On the Optimal Majority Rule

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Abstract

We develop a simple model that accounts for the widely spread intuition that as committees get large, (well chosen) majority rules are preferable to unanimity. The model is one of collective search in which members do not control the proposal put to a vote. The main drawback of unanimity is that it makes it too difficult to find a proposal acceptable by all, which in turn induces extra costly delays in comparison with less demanding majority rules. The best majority rule is the one that best solves the trade-off between speeding up the decision process and avoiding the risk of adopting too inefficient proposals.

1 Introduction

It is widely accepted that when a committee is large, unanimity is a source of inaction and immobility.¹ This is, in essence, what led the EU to adopt

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¹This view is expressed in various ways in a number of classic writings. For example, Black (1958, page 99) writes:

"The larger the size of majority needed to arrive at a new decision on a topic, the smaller will be the likelihood of the committee reaching a decision that alters the existing state of affairs."
the Lisbon treaty in recent years. As the EU has grown larger, it has become clear that maintaining the requirement that decisions should be approved by unanimity would lead to much inaction, and the Lisbon treaty was precisely proposed to correct this deficiency (by lowering the majority requirement for a number of decisions).

Despite the wide acceptance of such a claim, the bargaining literature has difficulties providing a rationale for it. For example, following the legislative bargaining literature (pioneered by Baron and Ferejohn (1989)) and viewing committees as bargaining over the division of a pie of fixed size, unanimity is found to be no worse than any other majority rule (in fact all majority rules are welfare equivalent).\(^2\) Even more striking: if one extends the basic Baron-Ferejohn’s setup to allow for the size of the pie to evolve according to a stochastic process, then the unanimity rule provides a welfare efficient outcome (see Merlo and Wilson (1998)) and other majority rules typically lead to welfare inferior outcomes (see Eraslan and Merlo (2002)).

We revisit this question by applying a collective search framework recently developed in Compte and Jehiel (2004-2010).\(^3\) In the collective search model, the members of the committee do not control the proposals put to a vote. Their strategic decision consists in voting on whether they are in favor of the proposal or whether they prefer waiting for a better alternative. If the proposal put to a vote receives the support of the required majority, it is implemented. Otherwise, the search process continues until a proposal is adopted.\(^4\)

\(^2\)This is because an agreement is reached immediately in all cases.
\(^3\)See Sakaguchi (1973), Kurano et al. (1980), Yasuda et al. (1982) and Ferguson (2005) for precursors of the collective search model developed in Operations Research and that focus on issues of existence and uniqueness of equilibrium (as opposed to welfare and other economic related properties). See also Albrecht et al. (2010) for another recent paper making use of this model, which is further discussed below.
\(^4\)This paper emphasizes the collective search perspective by assuming that committee
Observe that the collective search model is flexible enough to accommodate the idea that the various possible proposals may correspond to different aggregate payoffs or welfare (the sum of utilities of the various committee members or the size of the pie in Eraslan and Merlo’s framework) so that it can accommodate the idea that the size of the pie may be stochastic (as in Merlo and Wilson’s work). A key difference between collective bargaining and collective search though is that under the collective bargaining approach all possible partitions of the pie are simultaneously available, while under the collective search approach, a proposal at a given date determines both the size of the pie and how the pie would be divided among the committee members: some other proposals for partitioning the pie will eventually arise, but only through later draws. With patient agents however, this difference between the two approaches would not seem to matter a great deal, as in principle each member could at little cost wait for a division he would like to see proposed. As this paper shows, this intuition appears to be incorrect: the two approaches lead to very different predictions, even in the limit of very patient agents.\footnote{Such an observation shares some similarities with Diamond’s observation that small search costs on consumers’ side would lead competing oligopolists to charge the monopoly price as if there were no competition, in sharp contrast with the analysis of the frictionless case (Bertrand competition). Here, we show that even as members are very patient (so that search frictions may be considered to be small), the collective search model and the collective bargaining model have very different welfare properties regarding the desirability of the various majority rules.}

To illustrate our basic ideas, we consider a setup such that for each member $i$, the value $u_i$ of adopting a particular proposal can be decomposed into two components: a common component $x$ that affects all members in the same way, and an idiosyncratic element $\theta_i$ that describes how that particular members have no control at all on the proposals put to a vote, but the main insights of this paper regarding the advantage of less tight majority rules would carry over to more balanced situations in which committee members have some imperfect control, thereby broadening the scope of application of the present insights.
proposal affects member \( i \), that is,

\[ u_i = x + \theta_i, \]

and in each period there is a new draw of \((x, \theta)\) corresponding to a new proposal put to a vote.

This is a simple (possibly the simplest) collective search setup in which the welfare of the various possible proposals may vary and individuals’ preferences are not perfectly homogeneous. The setup is used to characterize simply the welfare (measured as the discounted sum of utilities) associated with the various majority rules, in the limit of committees of arbitrarily large size.

