Information Aggregation in Large Elections

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I Large Elections

One dimensional spatial model with two given alternatives: $-1 < B < A < 1$

Voters: n finite, large

States: $\Omega = [0, 1]$ with prior beliefs described by a pdf $g(\omega)$ with full support

Voters identified by *type*: $t \in [-1, 1]$ distributed according to cdf $H(t)$; $dH = h$ has full support.

Distribution h common knowledge; realization of type private information

Utility difference between A and B for voter t in state ω is,
$U(t,\omega) = u(A,t,\omega) - u(B,t,\omega)$

Assume: U continuous and strictly increasing in t and ω with $U(-1,\omega) < 0 < U(1,\omega)$ for all ω

Each individual i observes a conditionally independent signal $s_i \in \{0,1\}$ about the true state:

$\forall i, \forall \omega,\ p(s_i|\omega) \in (0,1)$

Assume: $p(1|\omega)/p(0|\omega)$ strictly increasing in ω (SMLRP)

Redefine (non-unanimous) q-rules such that $0 < q < 1$ and B chosen iff strictly more than nq voters vote B

Symmetric voting strategy (& no abstention),
$v : [-1,1] \times \{0,1\} \to [0,1]$; $v(t,s) \equiv \Pr[\text{type } t \text{ voter votes } B|s]$.

Definition

A *voting equilibrium* is a symmetric and weakly undominated Nash equilibrium $\mathbf{v}^*$

Theorem (Feddersen & Pesendorfer 1997)

For every q-rule there exists a voting equilibrium $\mathbf{v}^*$*. Every voting equilibrium is characterized by cutpoints* $-1 < t_1 < t_0 < 1$ *such that* $E[U(t_j, \omega) | \mathbf{v}^*, t_j\text{-pivotal}] = 0$*; all types* $t < t_1$ *[*$t > t_0$*] vote surely for B [A] independent of their signal; and all types* $t \in (t_1, t_0)$ *vote informatively. Furthermore, the probability that any randomly chosen individual votes for B [equivalently, the vote share for B] in equilibrium is strictly interior and decreasing in* ω*.*

The theorem (and that to follow) generalize to having $K > 1$ possible sources of signals, with different types having access to different sources.

Let ω^* be the state at which t_q, the q^{th} percentile voter, is indifferent between alternatives:

$$\int_{-1}^{t_q} h(t)dt = q \text{ and } \omega^* = \arg\min_{\omega \in \Omega} |U(t_q, \omega)|$$

Definition

A sequence of strategy profiles $(\mathbf{v}_n)_{n \to \infty}$ satisfies *full information equivalence* (FIE) if for all $\eta > 0$ there is an n such that the following holds for $n' > n$: if $\omega < \omega^* - \eta$ then B is elected with probability greater than $1 - \eta$; if $\omega > \omega^* + \eta$ then A is elected with probability greater than $1 - \eta$.

Theorem (Feddersen & Pesendorfer 1997)

Fix a q-rule. Every sequence of voting equilibria $(\mathbf{v}_n^*)_{n \to \infty}$ under the rule satisfies FIE.

Note: This result extends the CJT to a private values setting.

There are two effects as n becomes large:

(1) the proportion of voters voting informatively in equilibrium becomes vanishingly small (pivot probabilities go to zero);

(2) the information reflected in the event of being pivotal approaches certainty (law of large numbers).

The proof shows that (1) & (2) necessarily imply FIE.

Proof [Sketch] Pick any sequence of voting equilibria $(\mathbf{v}_n^*)$; by F&P's existence theorem, this sequence is characterized by a corresponding sequence of cutpoints (t_1^n, t_0^n). Let $\Delta_n \equiv t_0^n - t_1^n \geq 0$.

Claim 1 $\lim_{n\to\infty} \Delta_n = 0$.

