# Moral Hazard in Dynamic Insurance, Classification Risk and Prepayment

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**Abstract**

This paper examines the effect of moral hazard on dynamic insurance contract. It models primary prevention in a two period model with classification risk. Agents' preferences appear to play an important role in the determination of preventive effort and prepayment. If absolute prudence is larger than twice absolute risk aversion, moral hazard increases prepayment of premium and classification risk . This highlights a tradeoff between prevention and prepayment that arises from the classification risk. An increase in the difference between prudence and twice risk aversion (that we define as the degree of "protectiveness") moreover makes dynamic insurance contracts more stable (when competing with spot insurance) if the cost of prevention is low enough when agents preferences exhibit CRRA. Under a formulated utility function with linear reciprocal derivative, we finally show that an increase in agents' degree of "protectiveness" enhances the stability of dynamic contract and the extent of prepayment.

**Key words**: Dynamic Insurance, Classification Risk, Moral Hazard, Prudence

**JEL Classification No.** : D81, D91, G22

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# 1 Introduction

Thanks to technological progress in medicine, people tend to live longer. If most individuals also live healthier[1], these technological progress moreover allows patients affected by chronic illness to have higher life expectancy. This raises a new issue in medical insurance, as insurers tend to charge this new class of agents – that needs to be covered for a long time against high expected heath cost of treatment – with high premia. It therefore creates – for agents that contract a chronic condition – a risk of being reclassified "high-risk" by his insurer and therefore to pay a high premium. Insurance literature refers to this risk as the classification risk.

One possible option for reducing these premia is the use of to dynamic insurance, that is of long-term insurance contract. This allows for risks mutualization through intra- and inter-generational insurance. On the one hand, younger agents can subsidize older ones, as they expect to benefit form similar subsidies when old. This subsidy corresponds to early payment of future premia, and is referred to as front-loading. On the other hand, when old, low risk may subsidize high risk agents. These two mechanisms tend to decrease the premium of old agents highly exposed to risk, and therefore reduce classification risk.

This process however introduces new issues. First, intra-generational insurance may lead to the exit of low risk policyholders. Insurance contract are characterized by one-side commitment and agents that turn out to be low risk may have an incentive to leave the contract if they find more profitable outside options (spot market for example). This phenomenon is sometimes refereed to as (cream)-skimming.

The reduction of classification risk moreover raises a moral hazard issue. Being insured against the risk of being considered high risk reduces the incentive to exert preventive efforts that decreases the probability of becoming more risky. In the following we will refer to such effort - that can correspond to safe behavior or to "hygieno-dietetic" regime for example - as primary prevention [2].

[1] The literature on medical science refers to this phenomenon as healthy ageing

[2] This terminology comes from medical science. The U.S. Preventative Services Task Forces' Guide to Clinical Preventive Services (2d edition, 1996) defines primary prevention as interventions that reduce the risk of disease occurrence in otherwise healthy individuals. Secondary prevention measures then corresponds to identifying and treating "persons who have already developed risk factors or preclinical disease but in whom the condition is not clinically apparent". It thus can

Recent initiatives on insurance market have highlighted the interest for insurers, and especially mutual insurers, for primary prevention. In 2005 and 2007 respectively, French mutual insurers AGF and MAAF have begun to reimburse some alimentary products designed to lower the cholesterol level. A similar program has also been introduced in 2005 by the Dutch insurer VGZ.

The aim of the present paper is to introduce the notion of primary prevention in dynamic insurance. We define the optimal dynamic contract under moral hazard on the probability of becoming high risk. We then analyze the impact of moral hazard on front-loading, classification risk and cream-skimming. This way we are able to infer the stability of dynamic insurance contract that account for preventive effort.

To do so we build a two-period model of mutual health insurance. During the first period, agents are identically exposed to health risk and can invest in primary prevention. In period 2, agents can either be high risk or low risk type. The amount of effort spent in period 1 reduces the probability of being high risk that is the probability of having a high probability of falling ill in period 2. When effort is observable and contractible upon, dynamic insurance fully insures against classification risk. However, when effort is unobservable, the insurance offered during the second period depends on risk type (that we assume to be observable and public information). This raises the issue of classification risk.