In general, the expected welfare associated with a particular majority rule depends on whether proposals with high common components \( x \) are selected, and how long it takes to reach a decision. The optimal majority rule precisely solves the tradeoff between selecting proposals with high common component and reducing the expected delay in reaching a decision. As one considers more stringent majority rules, higher welfare levels are obtained upon reaching a decision, but at the cost of longer delays. And as one contemplates less stringent majority rules, decisions are reached faster but at the cost of agreeing on proposals with lower welfare ex post. In this simple model, the optimal majority rule is related to the dispersion of the idiosyncratic part only (and not to the distribution of the common part). More precisely, consider the fraction of agent for whom \( \theta_i \) is greater than the average value of \( \theta_i \). This fraction can be interpreted as an intrinsic popularity index. We find that the optimal majority rule precisely coincides with that intrinsic popularity index.\(^6\)

\(^6\)If individual preferences are perfectly homogeneous, the collective search problem is identical to a single agent search problem no matter what the majority rule is. And if all proposals are welfare equivalent, the best for expected welfare is to reach a decision as quickly as possible, which is achieved for lowest majority requirements (see claim in Section 2).
It should be mentioned that the inefficiency obtained for majority requirements less stringent than the optimal majority rule bears some similarity with the inefficiency derived in the collective bargaining model of Eraslan and Merlo (2002) (for rules other than unanimity). In our case as in Eraslan and Merlo (2002), a decision is reached too early (as compared with the first-best) because there is a sufficiently large share of members who fear that other members might agree on a less favorable outcome later on. But, the inefficiency obtained under majority requirements more stringent than the optimal one is specific to the collective search approach of our model, and does not arise in the collective bargaining approach. In our case, unlike in the bargaining formulation, a decision is reached too late because of the difficulty of finding proposals that please a sufficiently large share of members.

In Section 2 we present the general collective search model. In Section 3 we analyze the various majority rules in the simple setup outlined above. We also compare the welfare so obtained with that obtained in an analogous bargaining framework. Finally, we suggest some extensions of the basic model. Section 4 concludes.

\[\text{\footnotesize\textsuperscript{7}}\] Aghion and Bolton (2003) consider a two-period collective bargaining model that shares some similarities with our basic insight. More precisely, when transfers are sufficiently costly, Aghion and Bolton find in their setup that the unanimity rule is not optimal as less stringent majority rules may allow smaller groups to adopt welfare improving projects without the consent of the hurt agents who would request (costly) compensations (thereby making the project non-profitable when the transfer costs are big enough).

Compared to their model, our collective search approach allows us to obtain similar insights about the inefficiency of unanimity even in the limit as search frictions get small, thereby dispensing with the assumption of exogenous large transfer costs.

\[\text{\footnotesize\textsuperscript{8}}\] Observe that this inefficiency is still present in expected welfare terms in the limit of very patient agents, thereby explaining why the two approaches differ even in the limit of very patient agents.
2 The Model

We consider a committee consisting of $n$ members, labeled $i = 1, ..., n$. At any date $t = 1, ...,$, if a decision has not been made yet, a new proposal is drawn and examined. A proposal is denoted $u$, where $u = (u_i)_{i \in \{1, ..., n\}}$ is a vector in $R^n$ that describes the utility $u_i$ that member $i$ gets if the proposal $u$ is adopted. The set of possible proposals is denoted $U$, and it is assumed to be a compact convex subset of $[u, \overline{u}]^n$. We also assume that proposals at the various dates $t = 1, ..$ are drawn independently from the same distribution with continuous density $f(\cdot) \in \Delta(U)$. We shall be more specific about that distribution over proposals at the end of this Section.

Decision rules. Upon arrival of a new proposal $u$, each member decides whether to accept that proposal. We consider various majority rules. Under the $k$-majority rule, the game stops whenever at least $k$ out of the $n$ members vote in favor of the proposal.

We normalize to 0 the payoff that parties obtain under perpetual disagreement, and we let $\delta$ denote the common discount factor of the committee members. That is, if the proposal $u$ is accepted at date $t$, the date 0 payoff of member $i$ is $\delta^t u_i$.\footnote{Observe that we allow that $u$ be negative, that is, we do not impose that proposals deliver payoffs above the status quo payoffs to all members.}

Strategies and equilibrium. In principle, a strategy specifies an acceptance rule that may at each date be any function of the history of the game. We will however restrict our attention to stationary equilibria of this game, where each member adopts the same acceptance rule at all dates.\footnote{To avoid coordination problems that are common in voting (for example, all players always voting "no"), we will also restrict attention to equilibria that employ no weakly dominated strategies (in the stage game). These coordination problems could alternatively be avoided by assuming that votes are sequential.}

Given any stationary acceptance rule $\sigma_{-i}$ followed by members $j$, $j \neq i$, we may define the expected payoff $\bar{v}_i(\sigma_{-i})$ that member $i$ derives given $\sigma_{-i}$.
from following his (best) strategy. An optimal acceptance rule for member
$i$ is thus to accept the proposal $u$ if and only if

$$u_i \geq \delta \bar{v}_i(\sigma_{-i}),$$

which is stationary as well (this defines the best-response of member $i$ to
$\sigma_{-i}$).