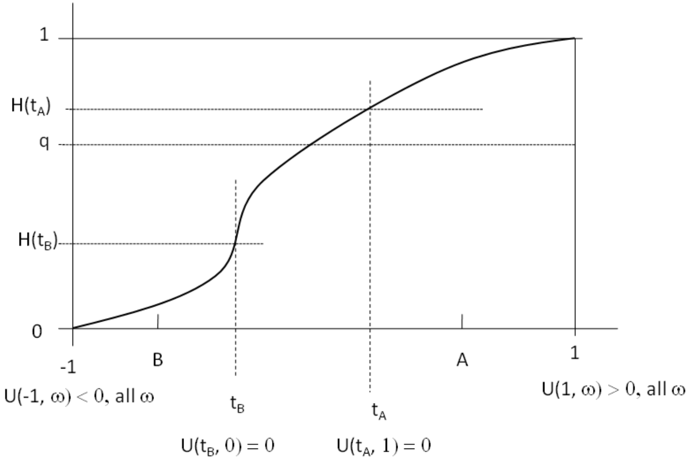
Argument: If not, $\lim_{n\to\infty} \Delta_n > 0$ and # *{informative voters}* increases without limit as $n \to \infty$.

Hence, the posterior belief over states conditional on the event "*i* is pivotal" converges a.s. to having mass-one on a particular state.

But then the marginal relevance of any given signal for an individual's beliefs - and therefore behaviour - is negligible, contradicting $\lim_{n\to\infty} \Delta_n > 0$. ||

Let t_A [t_B] be the type indifferent between A and B at $\omega = 1$ [$\omega = 0$]. Let $V_\omega(\mathbf{v}_n)$ be the expected vote share of B at ω given $\mathbf{v}_n$ and suppose $H(t_A) > q > H(t_B)$.

Premises for Claim 2



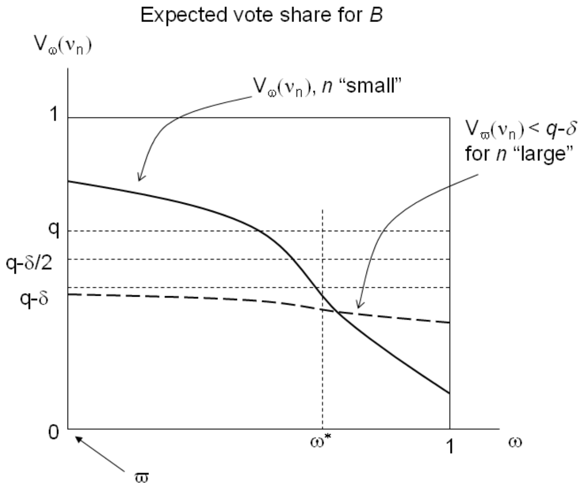
Claim 2 If $H(t_A) > q > H(t_B)$ *then, for sufficiently large* n, $V_\omega(\mathbf{v}_n^*) \approx q$, *all* ω.

Argument: If not, $V_{\bar{\omega}}(\mathbf{v}_n^*) < q - \delta$ for some $\delta > 0$ at some state $\bar{\omega}$, all n.

By Claim 1, $V_\omega(\mathbf{v}_n^*)$ is approximately constant for n sufficiently large; so, $V_\omega(\mathbf{v}_n^*) < q - \frac{\delta}{2}$ at every $\omega \in \Omega$.

By the existence theorem, $V_\omega(\mathbf{v}_n^*)$ is decreasing in ω; hence, $\forall\omega$, $q - V_\omega(\mathbf{v}_n^*) > q - V_0(\mathbf{v}_n^*)$.

Therefore, by Claim 1, $\bar{\omega} = 0$ a.s. conditional on being pivotal. But $H(t_A) > q$ contradicting $V_{\bar{\omega}}(\mathbf{v}_n^*) < q$. $||$



Expected vote share for *B*
$V_\omega(\nu_n)$
$V_\omega(\nu_n)$, *n* "small"
$V_\varpi(\nu_n) < q-\delta$ for *n* "large"
1
q
q-δ/2
q-δ
0
ω*
1
ω
ϖ

For sufficiently large n, Claim 1 implies that, conditional on being pivotal, individuals place almost all probability mass on some state ω_n. Now suppose (for convenience only) ω^*, defined above, is interior to $[0, 1]$.

