully and make appropriate decisions, you can earn a considerable amount of money.

The revenues made during the experiment are counted in ECU ("experimental currency units"). After the end of the experiment all the revenues you made during the experiment will be added up and paid to you in cash. For every ECU you will receive 0.7 Eurocent.

Any communication between the participants is strictly forbidden.

In every round of the experiment you have to choose between three different options, A, B, or C. To do so you click on the appropriate button for decision A, B, or C. Then you confirm your decision by clicking on the OK button. In each round you have half a minute to make this decision.

In this experiment there are two types of participants, participants of Type I and participants of Type II. You are of Type I. All in all, there are 7 participants of Type I and 7 participants of Type II within the group relevant for you. You will not be informed about the identity of the other group members, and the other group members will not be informed about your identity.

The revenues you make in a round depend on the number of other group members of each type who make the same decision as you. Assume that you have made decision A, three other

Type I participants have made the same decision, and 5 Type II participants have chosen A, too. In this case 4 Type I participants and 5 Type II participants have chosen A. As you can see from the attached revenue matrix, your revenues are 21 ECU in this case.

Another example: You have chosen B, one other Type I participant has made the same decision, and 3 Type II participants have chosen B, too. In this case your revenues are 20 ECU.

After all members of your group have made a decision, you will be informed about the number of participants of each type that have chosen A, B, and C. You will also be informed about their revenues, and about the sum of revenues you have earned so far in the whole experiment.

After that, a new round will start, in which you will have to decide between A, B, and C, again. Overall there will be 90 rounds.

Control Questionnaire.

1. Suppose that you have made decision B, 2 other Type I participants have made the same decision, and 2 Type II participants have also chosen B. What are your revenues?

   Suppose that 2 Type I participants and 3 Type II participants have chosen A. What are the revenues of those Type I participants who have chosen A?

   Suppose the remaining 2 Type I participants and the remaining 2 Type II participants have chosen C. What are the revenues of those Type I participants who have chosen C?

2. Suppose that you have made decision C, 3 other Type I participants have made the same decision, and 2 Type II participants have also chosen C. What are your revenues?

   Suppose that 1 Type I participant and 2 Type II participants have chosen B. What are the revenues of the Type I participant who has chosen B?

   Suppose the remaining 2 Type I participants and the remaining 3 Type II participants have chosen A. What are the revenues of those Type I participants who have chosen A?

Institution A: Market Clearing.

Buyers									Sellers							
$n\backslash m$	0	1	2	3	4	5	6	7	$n\backslash m$	1	2	3	4	5	6	7
									0	10	10	10	10	10	10	10
1	10	20	30	31	32	32	32	32	1	18	12	11	11	10	10	10
2	10	12	20	28	30	31	31	32	2	32	18	14	12	12	11	11
3	10	11	13	20	27	29	30	31	3	47	25	18	15	13	12	12
4	10	11	12	14	20	26	28	30	4	60	32	23	18	16	14	13
5	10	10	11	13	15	20	25	27	5	72	40	27	21	18	16	15
6	10	10	11	12	13	16	20	24	6	82	47	32	25	21	18	16
7	10	10	11	11	12	14	16	20	7	91	54	37	29	23	20	18

Institution B: Stochastically Stable but not Market Clearing ($\beta = 0.8$)

Buyers									Sellers							
$n\backslash m$	0	1	2	3	4	5	6	7	$n\backslash m$	1	2	3	4	5	6	7
									0	10	10	10	10	10	10	10
1	10	23	30	31	32	32	32	32	1	15	12	11	10	10	10	10
2	10	13	23	29	30	31	31	31	2	24	15	13	12	11	11	11
3	10	11	15	23	28	29	30	31	3	34	20	15	13	12	12	11
4	10	11	13	16	23	27	29	30	4	42	24	18	15	14	13	12
5	10	11	12	14	18	23	26	28	5	50	29	21	17	15	14	13
6	10	10	11	13	15	18	23	26	6	56	34	24	20	17	15	14
7	10	10	11	12	13	16	19	23	7	62	38	27	22	19	17	15

Institution C: Not Stochastically Stable ($\beta = 0.4$)

Buyers									Sellers							
$n\backslash m$	0	1	2	3	4	5	6	7	$n\backslash m$	1	2	3	4	5	6	7
									0	10	10	10	10	10	10	10
1	10	12	14	15	16	17	17	17	1	11	10	10	10	10	10	10
2	10	11	12	13	14	15	15	16	2	14	11	11	10	10	10	10
3	10	11	12	12	13	14	14	14	3	16	12	11	11	11	10	10
4	10	11	11	12	12	13	13	14	4	18	14	12	11	11	11	11
5	10	11	11	11	12	13	13	13	5	20	15	13	12	11	11	11
6	10	10	11	11	12	12	12	13	6	22	16	14	12	12	11	11
7	10	10	11	11	11	12	12	12	7	23	17	14	13	12	12	11

Table 4: Experimental Payoff Tables.

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