Generalized Dynamic Factor Models, Cointegration, and Error Correction Mechanisms

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Abstract

In this work, we study the Generalized Dynamic Factor model (GDFM) in the case of non-stationary I(1) variables. In GDFMs, each observable is the sum of a common and an idiosyncratic component. The vector of the common components is highly singular, i.e. is driven by a number of shocks that is much smaller than its dimension. We use the results of Anderson and Deistler (2008) on singular stationary vectors to generalize the Granger Representation Theorem to singular I(1) vectors. This new theorem provides a representation for the GDFM when the variables are driven by both common trends and common cycles.

We then turn to determining the number of common trends and cycles. It is well known that if a stationary panel admits an approximate factor structure, then there are \( q \) eigenvalues of the spectral density matrix diverging as the cross sectional dimension increases, while the others stay bounded. If in addition only \( q - d \) common shocks have permanent effects on the data, then the spectral density matrix at the zero frequency has only \( q - d \) diverging eigenvalues, where \( d \) is the number of common cycles. On the basis of this result, we propose a modified version of the criterion by Hallin and Liška (2007) to determine the number of common trends and common cycles in large panels. The performance of this method is analyzed by means of a simulation study. Finally, estimation of the GDFM for I(1) variables is discussed.

\textit{JEL classification:} C0, C01, E0.

\textit{Key words:} Generalized Dynamic Factor Models, I(1) variables, Cointegration, Granger Representation Theorem.

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