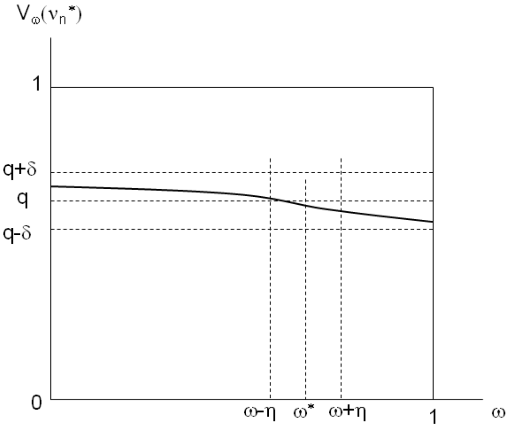
Claim 3 $\lim_{n\to\infty} \omega_n = \omega^*$.

Argument: If not and, say, $U(t_q, \omega_n) > \varepsilon > 0$ all n, then $V_{\omega_n}(\mathbf{v}_n^*) < q$; contradicting Claim 2. $||$

Together, Claims 1, 2 and 3 imply that, for sufficiently large n, the election can be tied only if $\omega \approx \omega^*$.

Since, for all n, the vote share $V_{\omega_n}(\mathbf{v}_n^*)$ is decreasing in ω, this is only possible for n large if B wins a.s. when $\omega < \omega^* - \eta$ and A wins a.s. when $\omega > \omega^* + \eta$. $\square$

Expected vote share for *B* with *n* “large”



II Preference Monotonicity

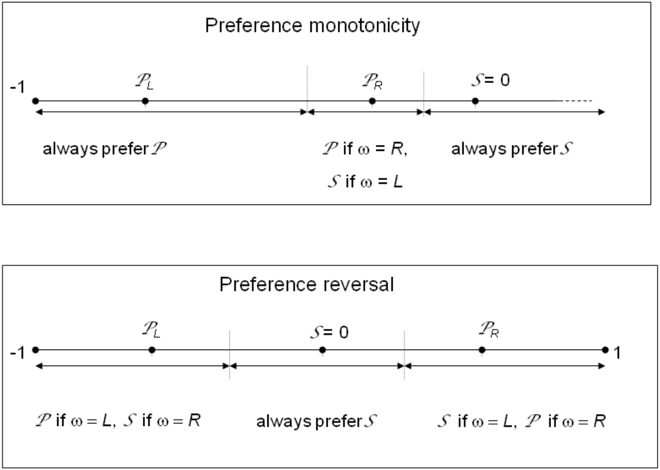
Key assumption for successful aggregation in the F&P model: aggregate preferences (and therefore expected vote shares) are *monotonic* in the state; i.e. $U(t, \omega)$ is strictly increasing in ω for all t.

Example Two states $\{L, R\}$, and alternatives $\{\mathcal{S}, \mathcal{P}\} \subset [-1, 1]$.

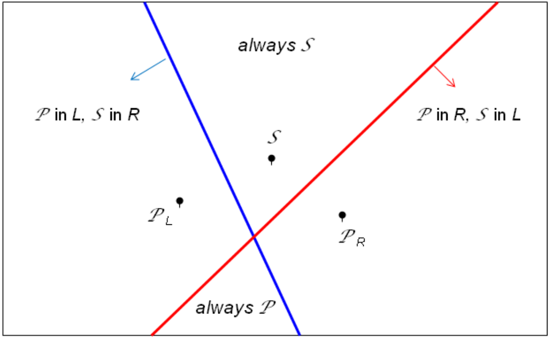
$\mathcal{S} = 0$ is a fixed status quo policy;

$\mathcal{P}$, the alternative, has two possible realizations, $\mathcal{P}_L, \mathcal{P}_R$ depending on the state.