Stationary equilibrium acceptance rules are thus characterized by a vector
$v = (v_1, ..., v_n)$ such that member $i$ votes in favor of $u$ if $u_i \geq \delta v_i$ and votes
against it otherwise. For any $k$-majority rule and value vector $v$, it will be
convenient to refer to $A_{v,k}$ as the corresponding acceptance set, that is, the
set of proposals that get support from at least $k$ members when failing to
agree today yields member $i$ a continuation payoff of $v_i$ (from the viewpoint
of next period):

$$A_{v,k} = \{ u \in U, \exists K \subset \{1, ..., n\}, |K| = k, u_i \geq \delta v_i \text{ for all } i \in K \}. \quad (1)$$

Equilibrium consistency then requires that

$$v_i = \Pr(u \in A_{v,k})E[u_i \mid u \in A_{v,k}] + [1 - \Pr(u \in A_{v,k})] \delta v_i \quad (2)$$

or equivalently

$$v_i = \frac{\Pr(u \in A_{v,k})}{1 - \delta + \delta \Pr(u \in A_{v,k})}E[u_i \mid u \in A_{v,k}]. \quad (3)$$

A stationary equilibrium is characterized by a vector $v$ and an acceptance
set $A_{v,k}$ that satisfy (1)-(2). It always exists, as shown in Compte and Jehiel
(2004-2010).

An illustration. One of the objective of this paper is to illustrate the cost
of having too stringent majority requirements. We provide below a simple

\footnote{For any finite set $B$, $|B|$ denotes the cardinality of $B$.}
illustration of this cost. Define $W(u)$ as the welfare (measured as the sum of utilities) associated with proposal $u$:

$$W(u) = \sum_i u_i$$

We fix $w > 0$ and assume that possible proposals all yield the same welfare level $w$, that is:

$$A1: U = \{u \mid W(u) = w, u_i \geq u \text{ for all } i\}$$

We have the following claim:

Claim: Assume A1 holds and that proposals are uniformly distributed on $U$. Then expected welfare increases when the majority requirement decreases.

The proof of this claim is in the Appendix. Intuitively, when proposals are welfare equivalent, the majority rule affects the expected welfare only to the extent that it speeds up the agreement. The claim holds because under less stringent majority rule, the acceptance set gets bigger.

In the rest of the paper, the possible proposals will not always yield the same welfare, and as we shall see, decreasing too much the majority requirement will be suboptimal because it would lead to the adoption of proposals with too low welfare level.

Distribution over proposals. In the rest of the paper, we assume that proposals do not all yield the same welfare and we make the following assumptions regarding the distribution over possible proposals. We assume that for each member $i$, the value $u_i$ of adopting a particular proposal $u$ can be decomposed into two components: a common component $x$ that affects all members in the same way, and an idiosyncratic element $\theta_i$:

$$u_i = x + \theta_i,$$
and that in each period, the common component $x$ is drawn according to a density $g(\cdot)$ on $[x, \overline{x}]$, while the idiosyncratic parts $\theta_i$ are drawn independently of one another and of $x$, according to a smooth density $h(\cdot)$ on $[-\overline{\theta}, \overline{\theta}]$.

We shall also assume that the conditional expectation $z \to E(x | x > z)$ is a smoothly differentiable function of $z$ with slope no greater than 1,\footnote{This holds true for the uniform distribution and for many more densities $g(\cdot)$ with bounded variations. This assumption will ensure the uniqueness of a stationary equilibrium.} and we normalize $\theta_i$ so that $E(\theta_i) = 0$.\footnote{This is just a normalization because if $E(\theta_i) \neq 0$ we can add $E(\theta_i)$ to $x$.}

### 3 Optimal majority rule in large committees

We wish to analyze how the various majority rules compare in terms of expected welfare as the number of members grows large. Specifically, we will compute the ex ante payoff obtained by every member in equilibrium under the various majority rules, for any given discount factor (possibly set close to 1 but not necessarily), and make the comparison taking the limit as the number of members grows arbitrarily large.

As we shall see, when the number of members grows large, whether a proposal is accepted or not depends almost exclusively on the realization of $x$ (this will be due to the law of large numbers). In subsequent results we refer to $\alpha = \frac{k}{n}$ as the majority rule where $k$ is the majority requirement defined in Section 2. For every $\alpha$ and $\delta$, there will be an equilibrium threshold $x^*$ such that, as $n$ grows large, only proposals such that $x > x^*$ are accepted. Our first objective is to characterize the equilibrium acceptance threshold $x^*(\delta, \alpha)$. 
3.1 Optimal acceptance threshold.

Before characterizing the equilibrium acceptance threshold, we derive the acceptance threshold that would maximize expected welfare, or, equivalently for large committees, the threshold that a single decision maker interested in $x$ would choose in a single agent search problem. We denote it by $x^{**}$.

Let us define $v(x_0, \delta)$ as the expected payoff that any member receives if all proposals such that $x > x_0$ are accepted and only such proposals are accepted. We have:

$$v(x_0, \delta) \equiv \frac{\Pr(x > x_0)}{1 - \delta + \delta \Pr(x > x_0)} E(x \mid x > x_0)$$  \hspace{1cm} (4)

The following figure draws $\delta v(., \delta)$ for a discount factor $\delta = 0.95$, assuming that $x$ is uniformly distributed on $[-1, 3]$, and $\theta_i$ is uniformly distributed on $[-1, 1]$.

