

Lecture 9. General Dynamic Games of Complete Information

Till now: Simple dynamic games and repeated games

Now: General dynamic games but with complete information (for dynamic games with incomplete information see lecture 10)

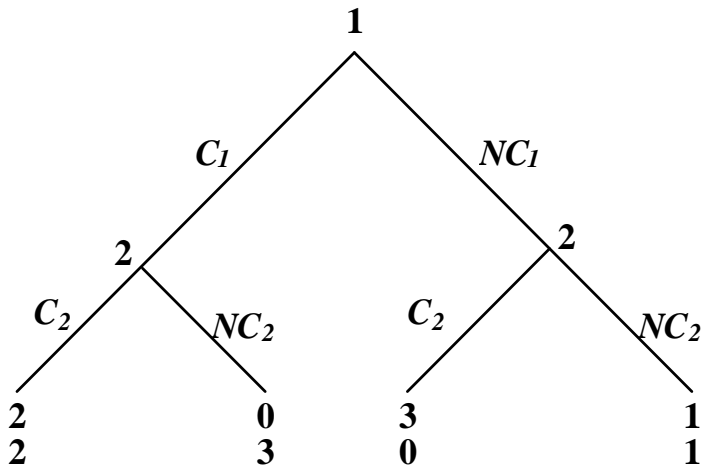
1. Extensive form games

Definition: The extensive form representation of a game specifies:

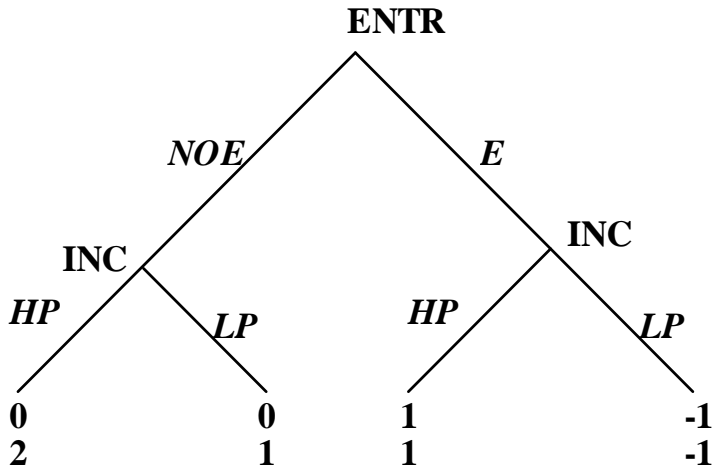
- i) the players of the game
- ii) when each player has to move ("which decision nodes are controlled by which player")
- iii) what each player can do when he has to move ("the set of feasible actions in each node")
- iv) what each player knows when he has to move ("information sets")
- v) the payoff received by each player for each combination of moves that could be chosen by each player ("payoffs at the endnodes").

Extensive form game typically drawn as "game tree"

Sequential cooperation game



Price war: First potential entrant decides whether to enter a market or not. After being informed about the entrant's decision, the incumbent decides about whether to set low or high prices.



Definition: A strategy for a player is a complete plan of action - it specifies a **feasible action** for the player **in every contingency** in which the player have to move.

Sequential cooperation game

Strategy of player 1: e.g. C_1

Strategy of player 2: e.g. (C_2, NC_2)

Strategy space of player 1: $\{C_1, NC_1\}$

Strategy space of player 2: $\{(C_2, C_2), (C_2, NC_2), (NC_2, C_2), (NC_2, NC_2)\}$

Price war:

Strategy of entrant: e.g. NOE

Strategy of incumbent: e.g. (HP, LP)

Strategy space of player 1: $\{E, NOE\}$

Strategy space of player 2: $\{(HP, HP), (HP, LP), (LP, HP), (LP, LP)\}$

Transformation of the extensive form game into a normal form game:
 Since a player's strategy describes his action at all possible contingencies, he might choose his strategy in advance \Rightarrow equivalent to normal form game with same strategy space and appropriate payoff-functions:
 appropriate definition of strategy in the extensive form game allow to transform the dynamic extensive form game into a static normal form game.

Sequential cooperation game

	(C_2, C_2)	(C_2, NC_2)	(NC_2, C_2)	(NC_2, NC_2)
C_1	2 2	2 2	0 3	0 3
NC_1	3 0	1 1	3 0	1 1

Price war game

	(HP, HP)	(HP, LP)	(LP, HP)	(LP, LP)
NOE	0 2	0 2	0 1	0 1
E	1 1	-1 -1	1 1	-1 -1

Definition: A Nash equilibrium of an extensive form game is a Nash equilibrium of the equivalent normal form game.

Unique NE of the sequential cooperation game: player 2 chooses strategy (NC_2, NC_2) , player 1 chooses strategy NC_1 .