Then aggregate preferences (and therefore expected vote shares) are not necessarily monotonic in the state and can be subject to *preference reversal*:


Preference monotonicity
-1
$\mathcal{P}_L$
$\mathcal{P}_R$
$\mathcal{S}$= 0
always prefer $\mathcal{P}$
$\mathcal{P}$ if ω = R,
$\mathcal{S}$ if ω = L
always prefer $\mathcal{S}$
Preference reversal
-1
$\mathcal{P}_L$
$\mathcal{S}$= 0
$\mathcal{P}_R$
1
$\mathcal{P}$ if ω = L, $\mathcal{S}$ if ω = R
always prefer $\mathcal{S}$
$\mathcal{S}$ if ω = L, $\mathcal{P}$ if ω = R

Preference reversal in multidimensional policy spaces



Bhattacharya (2008) identifies the importance of preference monotonicity in a general framework (see also Bhattacharya 2007).

Assume states of the world and the signal technology as for CJT:

$$\Omega = \{A, B\};$$

$$s \in \{a, b\} : \Pr[a|A] = p_A \in (\tfrac{1}{2}, 1), \Pr[b|B] = p_B \in (\tfrac{1}{2}, 1).$$

Alternatives $\{\mathcal{S}, \mathcal{P}\}$, with $\mathcal{P}$ chosen iff it receives more than nq votes.

For each $\omega \in \Omega$, an arbitrary voter t's utility difference, $U(t, \omega) = u(\mathcal{P}, t, \omega) - u(\mathcal{S}, t, \omega)$, is an iid draw from the reals

Let $\mu \in [0, 1]$ be a belief that $\omega = A$ and define four types of voter:

(1) $\mathcal{P}$-partisan: $\forall \mu$, $E[U(t, \omega) | \mu] \geq 0$

(2) $\mathcal{S}$-partisan: $\forall \mu$, $E[U(t, \omega) | \mu] \leq 0$

(3) $\mathcal{P}^r$-independent: $\exists \mu_0 \in (0, 1)$ such that

$$E[U(t, \omega) | \mu > \mu_0] > 0 \text{ and } E[U(t, \omega) | \mu < \mu_0] < 0$$

(4) $\mathcal{P}^l$-independent: $\exists \mu_0 \in (0, 1)$ such that

$$E[U(t, \omega) | \mu > \mu_0] < 0 \text{ and } E[U(t, \omega) | \mu < \mu_0] > 0$$

Let $\gamma_x > 0$ be the probability a voter is an x-partisan, $x = \mathcal{S}, \mathcal{P}$

Let $\gamma_I > 0$ be the probability a voter is an independent.

An independent voter's preferences characterized by a pair $(\mu, d) \in [0, 1] \times \{r, l\}$

Assume (μ, d) is a random draw from a nicely behaved joint distribution $H(\mu, d)$.

The probability that a randomly drawn independent with belief $\Pr[A|\cdot] = \pi$ votes for $\mathcal{P}$ is therefore

$$v(\pi) = \gamma_I \left[H(\pi, r) + H(1, l) - H(\pi, l)\right] + \gamma_{\mathcal{P}} \in (0, 1)$$

Given (p_A, p_B) and a common belief $\Pr[A] = \pi$, a symmetric (signal and type dependent) voting strategy profile $\mathbf{v}$ yields an expected vote share for $\mathcal{P}$ in state $\omega = A, B$, $V_\omega(\pi)$.

Given n voters and a q-rule, the expected vote shares imply a state-contingent pivot probability for any individual which, in turn, induce an updated belief about the state,

$$\Pr[A | piv(\mathbf{v}), \pi, n, q] = \pi'.$$

A strategy profile $\mathbf{v}^*$ and beliefs π^* are an equilibrium iff

$$\Pr[A | piv(\mathbf{v}^*), \pi^*, n, q] = \pi^*.$$

Given finite n and any q, there exists a solution to the equilibrium condition, say $\pi^*(n, q)$.

Define the *limiting equilibrium condition*, $\pi^*(q) = \lim_{n \to \infty} \pi^*(n, q)$.

