Lecture 10. Dynamic Games of Incomplete Information

1. Extensive form games with incomplete information

An extensive form game with incomplete information is like an extensive form game with complete information, but

i) at very the beginning of the game an additional player, called "nature" or "0", "chooses" between different actions according to an *exogenous* probability distribution. The set of options for player 0 is denoted by A_0, and $\theta(a_o)$ is the exogenous probability that nature "chooses" action a_o.

ii) the payoffs at the endnodes do not only depend on the strategy combination chosen by the real players, but also on the move of nature. $e_i(s, a_o)$ denotes $i's$ payoff when the players choose combination s and nature chooses a_o.

Note: Like in the game with complete information, a strategy of a player describes an action for the player at each information set the player controls.

Example: Market for lemons (Akerlof 1967)

Player 1 has a used car, which he might try to sell to a potential buyer, player 2.

The car can be of good quality (g) or of bad quality (b). 1 knows the true quality of the car. 2 knows only that the car is of high quality with probability θ.

For both players, the value of the car depends on the quality. Denote by v_1^g, v_1^b seller's valuation of the car (with $v_1^g > v_1^b$), and by v_2^g, v_2^b buyer's valuation of the car (with $v_2^g > v_2^b$). If the quality would be known, mutually beneficial trade would be possible for both types of cars: $v_2^g > v_1^g, \quad v_2^b > v_1^b$.

After nature has chosen the quality, player 1 (knowing the quality) might demand a high price p_h or a low price p_l, or just not offer the car at all.

The possible prices are such that the high price the seller wants to sell both types of cars, but the buyer wants to acquire only the good car ($p_h > v_1^g$, $v_2^g > p_h > v_2^b$). At the low price the buyer wants to acquire both types of cars, but the seller wants to sell only the bad car ($p_l < v_2^b$, $v_1^g > p_l > v_1^b$).

Denote by p_h^g when the seller demands the high price for the good car, by p_h^b when the seller demands the high price for the bad car, by p_l^g when the seller demands the low price for the good car, by p_l^b when the seller demands the low price for the bad car.

The seller might also decide not to offer the car at all. Denote by no^g when he does not offer the good car and no^b if he does not offer the bad car.

Player 2, knowing the demanded price, but not the quality, decides whether to accept the demanded price or not. Denote by y_h when the buyer accepts the high price, by y_l when the buyer accepts the low price, by n_h when the buyer rejects the high price, and by n_l when the buyer rejects the low price.

In this game, the strategies spaces are given by

Player 1 :

$$\begin{aligned} S_1 &= \{(p_h^g, p_h^b), (p_h^g, p_l^b), (p_l^g, p_h^b), (p_l^g, p_l^b), (no^g, p_h^b), \\ &\quad (no^g, p_l^b), (p_h^g, no^b), (p_l^g, no^b), (no^g, no^b)\} \end{aligned}$$

Player 2 :

$$S_2 = \{(y_h, y_l), (n_h, y_l), (y_h, n_l), (n_h, n_l)\}$$

Note (check by applying SPE): If the game would be of complete information, player 2 would accept both prices for the good car, and reject the high price for the bad car. Player 1 would demand the high price for the good and the low price for the bad car, Hence, in case of complete information, both types of cars would be sold.

2. Normal form representation

Like the extensive form game with complete information, also the extensive form game of incomplete information can be represented by a normal form game. The strategy sets are as previously defined. The payoffs of the representative normal form game is given by

$$u_i(s) = \sum_{a_0 \in A_0} \theta(a_0).e_i(s, a_o).$$

Definition: The Nash equilibria of an extensive form game with incomplete information are the Nash equilibria of the normal form representation of the extensive form game.

Lemon game

The outcome of the complete information game (player 1 demands high price for good and low price for bad quality, player 2 accepts both demands) is no longer an equilibrium outcome. Given that player 2 accepts a high price, player 1 has an incentive to demand for both types of cars a high price in order to pretend that the car is good, even if it is actually bad.

If $\theta(v_2^g - p_h) + (1-\theta)(v_2^b - p_h) < 0$, then there exists no equilibrium where the good car is sold. If player 2 accepts a high price, player 1 has an incentive to demand a high price even for the bad car, which in turn gives player 2 an incentive to turn down the high price. If player 2 does not accept the high price, player 1 is always better off if he keeps the good car.

$\{(no^g, p_l^b), (n_h, y_l)\}$ is the only pure strategy NE - in equilibrium the good car is not even offered. Hence, only bad cars are traded.

This equilibrium outcome is inefficient - mutually beneficial trade is in principle also possible for a good used car $(v_2^g > p_h > v_1^g)$, but not conducted due to informational problems.

Lemon market is a typical example of a particular type of informational problem, called "adverse selection":

Take a situation where one market side has systematically better information about some payoff relevant feature, and assume that this feature is not chosen by the players, but by nature. Then a systematic selection bias can result, which leads to inefficiencies.

E.g. insurance market, where clients typically have better information about their risk then insurance companies. Can explain underinsurance, when insurance is voluntary, and hence justify mandatory insurance (like health insurance in most European countries).