Three NEs of the price war game:

$\{E, (HP, HP)\}, \{E, (LP, HP)\}, \{NOE, (HP, LP)\}$. Third of these equilibria incorporates threat of price war.

Till now: Every player knows all the decisions already made by other players: Games with perfect information

Now: Games with imperfect information \Rightarrow information sets.

Definition: An information set for a player is a collection of decision nodes satisfying:

- i) the same player has to move at every decision node in the information set.
- ii) when the play of the game reaches a node in the information set, the player who has to move does not know which node in his information set has (or has not) been reached.

Part ii) implies that the moving player has the same number of feasible actions at each decision node in an information set.

Definition: In an extensive form game a strategy of player i determines a feasible action at every information set player i controls.

Simultaneous cooperation game

Strategy space of player 1: $\{C_1, NC_1\}$

Strategy space of player 2: $\{C_2, NC_2\}$

Note: Sequential versus simultaneous determined by also information, not only by actual timing.

4-players game

Strategy space of player 1: $\{L_1, R_1\}$

Strategy space of player 2: $\{L_2l_2, L_2r_2, R_2l_2, R_2r_2\}$

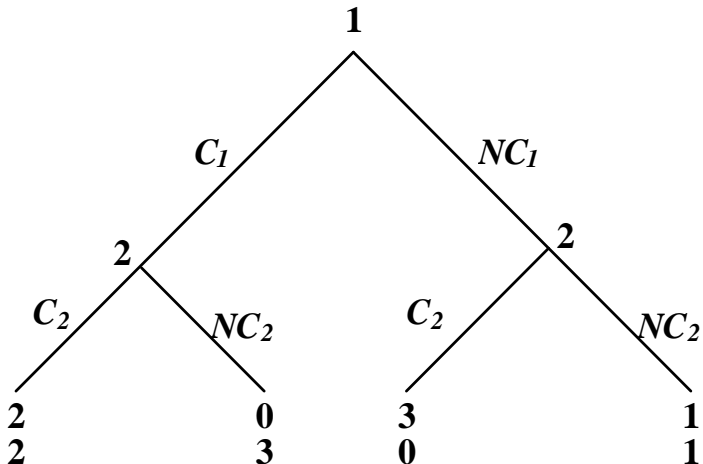
Strategy space of player 3: $\{L_3, R_3\}$

Strategy space of player 4: $\{l_4, r_4\}$

2. Subgame-Perfect Nash Equilibrium (*SPE*)

Definition: A subgame in an extensive form game

- i) begins at a decision node n that is a singleton information set.
- ii) includes all the decision nodes and terminal nodes that follow n in the game tree
- iii) does not cut any information sets - if a decision node n' follows n , then all decision nodes which are in the same information set as n' must also follow n .

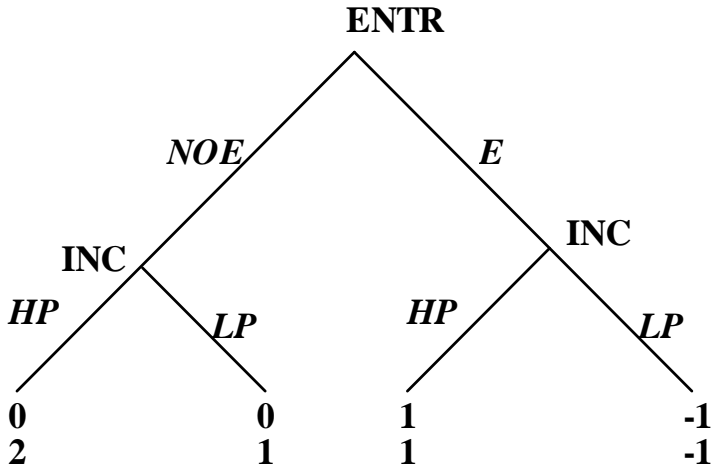


subgames of sequential cooperation game:

whole game

part of game starting with left hand node of the nodes controlled by 2

part of game starting with right hand node of the nodes controlled by 2



subgames of price war game

whole game

part of game starting with left hand node controlled by incumbent

part of game starting with right hand node controlled by incumbent

subgames of simultaneous cooperation game

whole game

subgames of 4-players game

whole game

part of game starting with node controlled by 4

Definition: A Nash-equilibrium is subgame perfect, if the players' strategies constitute a Nash-equilibrium in every subgame.

SPE of sequential cooperation game = unique *NE* of the game:

player 2 chooses strategy (NC_2, NC_2)

player 1 chooses strategy NC_1

SPE of the price war:

incumbant chooses (HP, HP)

entrant chooses E

Note: Two *NEs* are not *SPEs* - threat of price war not credible.

In general: Any *SPE* is a *NE*, but not necessarily the other way round.

SPEs of 4-players game:

SPE1: $L, (L', r'), R'', r''$

SPE2: $R, (R', r'), R'', r''$

Note: SPE needs not be unique