For any belief π_0, let $Q(\pi_0) = \{q \in [0, 1] \, | \pi^*(q) = \pi_0\}$.

Theorem (Bhattacharya 2008)

Subject to some mild restrictions on the preference distribution, for almost any belief π_0*,* $Q(\pi_0) \neq \emptyset$*. Moreover,* $Q(\pi_0)$ *is singleton iff* $V_A(\pi_0) \neq V_B(\pi_0)$*.*

Definition

For each alternative $x \in \{\mathcal{S}, \mathcal{P}\}$, say a q-rule is *x-trivial* iff x wins in both states under full information and the rule is *consequential* otherwise.

Let V_ω be the share of voters who prefer $\mathcal{P}$ in state ω and, wlog, assume $V_A > V_B$.

Then: $q < V_B$ is $\mathcal{P}$-trivial; $q > V_A$ is $\mathcal{S}$-trivial; and $q \in (V_B, V_A)$ is consequential.

Definition

The distribution H satisfies *strong preference monotonicity* if a change in signal from b to a induces a strictly larger probability of voters switching from $\mathcal{S}$ to $\mathcal{P}$ than from $\mathcal{P}$ to $\mathcal{S}$ for any increase in belief on state A.

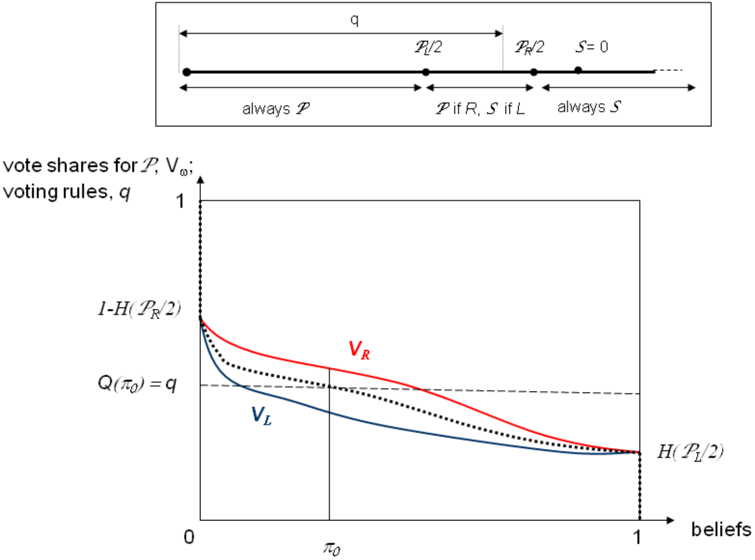
Theorem (Bhattacharya 2008)

The following statements are equivalent:
(1) H satisfies strong preference monotonicity.
(2) For all (p_a, p_b, π), voting satisfies FIE for any non-unanimous q-rule.

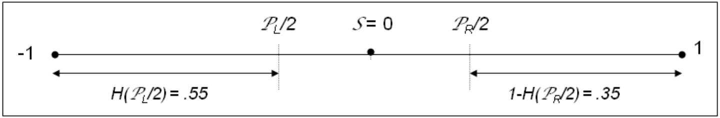
Note: *weak preference monotonicity* limits SPM only to obtain at the given (p_a, p_b, π). In this case, full information equivalence holds for a consequential q-rule iff H satisfies WPM.

Recall the spatial example with $\Omega = \{L, R\}$ and $\{\mathcal{S}, \mathcal{P}\} \subset [-1, 1]$; let $\Pr[L] = \pi$:

Monotonic preferences



Non-monotonic preferences

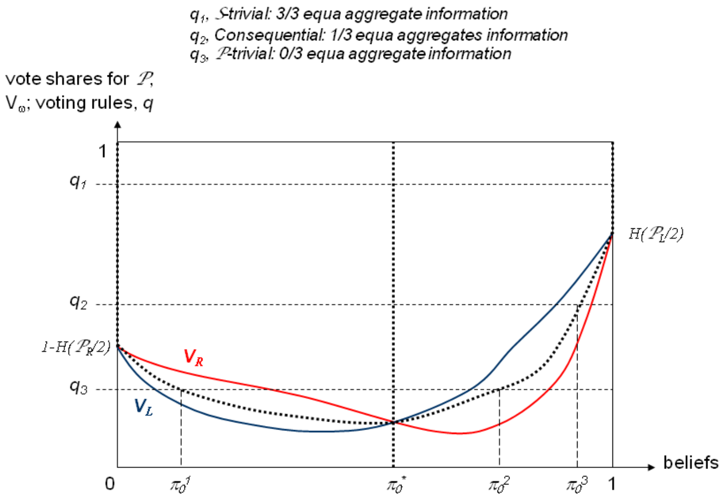


Full information collective choices under a q-rule:

If $q > 55\%$ then $\mathcal{S}$ wins in both states ($\mathcal{S}$-*trivial* rule)

If $35\% \leq q \leq 55\%$ then $\mathcal{P}$ wins if $\omega = L$ and $\mathcal{S}$ wins if $\omega = R$ (*Consequential* rule)

If $q < 35\%$ then $\mathcal{P}$ wins in both states ($\mathcal{P}$-*trivial* rule)


q1, S-trivial: 3/3 equa aggregate information
q2, Consequential: 1/3 equa aggregates information
q3, P-trivial: 0/3 equa aggregate information
vote shares for P,
Vω; voting rules, q
1
q1
q2
q3
1-H(PR/2)
H(PL/2)
VR
VL
0
π0¹
π0*
π0²
π0³
1
beliefs

Although fundamental, preference nonmonotonicity is not the only problem for successful information aggregation.

If there is any additional source of uncertainty correlated with expected vote shares, the mapping connecting vote shares and the state is almost always not invertible and FIE breaks down.

Example (F&P 1997) Unknown distribution of preferences in the F&P model: e.g.

$$H_\lambda = \lambda H_0 + (1-\lambda)H_1$$

with H_λ "consequential" for all $\lambda \in [0,1]$, λ unknown.

Then there exists a nondegenerate interval of states J such that, $\forall \omega \in J$, $\exists \lambda \in [0,1]$ at which ω is consistent with being pivotal.

Hence, FIE is no longer assured as $n \to \infty$. $\square$

Example (Mandler 2008) Unknown signal precision in the Condorcet model: e.g.

$$\begin{array}{lrcl} (1) & \Pr[p_A & = & p_B \approx .5] = z \\ (2) & \Pr[p_A & \gg & p_B] = 1 - z. \end{array}$$

$z \equiv 1$ and pivotal implies informative voting is a best response;

$z \equiv 0$ and pivotal, implies voting A for all signals is a best response.

If $z \in (0, 1)$, the relative likelihood of (2) conditional on being pivotal becomes negligible as $n \longrightarrow \infty$

So there is informative voting in the limit and FIE fails when (2) obtains. $\square$

III Abstention

A resilient question: the probability any given individual is pivotal in a large election is negligible, yet voting costs are positive; so why do rational people vote?

A pragmatic response: in most mature democracies, voting costs are trivial.

However,

(1) observed turnout patterns are broadly consistent with strategic behaviour;

(2) there is significant "roll-off" in multi-race elections;

(3) if voting costs are trivial, why do rational people abstain?

(1), (2) and (3) motivate Feddersen & Pesendorfer (1996) (see also Palfrey & Rosenthal 1983, 1985)

Example Condorcet Jury problem with $N = \{1, 2, 3\}$, and $u(x, \omega) = 1$ if $x = \omega$ and zero otherwise.

Individuals are either uninformed, with prior $\Pr[\omega = A] = \pi > 1/2$, or fully informed. Types are private information.

Suppose $i \neq j$ votes A if uninformed and votes 'correctly' if informed; assume j uninformed;

j pivotal $\Rightarrow$ an informed type is voting $B \Rightarrow j$ voting for A is a mistake.

