Lecture 12. Further Applications and the Sequential Equilibrium

1. Application - Cheap-Talk Games

Special version of signaling games

signal has no impact on payoffs, messages are pure cheap talk

Sender: "I am of type x"

If all types of senders have same preferences over receiver's actions: Cheap-talk not credible.

Example: Signalling of talent: all types of potential employees want job that requires talent $\Longrightarrow$ every type will claim that he is highly talented $\Longrightarrow$ costly signal (like university education) necessary for signal to be credible.

If sender's and receiver's preferences are completely opposed, cheap talk is never credible, too.

Example: Two types of sender, t_l and t_h, and two actions of receiver, a_l and a_h.

payoffs

$$
\begin{aligned}
u_S(t_l, a_l) &> u_S(t_l, a_h);\ u_S(t_h, a_h) > u_S(t_h, a_l) \\
u_R(t_l, a_l) &< u_R(t_l, a_h);\ u_R(t_h, a_h) < u_R(t_h, a_l)
\end{aligned}
$$

Cheap talk can make difference, if not all types of sender have same preferences about receiver's actions, and if preferences of sender and receivers are not completely opposed.

General structure of the cheap-talk game

Nature draws a type t_i for the sender from a set of feasible types T according to probability distribution $q(t_i)$.

Sender observes t_i, and then decides to send a signal m_j sends a signal from a set of feasible signals M.

Receiver observes m_j (but not t_i), and then chooses an action a_k from his action space A.

Difference to signalling game - **payoffs do not depend on messages:** $u_S(t_i, a_k)$, $u_R(t_i, a_k)$.

Result: In a cheap talk game, a pooling equilibrium, where all types send the same message, always exists.

Sketch of the proof: Assume that for all messsages R does not change his beliefs. Given this belief of R, sending the same message m_j is optimal for all types of senders, since the message does not directly influence the sender's payoff.

On the other hand, not changing beliefs fulfills the requirements of perfect Bayesian equilibrium. For m_j, which is the only message observed in equilibrium, the holding the a-priori belief is consistent with Bayesian updating. For out of equilibrium messages Bayesian updating allows for everything (and hence also for letting beliefs unchanged).

Example: 2 types, 2 actions

nature chooses whether sender is type h or type l; probability of l is denoted by q, $0 < q < 0.5$.

sender decides whether to send message m_l or m_h.

receiver observes message, and decides whether to to choose a_h or a_l.

payoffs:

$$\begin{aligned} u_S(t_l, a_l) &= x;\ u_S(t_l, a_h) = z;\ u_S(t_h, a_h) = w;\ u_S(t_h, a_l) = y \\ u_R(t_l, a_l) &= 1;\ u_R(t_l, a_h) = 0;\ u_R(t_h, a_h) = 1;\ u_R(t_h, a_l) = 0 \end{aligned}$$

Note: Receiver prefers action a_l when sender is t_l, and a_h when sender is t_h.

Case 1: $x > z$ and $y > w$.

Both types of senders prefer a_l over a_h. Assume that receiver's choice depends on message, e.g. he chooses a_h after message m_h and a_l after message m_l. In order that this behavior is optimal, reciever's beliefs have to fulfill the following conditions

$$\begin{aligned} prob(t_l \,|m_l\,) &\geq 0.5 \\ prob(t_h \,|m_h\,) &\geq 0.5. \end{aligned}$$

On the other hand, with such a behavior of the receiver both types of senders will send message m_l. But then Bayesian updating requires

$$prob(t_l \,|m_l\,) = q < 0.5,$$

a contradiction. Hence, it cannot be that receiver's action depends on message - in equilibrium receiver ignores messages

Case 2: $z > x$ and $y > w$

Preferences of sender and receiver are completely opposed. Assume again that receiver's choice depends on message, e.g. he chooses a_h after message m_h and a_l after message m_l. In order that this behavior is optimal, reciever's beliefs have to fulfill the following conditions

$$\begin{aligned} prob(t_l \,|m_l\,) &\geq 0.5 \\ prob(t_h \,|m_h\,) &\geq 0.5. \end{aligned}$$

On the other hand, with such a behavior of the receiver type l will send message m_h and type h will send message m_l. But then Bayesian updating requires

$$\begin{aligned} prob(t_l \,|m_l\,) &= 0 \\ prob(t_h \,|m_h\,) &= 0, \end{aligned}$$

a contradiction. Hence, it cannot be that receiver's action depends on message - in equilibrium receiver ignores messages.

Case 3: $x \geq z$ and $y \leq w$

Preferences of both player are completly aligned: For type l both players want action a_l, and for type h both player want a_h.

Take the following strategy combination and beliefs:

Sender's strategy	: type	t_l sends m_l, type t_h m_h.
Receiver's strategy	: action	a_h after m_h and a_l after m_l.
Beliefs	:	$prob(t_l \| m_l) = 1$, and $prob(t_h \| m_h) = 1$

Obviously, beliefs fulfill Bayesian updating

Given these beliefs, receiver's strategy is optimal, and given receiver's strategy, sender's strategy is optimal, too $\Longrightarrow$ perfect Bayesian equilibrium

For continuous message and strategy spaces: As long as preferences are not perfectly aligned, partial pooling equilibrium exists.

2. Sequential equilibrium

Equilibrium concept for any type of extensive form game (with or without complete information): Sequential equilibrium

E is any finite extensive form game (possibly with incomplete information)

Each player i is endowed with set S_i of pure strategies (recall definition of strategy in extensive form games).

q_i: a mixed strategy of i.

$\widehat{q}_i$ denotes a completely mixed strategy of i, and $\widehat{q}$ a profile of completely mixed strategies.

p_i^I : belief of player i for each information set I: a probability distribution over the nodes belonging to the informtion set.

p : belief profile: for each information set pobability distribution over the nodes.

Note: A completely mixed strategy profile $\widehat{q}$ and Bayes rule induce for any information set a unique probability distribution over the nodes belonging to the information set. Denote this induced distribution by $\widehat{p}(\widehat{q})$.

Definition: An assessment consisting of a strategy profile and a belief belief, (q, p), is consistent, if there exists a sequence of profiles of completely mixed strategies, $\widehat{q}^k$, $k = 1, 2, ...$, such that

$$\begin{aligned} \lim_{k \to \infty} \widehat{q}^k &= q \\ \lim_{k \to \infty} \widehat{p}(\widehat{q}^k) &= p. \end{aligned}$$

Definition: An assessment (q, p) is sequentially rational, if for every information set I controlled by player i it holds that given the belief profile p and the strategies of the other players q_{-i} player i cannot increase his expected payoff by deviating from q_i.

Definition: An assessment (q^*, p^*) is a sequential equilibrium if it is consistent and sequentially rational.

Theorem: For any finite extensive form game there exists a sequential equilibrium (q^*, p^*). Furthermore, q^* is a subgame perfect and (q^*, p^*) a perfect Bayesian equilibrium (but not necessarely the other way round).