This paper highlights a trade-off between two behaviors toward this future risk.

On the one hand, thanks to dynamic insurance, agents can transfer wealth between the two periods through prepayment of premia (i.e. intergenerational insurance). By paying a higher premium during the first period, they can reduce second period premia and classification risk. Such a mechanism can therefore be related to pain disaggregation and to the notion of prudence (see Eeckhoudt and Schlesinger 2006). However, as it allows an unequal repartition of the prepaid premium between the two states of the second period[3], this mechanism also appears to be linked to risk aversion.

be related to "self-protection" (see Ehrlich and Becker 1972) or "loss prevention" (see Mehr and Commack 1966). Finally, tertiary prevention concerns "care of established disease, with attempts made to minimize the negative effects of disease" and thus correspond to "self-insurance" in Ehrlich and Becker 1972 or "loss protection" in 1966.

[3]In this sense, prepayment of premia appears to be more flexible than precautionary saving

On the other hand, to reduce the classification risk, agents can exert effort of primary prevention. As shown by Chiu (2000) and Eeckhoudt and Gollier (2005) in the case of self-protection, prevention can also be related to both prudence and risk aversion.

This suggests that the trade-off between the two means of reducing classification risk depends on prudence and risk aversion.

The present paper confirms this intuition and shows that the critical level of the ratio (absolute) prudence to (absolute) risk aversion is 2. If absolute prudence is larger than twice absolute risk aversion, to respond to future uncertainty, agents transfer wealth in second period rather than exert effort, that is they protect themselves rather than prevent. We will then refer to agents with these preferences as being "protective". We show in this paper that, when agents are protective, moral hazard (through the unobservability of preventive efforts) increases the first period premium (and hence enhances prepayment). On the contrary, if/when agents are not protective, that is if they favor prevention (rather than inter-period transfers), moral hazard reduces classification risk and intergenerational insurance. As a scope for public action, we also prove that the classification risk can be reduced by a decrease in the cost of prevention (whatever the degree of protectiveness) or by increasing the effectiveness of prevention when agents are protective.

With CRRA (Constant Relative Risk Aversion) preferences, it appears that an increase in agents' degree of protectiveness leads to a decrease in the premium offered to low risk agents in second period, if the cost of effort is low enough. Then, the more protective its policyholders, the less contestable (by spot insurers) a mutual insurer that offers long-term contracts. After having defined a suitable utility function - that satisfies the simplifying property of having a linear reciprocal derivative - we moreover show that the various degrees of front-loading and lapsation observed in insurance contracts (see Hendel and Lizzeri 2003) can be explained by heterogeneity in agents' preferences. Lastly, we highlight the fact that cross-subsidization allows to increase front-loading for agents with a strong preference for present and therefore to improve the stability of dynamic contracts.

The issue of classification risk has received significant attention in the literature on long term insurance. To reduce this risk that may make insurance unaffordable for most risky agents, Pauly et al. (1995) propose guaranteed renewable insurance policies, that consists in a declining schedule of premia over time. They construct a scheme in which the premium is always lower than the expected future lifetime expenses of the lowest risk buyers. Frick (1998) however points out that such solution may not be observed as it involves high premia early in life. If agents are too impatient and if they face borrowing constraint, they will at most purchase partially guaranteed renewable insurance.

Alternatively, Cochrane (1995) proposes time-consistent insurance contracts that provide insurance against classification risk using severance payments. When an agent turn out to be high risk, she receives a lump sum equals to the increased present value of his premium. Severance payments compensate for changes in premium and allow every consumer to purchase insurance at his actuarially fair premium. Pauly et al. (1998) argue that the effectiveness of this scheme highly relies on the assumption of perfect credit market, and Hendel and Lizzeri (2003) point out that such contracts can not be implemented in life insurance for legal reasons.

In a two-period model similar to ours, Hendel and Lizzeri (2003) analyzes to what extend prepayment of premia (front-loading) can reduce classification risk when accounting for cream-skimming. They state that front-loading allows reducing both cream-skimming (low risk are insured at their fair premium in second period) and classification risk (agents with different types have the same insurance contract)[4]. On the basis of these findings, they moreover argue that the various degrees of front-loading and lapsation observed in insurance data can be explained by heterogeneous costs of front-loading (that is by heterogeneous profiles of income growth).