But uninformed voters voting for B also yields a contradiction ... the "*swing voter's curse*".

Solution: uninformed types should *abstain* and delegate the decision to informed players, even if $\pi \approx 1$. $\square$

F&P (1996) extend the swing voter's curse to a 2-state/2-alternative model in which:

voters are either x-partisans, $x \in \{A, B\}$, uninformed independents, or informed independents;

independents have "Condorcet preferences", x-partisans strictly prefer x in all states;

types are private information with the probability of a randomly chosen voter being any particular type assumed positive and common knowledge;

the number of voters is uncertain and follows a binomial distribution with parameters $(N+1, r)$; N even, $r \in (0, 1)$.

$\gamma_x > 0$ is the probability a randomly chosen voter is an x-partisan; let $\Delta_{AB} \equiv \gamma_A - \gamma_B$ and, wlog, suppose $\Delta_{AB} \geq 0$.

$\gamma_I > 0$ ($\gamma_K > 0$) is the probability a randomly chosen voter is an uninformed (informed) independent.

x-partisans surely vote x and informed independents always vote for the best alternative given the state.

Theorem (Feddersen & Pesendorfer 1996)

Consider equilibrium (symmetric, undominated) voting behaviour in the limit as $N \to \infty$.
(1) If $\gamma_I < \Delta_{AB}$, all uninformed voters vote for B.
(2) If $\gamma_I \geq \Delta_{AB} > 0$, all uninformed voters mix between voting for B and abstaining.
(3) If $\Delta_{AB} = 0$, all uninformed voters abstain.
Moreover, if $(\gamma_K + \gamma_I) \neq \Delta_{AB}$, voting satisfies FIE.

Intuition Suppose $\omega = A$. Then informative voting & N large $\Rightarrow$

$\Pr[A \text{ ahead by one } | i \text{ pivotal}] > \Pr[A \text{ behind by one } | i \text{ pivotal}] \Rightarrow$

Uninformed voters relatively less likely to vote 'correctly' when pivotal (swing voter's curse).

So uninformed independents allocate votes to maximize the opportunity for informed independents to be pivotal in the election, an event that almost surely occurs in the limit.

F&P (1999) extend the theorem to diverse preferences and show that a biased distribution of information yields a biased voting distribution but does not upset FIE.

However, they also observe a necessary condition for abstention in the limit is that the state space is coarse: cf F&P (1997).

McMurray (2008) proves FIE in a common values model with types defined by signals and signal-precision (a random draw from $[\frac{1}{2}, 1]$).

Equilibrium in the n-voter model is characterized by a critical precision $\bar{p}(n) < 1$ such that voters abstain iff $p_i < \bar{p}(n)$, with $\bar{p}'(n) > 0$.

Moreover, although the likelihood of A losing by one vote (given pivotal) declines with n, it does so at a sufficiently decreasing rate that $\lim_{n \to \infty} \bar{p}(n) < 1$.

Bouton & Castanheira (2008) provide a first effort to include multiple candidates, comparing approval voting with plurality rule.

Kim & Fey (2006) and Oliveros (2007) combine abstention with nonmonotonic preferences and show FIE can fail.

IV Costly Information

Suppose signals are costly in the Condorcet model ($X = \Omega = \{A, B\}$) with preferences

$$u(x, \omega) - c(z)$$

where $u(x, \omega)$ is defined as usual:

$$u_i(x,\omega) = \begin{array}{c|c|c|} & A & B \\ \hline A & 0 & -(1-t) \\ \hline B & -t & 0 \\ \hline \end{array}$$

and $c(z)$ is the cost of a noisy signal $s \in \{a, b\}$ with precision $\frac{1}{2} + z$, $z \in [0, \frac{1}{2})$.

Assume: $c(0) = 0$, $\lim_{z \to \frac{1}{2}} c(z) = \infty$ and $c' \geq 0$, $c'' \geq 0$.