![Graph of $v(., \delta)$](image)

The function $v(., \delta)$

The figure illustrates that starting from $x_0$, a more stringent acceptance threshold $x_0$ increases welfare, up to the point where this would reduce so much the probability of acceptance that welfare starts decreasing.\(^{14}\)

The optimal acceptance threshold is the point $x^{**}$ where $\delta v(\delta, x)$ is maximum. Since rejecting a proposal $x$ yields $\delta v(\delta, x^{**})$, and since $\bar{x} > 0$, the

\(^{14}\)At the limit where $x_0$ approaches $\bar{x}$, no proposals are accepted so $v(\delta, x_0)$ approaches 0.
optimal acceptance threshold is positive and characterized, as in a standard one person search problem by the condition:

$$x^{**} = \delta v(\delta, x^{**}),$$

(5)

and it is determined graphically by the point at which $\delta v(\delta, .)$ and the $45^\circ$ line cross.

### 3.2 Equilibrium acceptance threshold

We now turn to the equilibrium acceptance threshold. For every $\alpha$, define $\theta^*(\alpha)$ as the threshold that solves:

$$\Pr(\theta_i > \theta^*(\alpha)) = \alpha.$$

The threshold $\theta^*(\alpha)$ is thus set so that each member $i$ has a probability $\alpha$ to have his idiosyncratic part $\theta_i$ exceed $\theta^*(\alpha)$. Note that as $\alpha$ increases, the threshold $\theta^*(\alpha)$ decreases.\(^\text{17}\)

Now assume that $v^*$ is the equilibrium expected payoff received by each member. A member votes in favor of a proposal with common component $x$ whenever

$$x + \theta_i > \delta v^*.$$  

(6)

For a given $x$, the number of members in favor of the proposal is thus a random variable: it corresponds to the number of realized value of $\theta_i$ for which (6) holds. As $n$ grows large, by the law of large number, the fraction of members that supports the proposal is approximately equal to

\(^\text{15}\)The optimal threshold exists and is uniquely defined: our assumption that $z \to E(x \mid x > z)$ has slope less than 1 implies that $x_0 \to \delta v(x_0, \delta)$ has slope less than 1. Also note that $\delta v(0, \delta) > 0$ and that $\delta v(\bar{x}, \delta) = 0 < \bar{x}$.

\(^\text{16}\)Equivalently, $\theta^*(\alpha)$ solves $1 - H(\theta^*(\alpha)) = \alpha$, where $H(\cdot)$ denotes the cumulative of $h(\cdot)$.

\(^\text{17}\)When $\alpha$ gets close to 1, $\theta^*(\alpha)$ gets close to $\bar{\theta}$, while when $\alpha$ gets close to 0, $\theta^*(\alpha)$ gets close to $\bar{\theta}$. For the uniform distribution for example, $H(\theta) = (1 + \theta)/2$ so $\theta^*(\alpha) = 1 - 2\alpha$. 

11
Pr(θ_i > δv^* - x) with probability close to 1. Thus, given the majority rule α, a proposal with common component x goes through if and only if Pr(θ_i > δv^* - x) > α, or equivalently whenever \( \theta^*(\alpha) > \delta v^* - x \). That is, whenever:

\[ x > \delta v^* - \theta^*(\alpha). \]

So as mentioned earlier, in equilibrium, as \( n \) grows arbitrarily large, a proposal with common component x is accepted if and only if x exceeds some threshold \( x^* \),\(^{18}\) with

\[ x^* \equiv \delta v^* - \theta^*(\alpha) \]

Also by definition of \( v(.,\delta) \) (see (4)) we must have:

\[ v^* = v(x^*,\delta), \]

implying that the threshold \( x^* \) solves:\(^{19}\)

\[ x^* + \theta^*(\alpha) = \delta v(x^*,\delta) \tag{7} \]

As mentioned earlier the analysis above has been made for the case where \( n \) is arbitrarily large. The next proposition makes a formal statement for cases where \( n \) is large but fixed. It is proven in Appendix.

**Proposition 1.** Fix \( \alpha \) and for every \( n \) consider the majority requirement \( k(n) = \text{Int}(\alpha n) \) where \( \text{Int}(\alpha n) \) denotes the integer that is closest to \( \alpha n \). For every \( \varepsilon > 0 \), there exists \( \bar{n} \) such that for every \( n > \bar{n} \), the expected equilibrium payoff obtained by every member under the \( k(n) \) majority requirement \( w(\delta, n) \) satisfies \( |w(\delta, n) - v(x^*, \delta)| < \varepsilon \) where \( x^* \) is the threshold defined in (7).

\(^{18}\)The statement only holds at the limit where \( n \) grows very large. Proposition 1 will make a precise statement for the case where \( n \) is large but fixed.