Through primary preventive effort we add moral hazard to this model. To be incentive compatible, the optimal contract then necessarily specifies different insurance schemes to different types. This offers an alternative explanation to the stylized fact highlighted by Hendel and Lizzeri (2003). Accounting for moral hazard, the observed variety of front-loading and lapsation, can be explained by heterogeneous behavior toward risk (risk aversion and prudence).

[4]Contrary to us, Hendel and Lizzeri (2003) allow for more than two risk types in second period

Our paper is not the first attempt to introduce moral hazard in dynamic insurance contract. Abbring et al. (2003) use dynamic insurance contract in their empirical study on the distinction between moral hazard and adverse selection. They however analyze moral hazard on the probability of accident, that is secondary prevention. We focus here on primary prevention. An important implication of the difference is that primary prevention - contrary to secondary prevention - leads to dynamic moral hazard. The effort exerted in current period reduces the probability of being high risk next period.

Nishimura (2004) analyzes the effect of primary prevention on front loading in Hendel and Lizzeri's life insurance model. He characterizes the condition under which the optimal contract involves front-loading. The emergence of front-loading depends on agents' risk aversion and on the effectiveness of prevention. He then studies capital market structure and the scope for government action. We however want to focus here on classification risk and on the conditions under which dynamic contracts can prevent from agents to leave the company when they turn out to be low risk.

In the next session, we define the notion of protectiveness and analyze in which context it arises in the economic literature. We present the model in section 3. The optimal dynamic contract under moral hazard is defined in section 4 and general results of comparative statics are provided in Section 5. To go further in the analysis of comparative protectiveness we then rely on specific utility function in section 6, and we interpret our results in term of sustainability of dynamic insurance with respect to spot insurance in section 7. A brief extension with cross-subsidization among agents heterogeneous in protectiveness is provided in section 8. We present a possible application of our model to unemployment and life insurance in Section 9, and eventually outline our conclusion and directions for future research in Section 9.

## 2 The notion of protectiveness

It is now well established that the inverse of marginal utility ($1/u'$) plays a preponderant role in models with model hazard. Our paper emphasizes the influence of the degree of concavity of this function, that is $(1/u')''$. This is not the first work where the second derivative of the inverse of marginal utility matters. In a principal-agent model with moral hazard, Newman (1995) shows that, if $1/u'$ is convex, an increase in wealth makes the incentive scheme more expensive for the principal. Then, the higher the initial wealth of the agent, the lower the expected profit of the principal. Thiele and Wambach (1999) generalize this condition and state that for this statement to hold, it is sufficient to assume that an agent with utility function $1/u'$ is less risk adverse that an agent with utility function $u$. This last condition corresponds to an absolute index of prudence (introduced by Kimball 1990) smaller than three times the absolute index of risk-aversion ($P \leq 3A$) when the convexity of the inverse of marginal utility writes $P \leq 2A$ [5]. In the general case, Amir and Czupryna (2004) prove that $P \geq kA$ is equivalent to (k-2)-concavity of the first derivative of the inverse utility function. Moreover, Eeckhouldt and Gollier (2000) show that two independent risks are substitutes if absolute prudence is decreasing and larger than twice the absolute risk aversion.

The second derivative of the inverse of marginal utility function is also crucial in models with uncertainty in the probability of damage. In studying environmental problems and the 'precautionary principle', Gollier et al. (2000) build a two-period model where the uncertain damage in second stage depends on the consumption of both periods. They moreover assume that researchers work on (Bayesianly) revising the beliefs on the distribution of uncertainty. One of their main results is that a better information structure (that is beliefs more dispersed) decreases (resp. increases) the efficient level of consumption in first stage if $P \geq 2A$ (resp. $P \leq 2A$). In this sense, if prudence is larger than twice risk aversion, progress induces precautionary savings.