The common prior on A is $\pi \in (0, 1)$; individuals simultaneously choose (private) signal precisions and then vote with no abstention.

Definition

An election is (1) *unbiased* if $\pi(1 - t) = (1 - \pi)t$; and (2) *asymptotically efficient* if both equilibrium voting satisfies FIE and the aggregate cost of information converges to zero as $n \to \infty$.

Theorem (Martinelli 2006)

(1) If the election is sufficiently close to being unbiased, then it is asymptotically efficient if and only if $c'(0) = c''(0) = 0$. (2) Suppose each individual's preference parameter t is an iid draw from the uniform distribution on $[0, 1]$; then asymptotic efficiency holds if and only if it is also true that $c'''(0) = 0$. In both (1) and (2), asymptotic efficiency is unavailable if $c''(0) > 0$.

Intuition Information is a public good: as $n \longrightarrow \infty$, the pivot probability becomes negligible so the quality of information acquired declines to zero.

But analogously to F&P (1997), given the conditions on $c(z)$ as $z \longrightarrow 0$, the rate at which individual information vanishes is smaller than the rate at which the number of informed persons increases with n.

V Signaling

Piketty (2000), Razin (2003), Shotts (2006), Meirowitz & Shotts (2009) consider multiperiod models of candidate policy choice under incomplete information (see also Gul & Pesendorfer (2009) for a one period election with policy choice).

The new feature in such models is that, conditional on being pivotal, a voter has two incentives:

vote to promote a most preferred candidate's election (pivot incentive)

vote to influence the winning candidate's de facto policy choice (signaling incentive).

The two incentives typically conflict and FIE is therefore not assured.

Meirowitz & Shotts (2009) ask which incentive dominates in the limit as $n \to \infty$. An answer is given by the following result

Assume n voters with symmetric single-peaked preferences on $[0, 1]$ and ideal points drawn (iid) from a differentiable cdf F.

F is common knowledge but ideal points are private information.

A first (majority) election is between two fixed alternatives, $A < B$; in the second, two candidates observe first election vote totals and freely choose policy platforms in $[0, 1]$, following which voters vote a second time.

Candidates are purely office-oriented and voter payoffs are the sum of the two period payoffs (no discounting).

Theorem (Meirowitz & Shotts 2009)

In the limit as $n \longrightarrow \infty$, voters vote in the first election purely on the basis of the signaling incentive.

Intuition Although the return to winning is potentially large in the first election, the pivot probability goes to zero quickly as $n \longrightarrow \infty$.

The marginal impact of one extra vote affecting candidate policy decisions in the second election likewise becomes negligible as $n \longrightarrow \infty$, but since candidates converge in this election, it continuously influences the final outcome whoever wins.

Therefore, in the limit, the signaling incentive dominates the pivot incentive determining voters' behaviour in the first election.

VI Sequential Voting

Callander (2007) builds a model of sequential voting with an arbitrarily small but positive benefit for voting for the winner, over and above the canonical Condorcet policy preferences. (See also Fey (1998) and Ali & Kartik (2008).)

Pivotal voters tradeoff voting to secure a winner and voting otherwise to permit further information to be aggregated into the final choice

Bandwagon voting and momentum occur with probability one (but not necessarily early in the sequence).

VII Conclusion

Two positive conclusions:

(1) Nonunanimous voting rules can successfully aggregate information under a variety of settings

(2) Informational models of abstention can account for a variety of empirical regularities in two candidate plurality elections.

Moreover, there is growing (and surprising?!) experimental evidence for strategic voting and swing voter curse behaviour (Guarnescelli et al 1998; Battaglini et al 2006, 2008).

And two caveats:

(1') Successful information aggregation seems unlikely in general (Bhattacharya, Feddersen and Pesendorfer, *in preparation*)

nonmonotonic preferences

multidimensional uncertainty

(2') Given the practical irrelevance of pivot events for individuals in large elections, group-based models seem likely to prove more useful for understanding turnout (Uhlaner 1989; Morton 1991; Feddersen & Sandroni 2006a,b,c).