\(^{19}\)Equation (7) has a unique solution because \( z \to \delta v(z, \delta) - z \) is decreasing. Note that (i) if \( \bar{x} + \theta^*(\alpha) \leq 0 \), then \( x^* \geq \bar{x} \), and there is perpetual disagreement; and (ii) if \( \bar{x} - \delta E(x) + \theta^*(\alpha) \geq 0 \), then \( x^* \leq \bar{x} \) and there is immediate agreement on the first proposal. In all other cases, the threshold \( x^* \) belongs to \( (\bar{x}, \bar{x}) \).
Going back to the above case of uniform distribution, the following figure explains graphically how the threshold $x^*$ is obtained for the unanimity rule $\alpha = 1$ (in which case $\theta^*(\alpha) = -1$), and for the majority rule $\alpha = 1/2$ (in which case $\theta^*(\alpha) = 0$).

Deriving $x^*$

As one decreases the majority requirement $\alpha$, $\theta^*(\alpha)$ increases, hence so does the line $x + \theta^*(\alpha)$. As the figure above illustrates, the threshold $x^*_\alpha$ shifts to the left. Under the assumptions of the figure ($\delta = 0.95$) and starting from the unanimity rule, this shift increases welfare.

3.3 Comparative statics

As the above Figure illustrates, Proposition 1 can be used to compare the welfare obtained under different majority scenarios. It will be convenient to define $\gamma$ as the probability that the idiosyncratic part is positive:\(^{20}\)

$$\gamma \equiv \Pr(\theta_i > 0).$$

With a large number of members, the parameter $\gamma$ corresponds to the fraction of members that get a draw $\theta_i$ above the average $\theta_i$. It may be interpreted as an intrinsic popularity index.

\(^{20}\)Note that when $\theta_i$ is symmetric around 0, we have $\gamma = \frac{1}{2}$. 

13
We first observe that when the majority rule \( \alpha \) coincides with \( \gamma \), then \( \theta^*(\alpha) = 0 \), so the equation (5) that determines the optimal acceptance threshold coincides with the equation (7) that determines the equilibrium acceptance threshold. In other words, we have:

\[
x^{**}(\delta) = x^*(\delta, \gamma)
\]

As the majority requirement is modified away from \( \gamma \), the equilibrium acceptance threshold, which is determined by (7) moves away from the optimal acceptance threshold \( x^{**}(\delta) \), so welfare is reduced. Besides, since \( z \rightarrow z - \delta v(z, \delta) \) is increasing, the solution to \( z - \delta v(z, \delta) = -\theta^*(\alpha) \) increases when \( \alpha \) increases, implying that the equilibrium acceptance threshold \( x^*(\delta, \alpha) \) increases with \( \alpha \).

So we have:

**Proposition 2.** Let \( \gamma = \Pr(\theta_i > 0) \). As the number of members gets large, expected welfare is maximized for the \( \gamma \) majority rule. Besides, the equilibrium acceptance threshold \( x^*(\delta, \alpha) \) increases with \( \alpha \).

When the majority requirement increases above \( \gamma \), \( \theta^*(\alpha) \) decreases (below 0), and proposals pass only if the difference \( x - \delta v^* \) is sufficiently large (by an amount at least equal to \( -\theta^*(\alpha) \)). The interpretation is that proposals need to get stronger support to pass, and that support is obtained when the common component is sufficiently high compared to the continuation value \( \delta v^* \). How high should the common component be is related to the dispersion in the idiosyncratic component, and \( -\theta^*(\alpha) \) is the relevant measure of that dispersion when the majority requirement is \( \alpha \). Welfare is reduced because compared to the first best, only excellent proposals are accepted, and it takes time to find out such proposals.

When the majority requirement decreases below \( \gamma \), \( \theta^*(\alpha) \) increases (above 0), and proposals pass even if \( x \) is below \( \delta v^* \) (so long as \( x - \delta v^* \geq -\theta^*(\alpha) \)). The interpretation is that proposals pass even when they get weak support,
and such weak support obtains even if $x$ is below $\delta v^*$. Once again, how far below $\delta v^*$ the common component can be (and the proposal still be adopted) depends on the dispersion of the idiosyncratic component, and $\theta^*(\alpha)$ is the relevant measure of that dispersion when the majority requirement is $\alpha$. Welfare is reduced because poor proposals are accepted.

To illustrate Proposition 2, we go back to our previous example (with uniform distributions), assuming that $\delta$ is close to 1. When players are patient and $x^*$ is not close to $\bar{x}$, $\delta v(\delta, x^*)$ is close to $E(x \mid x > x^*)$, and the optimal threshold is thus determined by:

$$x^* + \theta^*(\alpha) = E(x \mid x > x^*). \quad (8)$$

The following figure summarizes how welfare varies as a function of $\alpha$.

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21 For uniform distributions, $\alpha_0 = 1/2$ and $\theta^*(\alpha) = 1 - 2\alpha$. When $\alpha > 1/2$, welfare is equal to $\bar{x} + \theta^*(\alpha)$. When $\alpha < 1/2$, welfare is equal to $x^* + \theta^*(\alpha)$, where $x^* + \theta^*(\alpha) = (\bar{x} + x^*)/2$, implying that $x^* + \theta^*(\alpha) = \bar{x} - \theta^*(\alpha)$. 

