Lecture 2. Static Games of Complete Information

1. Normal Form Games

Static strategic situation described by

n players

each player i chooses simultaneously a strategy s_i from his set of feasible strategies S_i (strategy space).

strategy profile $s = (s_1, s_2, ...s_n) \in S = \prod_{i=1}^{n} S_i$

each player i is endowed with a payoff function

$$u_i : S \to \mathbb{R}$$

with $u_i(s_1, s_2...s_n)$ denoting $i's$ payoff if the players choose profile $(s_1, s_2...s_n) \Longrightarrow$

each player gets a payoff u_i that depends on the choices of all players.

Definition: The normal-form representation of an n-player game specifies the players' strategy spaces $S_1, S_2, ..S_n$ and their payoff functions $u_1, u_2, ...u_n$. This game is denoted by $G = \{S_1, S_2, ..S_n; u_1, u_2, ...u_n\}$.

Example: cooperation game (prisoner's dilemma)

	C_2		NC_2	
C_1	2	2	0	3
NC_1	3	0	1	1

Notation:

$s_{-i} = (s_1, s_2..s_{i-1}, s_{i+1}, ...s_n)$ specifies a strategy for all players but i

S_{-i}: set of all strategy profiles for all players but $i. s_{-i} \in S_{-i}$.

$u_i(s_i, s_{-i})$: $i's$ payoff if he chooses s_i and the others s_{-i}.

2. Iterated Elimination of Strictly Dominated Strategy

Strictly dominated strategy: If a specific strategy s_i gives player i a strictly worse outcome than another strategy $\overline{s_i}$ regardless of what the other players are choosing, then i should never choose s_i.

Definition: In the normal-form game G the strategy $s_i \in S_i$ is strictly dominated by $\overline{s_i} \in S_i$, if it holds:

$$u_i(s_i, s_{-i}) < u_i(\overline{s_i}, s_{-i}) \text{ for all } s_{-i} \in S_{-i}$$

Iterated elimination of strictly dominated strategy

Step 1) Since the other players are rational and since they know that i is rational, the other players should not take a strictly dominated s_i into account when thinking about their strategy choice $\Longrightarrow$
nobody takes s_i into account, and hence it can be eliminated from the game. This holds for any player $\Longrightarrow$
In a first round all strictly dominated strategies of all players are eliminated from the game, resulting in a reduced game.

Step 2) In the reduced game where all strictly dominated strategies of the original game are eliminated, some players may suddenly find that previously undominated strategies are now dominated. Eliminate now these strategies form the game, look whether this leads to new strictly dominated strategies, etc.

Examples: cooperation game (prisoners' dilemma)

	C_2		NC_2	
C_1	2	2	0	3
NC_1	3	0	1	1

OPEC game:

	$KQ1$		$KQ2$	
$SAQ4$	64	16	48	24
$SAQ5$	60	12	40	16

G p. 6

	L		M		R	
U	1	0	1	2	0	1
D	0	3	0	1	2	0

Problem with this procedure: In most games, there remain many uneliminated strategies $\Longrightarrow$ imprecise predictions

Example

	L		M		R	
U	0	4	4	0	5	3
C	4	0	0	4	5	3
B	3	5	3	5	6	6

3. Nash equilibrium

Idea: Find a strategy combination such that no player has an incentive to unilaterally deviate from this strategy combination - this strategy combination is self-enforcing.

Definition: In the normal-form game G the strategy combination $s^* = (s_1^*, s_2^*, ..s_n^*) \in S$ is a Nash equilibrium (NE), if for all players i it holds:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \text{ for all } s_i \in S_i$$

Note: If a strategy combination s is not a NE, then at least one player wants to deviate from this strategy combination.

Examples: G p. 7

	L		M		R	
U	0	4	4	0	5	3
C	4	0	0	4	5	3
B	3	5	3	5	6	6

cooperation game

	C_2		NC_2	
C_1	2	2	0	3
NC_1	3	0	1	1

battle of the sexes

	FB		FO	
MB	2	1	0	0
MO	0	0	1	2

Note: NE-outcome can, but need not be paretoefficient.
NE can, but need not be unique.

4. Relation between NE and strictly dominated strategies

Proposition: In the normal-form game G, any strategy s_i^* that is part of a NE $s^* = (s_1^*, s_2^*, ..s_n^*)$ survives iterated elimination of strictly dominated strategies.

Proposition: In the normal-form game G, if for each player only one strategy s_i^* survives iterated elimination of strictly dominated strategies, then the strategy combination $s^* = (s_1^*, s_2^*, ..s_n^*)$ is the only Nash equilibrium of the game.