Comment. Proposition 2 holds even when players are patient. So the conclusion of Proposition 2 may seem at odds with the observation made in Albrecht et al. (2010) that as members get very patient, the best rule is the unanimity rule (see also Compte and Jehiel (2004) for an early statement
of the same insight in a less general class of examples). Yet, our conclusion holds for any fixed $\delta$ (possibly close to 1), in the limit as $n$ goes to infinity. For a fixed $n$, unanimity would be optimal in the limit as players become arbitrarily patient.

### 3.4 Collective search vs collective bargaining

In our collective search framework, the best majority rule is interior, as we have just shown. For comparative purpose, we now consider the corresponding collective bargaining framework, and we will show there that the more stringent the majority requirement the better (thereby confirming insights from Merlo and Wilson (1998) and Eraslan and Merlo (2002)).

The collective bargaining we consider is as follows. A pie is to be divided among the various committee members. As long as agreement on how to partition the pie has not been reached, a new pie is drawn. In every period, the size $z$ of the pie is drawn at random from a distribution with density $g(\cdot)$ on $[\bar{z}, \bar{z}]$, and we assume as before that $z_0 \to E[z \mid z > z_0]$ is a smooth increasing function with slope no greater than 1. The draws at the various periods are independent of each other. Members are equally patient (with discount factor $\delta$), and they are each selected to make a proposal with the same probability. A proposal consists of a splitting of the current pie among the various members with the constraint that every member should receive a non-negative share of the pie. After the proposal is made, there is a vote. The sharing is implemented and bargaining stops if the proposal receives the support of at least $k = \text{Int}(\alpha n)$ members. Otherwise, one moves to the next period, which has the same structure.

We consider stationary equilibria of the above collective bargaining game. As in the case of collective search, we consider the case of large $n$.

First consider the first best rule. It consists of agreeing on a partition whenever the pie is larger than some optimal acceptance threshold $z^{**}$. To
determine $z^{**}$, call $w^{**}$ the optimal welfare and define $w(z_0, \delta)$ as the welfare obtained when only pies of size at least equal to $z_0$ are shared. When a proposal is rejected, the continuation welfare is $\delta w^{**}$, so the optimal acceptance threshold satisfies:

$$z^{**} = \delta w(z^{**}, \delta)$$

Now consider a stationary equilibrium obtained under the $\alpha$-majority rule, and call $w^b$ the expected equilibrium welfare obtained by members. In equilibrium, pies of size $z$ get implemented whenever a fraction $\alpha$ of the members can each be allocated a payoff equal to $\delta w^b/n$, that is, taking the limit as $n$ grows large, whenever:

$$z > \delta \alpha w^b.$$ 

The equilibrium acceptance threshold $z^*$ thus satisfies:

$$\frac{z^*}{\alpha} = \delta w(z^*, \delta),$$

and the equilibrium acceptance threshold yields the first best if and only if $\alpha = 1$. Besides, since $z \to z - \delta w(z, \delta)$ is strictly increasing, expected equilibrium welfare is an increasing function of $\alpha$. To summarize,

**Proposition 3.** In the collective bargaining model, expected welfare is maximized for the unanimity rule. The equilibrium acceptance threshold $z^*(\alpha, \delta)$ increases with $\alpha$, and expected welfare is an increasing function of the majority rule $\alpha$.

In other words, majority rules less stringent than unanimity do not maximize welfare because agreement obtains even for pies of low size.

To illustrate Proposition 3, assume that $z$ is uniformly distributed on $[\underline{z}, \bar{z}]$, and that $\delta$ is close to 1. We have that $z^*(\alpha) = \max(\frac{\alpha}{2-\alpha}, \bar{z}, \underline{z})$ and the

$^22$This is because $z_0 \to E[z \mid z > z_0]$ is increasing with slope no greater than 1.
corresponding welfare \( w^b(\alpha) = E(z \mid z > z^*(\alpha)) \) is depicted in the following figure

The welfare function \( w^b(\alpha) \)

The contrast between Propositions 2 and 3 is striking. The collective search model explains why unanimity is undesirable in large committees, and the collective bargaining model does not.

3.5 Optimal majority, first-best and impatience

The main objective of the paper has been to formalize the trade-off induced by a tighter majority requirement regarding delay and expropriation and show the contrast with bargaining models.

Our analysis however reveals two further properties. First, the optimal majority rule achieves the first-best acceptance threshold. Second, this optimal majority rule is independent of the discount factor \( \delta \). These two properties would not necessarily carry over to more general settings though. In fact, there are two directions in which the setting of Section 3 can naturally be extended. First, the distribution of \( \theta_i \) need not be deterministically the same in every period. Second, this distribution may be correlated with \( x \). The trade-off we identified would still be relevant. However the optimal majority rule may no longer induce the first-best acceptance decision and the optimal majority rule may depend on the impatience of members.
A complete analysis of these extensions is beyond the scope of the present paper. Yet, to illustrate the possible effects, consider the following scenario.

Assume that a proposal is characterized as before by a common component $x$, and idiosyncratic elements $\theta_i$. In a given period, these idiosyncratic elements are all drawn from the same distribution with density $h(.)$, with $E\theta_i = 0$, but we now assume that $h(.)$ is one of two possible densities $h_1(.)$ or $h_2(.)$. A random variable $\nu \in \{1, 2\}$ determines in each period (independently across periods) which of these two densities applies, say with equal probability. We also let $\gamma_\nu = Pr(\theta_i > 0)$ and assume that $\gamma_1 < \gamma_2$.

The proposed model thus extends the previous one in assuming that proposals do not all have the same popularity index: when a proposal gets idiosyncratic elements drawn from $h_2$ rather than $h_1$, there is a larger fraction of members that get a positive $\theta_i$.

In that model, the optimal acceptance threshold $x^{**}$ is determined as before. Implementing such an acceptance threshold however would require to set a majority rule $\gamma_1$ in periods where the idiosyncratic elements are drawn from $h_1$, and a majority rule $\gamma_2$ in periods where the idiosyncratic elements are drawn from $h_2$. When the majority rule is fixed throughout the decision process, welfare is necessarily below that obtained under the optimal acceptance threshold.

Still an optimal majority rule $\alpha^*$ exists, and it can be shown that $\alpha^* \in (\gamma_1, \gamma_2)$. However, in periods when the $\theta_i$’s are drawn from $h_1$, the optimal majority rule $\alpha^*$ induces inefficient rejections, and in periods when the $\theta_i$’s are drawn from $h_2$, $\alpha^*$ induces inefficient approvals. The optimal majority rule will depend on $\delta$ and it is not too hard to see that it should get close to $\gamma_2$ as $\delta$ goes to 1.
4 Conclusion

This paper has provided a very simple model that accounts for the widely spread intuition that as committees get large, (well chosen) majority rules are preferable to unanimity. Unlike the well developed models of collective bargaining (with transferable utility) which would unambiguously favor unanimity, our model assumes that members do not control the proposal put to a vote. The main drawback of unanimity in such collective search settings is that it makes it too difficult to find a proposal acceptable by all, which in turn induces extra delay costs in comparison with majority rules. The majority rule should not be too low though, as it would result in the acceptance of too inefficient proposals. The best majority rule is the one that solves best this trade-off.\textsuperscript{23}

It should also be mentioned that even though in our setup there was room for only one project, our insights regarding the advantage of less stringent majority rules (and also regarding the determination of the optimal majority rule) would carry over to settings where adopting a proposal would reduce for some time the chance of getting the funds to implement a new one. This extension would fit well with many real situations where financial capacities are scarce, and where implementing a tentative proposal or financing a tentative project would come at the cost of making future proposals harder (or even impossible) to implement.

Our analysis has also revealed that the best majority rule is related to

\textsuperscript{23}A very different argument in favor of majority as opposed to unanimity follows the line of the Condorcet jury theorem by suggesting that majority rules may better aggregate information (this has been formalized recently by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996)).

Our setup offers a different perspective by emphasizing the delay costs attached to unanimity (and by removing the common value uncertainty present in such models - in our setting unlike in those settings, every member knows how much he values the proposal put to a vote, hence our setting falls in the category of private value uncertainty).
the proposals’ intrinsic popularity index, namely, the extent to which the benefits of a proposal are shared across many (or few) members. While we have kept this popularity index exogenous, it is reasonable to think of transfer mechanisms across members as modifying (and typically increasing) the proposals’ popularity index. Our analysis thus suggests that stronger transfer mechanisms should go along with tighter majority requirements.

References


Proof of Claim: Observe that by symmetry, the acceptance threshold $\delta v_i$ is the same for all members and depends only on the majority requirement $k$. Denote by $v^*_k$, the per-member welfare obtained in equilibrium under the $k$–majority rule, and by $\pi^*_k$ the equilibrium probability of agreement. Assume by contradiction that one can have $k_1 > k_2$ and $v^*_{k_1} \geq v^*_{k_2}$. Given that $\Pr(u \in A_{v,k})$ is decreasing with $k$ and $v$, we would have $\pi^*_{k_1} = \Pr(u \in A_{v^*_{k_1},k_1}) < \pi^*_{k_2} = \Pr(u \in A_{v^*_{k_2},k_2})$. Since the expected welfare $v^*_k$ is an increasing function of the probability of agreement $\pi^*_k$, we must have $v^*_{k_2} > v^*_{k_1}$, contradicting the premise that $v^*_{k_1} \geq v^*_{k_2}$.

Proof of Proposition 1. For any $w$ we define $\pi(w) \equiv \Pr(x > -\theta^*(\alpha) + \delta w)$, and $u(w) = E[x \mid x > -\theta^*(\alpha) + \delta w]$. Also define $w^*$ as the value satisfies:

$$w = \frac{\pi(w)}{1 - \delta + \delta \pi(w)} u(w)$$  \hspace{1cm} (9)

Note that by construction, we have $w^* = v^*(x^*, \delta)$ and $x^* = -\theta^*(\alpha) + \delta w^*$.

We show below that the equilibrium value must be close to $w^*$ when $n$ gets large. The argument makes use of the following lemma (some form of the law of large numbers). Define the events $B_\varepsilon, C_\varepsilon, D_\varepsilon$ as:

$$B_\varepsilon = \#\{i, \theta_i > \theta^*(\alpha) + \varepsilon\}/n > \alpha$$
$$C_\varepsilon = \#\{i, \theta_i < \theta^*(\alpha) - \varepsilon\}/n > 1 - \alpha$$
$$D_\varepsilon = \left\{ \frac{1}{n} \sum_i \theta_i \notin [-\varepsilon, +\varepsilon] \right\}$$  \hspace{1cm} (10)

and let $E_\varepsilon$ denote the event complement to $B_\varepsilon \cup C_\varepsilon \cup D_\varepsilon$.

Lemma. $\forall \varepsilon, \exists \pi$ such that for all $n > \pi, \Pr\{E_\varepsilon\} > 1 - \varepsilon$.

\hspace{1cm} 24This is because $E[u_i \mid u \in A_{v,k}] = w/n$ so (3) implies $v^*_k = \frac{v^*_k}{1 - \frac{\pi^*_k}{\pi}} \frac{w}{n}$.
Now choose \( \varepsilon \) small, and \( n \) large enough so that the inequality of the Lemma holds \((n > \bar{n})\).

Assume now that the equilibrium value is \( \omega \). We are going to establish bounds on \( \omega \). To this end, it is convenient to denote by \( A \) the event where the current proposal passes. It is also convenient to denote by \( F_{w,\varepsilon}^+ \) the event \( \{ x > -\theta^*(\alpha) + \delta w + \varepsilon \} \), by \( F_{w,\varepsilon}^- \) the event \( \{ x < -\theta^*(\alpha) + \delta w - \varepsilon \} \), by \( F_{w,\varepsilon}^0 \) the event complement to \( F_{w,\varepsilon}^+ \cup F_{w,\varepsilon}^- \). Note that since the distribution over proposals has a continuously differentiable density, there exists \( h \) such that \( \Pr(F_{w,\varepsilon}^+ < h\varepsilon) \)

Observe that under \( F_{w,\varepsilon}^+ \), member \( i \) accepts \( x \) if \( \theta_i + x > \delta w \), hence a fortiori if \( \theta_i > \theta^*(\alpha) - \varepsilon \). Under event \( E_\varepsilon \), there is a fraction \( \alpha \) of members for which this is true, hence such proposals \( x \) must pass. Similarly, under event \( E_\varepsilon \cap F_{w,\varepsilon}^- \), proposals cannot pass. It follows that in any period, a proposal passes with probability at least

\[
\pi^- \equiv \Pr(E_\varepsilon \cap F_{w,\varepsilon}^+) = (1 - \varepsilon)\pi(w + \varepsilon/\delta)
\]

and at most

\[
\pi^+ \equiv 1 - \Pr(E_\varepsilon \cap F_{w,\varepsilon}^-) = 1 - (1 - \varepsilon)(1 - \pi(w - \varepsilon/\delta)).
\]

We now derive bounds on the expected payoff that any given member \( i \) obtains, conditional on the event \( A \) where the current proposal passes.

Observe that by symmetry, for all \( x \),

\[
E[\theta_i \mid A] = E\left[\frac{1}{n} \sum_i \theta_i \mid A_x\right]
\]

which implies that

\[| E[\theta_i \mid A \cap E_\varepsilon] | < \varepsilon. \]

Finally we have seen that \( E_\varepsilon \cap F_{w,\varepsilon}^+ \subset A \) and that \( E_\varepsilon \cap F_{w,\varepsilon}^- \cap A = \emptyset \), thus:

\[
\Pr(E_\varepsilon \cap F_{w,\varepsilon}^+ \mid A) > 1 - \Pr(E_\varepsilon^c - \Pr F_{w,\varepsilon}^0 > 1 - (1 + h)\varepsilon
\]

24
It follows that $E[x + \theta_i \mid A]$ is bounded above by

$$u^+ \equiv (1 - (1 + h)\varepsilon)E[x \mid F_{w,\varepsilon}^+] + (1 + h)\varepsilon \bar{u}$$

and bounded below by:

$$u^- \equiv (1 - (1 + h)\varepsilon)E[x \mid F_{w,\varepsilon}^-] + (1 + h)\varepsilon \underline{u}$$

where $\bar{u}$ and $\underline{u}$ are bounds on the payoff that any member may get. The equilibrium value must satisfy:

$$\frac{\pi^-}{1 - \delta + \delta \pi^-} u^- < w < \frac{\pi^+}{1 - \delta + \delta \pi^+} u^+$$

As $\varepsilon$ get small, $\pi^+$ and $\pi^-$ converge to $\pi(w)$, and $u^+$ and $u^-$ converge to $u(w)$. Hence $w$ must converge to the (unique) solution of (9), that is, $w^*$. 

Q. E. D.