[5] With constant relative risk aversion utility functions these condition is equivalent to a degree of relative risk aversion larger than 1/2 and 1 respectively

Gollier (2002) finds a similar result when analyzing a dynamic model of prevention with uncertainty on the probability of loss and bayesian revisions. He assumes that, in both periods, agents can exert an effort of (tertiary) prevention that reduces the amount of loss in case of damage and have the possibility to save during the first period. Moreover, agents revise their belief about the probability of loss in the second period on the basis of what they have observed during first stage. His main results are that (i) an increase in the expected probability of loss increases (resp. decrease) the marginal value of savings if $1/u'$ is convex (resp. concave) and (ii) the uncertainty on the probability of loss increase (resp. decrease) the efficient level of first stage effort if $P \leq 2A$ (resp. $P \geq 2A$). Our model differs from Gollier's in at least two respects. First of all we allow for insurance in both stages when Gollier (2002) only models savings in first stage. In the present paper, risk (of classification) is then endogenous as it depends on insurance offered in both states of nature in second period. Moreover, whereas Gollier studies self protection and bayesian revision of probability we analyze in this paper the effort of primary prevention that is an effort that impacts the probability of having a high probability of damage.

This brief review of literature highlights the trade-off that arises from an increase in future uncertainty, between a decrease in present consumption (that is an increase in savings) and an increase in effort. If $P \geq 2A$, that is if $1/u'$ is concave, the precautionary motive (or the preference for pain disaggregation) dominates and to face uncertainty, the agents save rather than exert the effort. We therefore define such agents, that protect themselves rather than prevent, as being "protective".

**Definition 1** *In the following, an agent will be said to be "protective" when the inverse of the marginal utility is concave, that is when the index of absolute prudence is larger that twice the index of risk aversion.*

## 3 The Model

To analyze the impact of moral hazard on prepayment and classification risk, we build an overlapping generation model (to capture the case of social insurance) with change in risk exposure during the life cycle. We model the simplest 2-period, 2-type case and assume that (homogeneous) newborn agents can affect their second period health status through primary prevention.

Consider overlapping generations (of same size) living for two periods $t = 1, 2$. At each period, identical agents receive a sure revenue $R$. During the first period, (young) individuals face the same risk, that is the same probability $q_1$ of suffering a loss $L$. Let us note $K_1 \equiv q_1 L$ the expected loss, that is the expected health cost in the case of health insurance. At $t = 2$, (old) agents may be of two types. Either, with probability $p$, they are low risk type and face a probability of loss $q_2^l$ ($K_2^l \equiv q_2^l L$) or, with probability $1 - p$, they are high risk type and suffer loss with probability $q_2^h$, with $q_2^l < q_2^h$ (therefore $K_2^l < K_2^h \equiv q_2^h L$).

Information about agents' risk type is revealed at the beginning of second period (for example through medical check-ups) and is then public information. Young agents can exert a primary preventive effort that reduces the probability of becoming high risk type in second period [6]. We assume that agents choose among two levels of prevention $\underline{e}$ and $\overline{e}$ ($\underline{e} < \overline{e}$) leading respectively to probabilities of being low risk $p(\underline{e}) \equiv \underline{p}$ and $p(\overline{e}) \equiv \overline{p}$, with $\underline{p} < \overline{p}$. Let us note $\Delta p \equiv \overline{p} - \underline{p}$.

Let $X_i^j$ be the wealth of agents of type $j$ in period $i$. In the absence of insurance, the income profile of a newborn agent can be schematized as follows

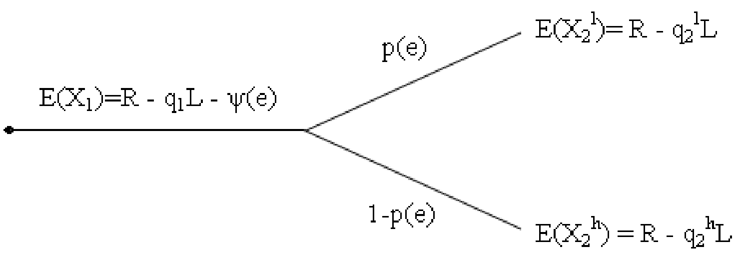


Figure 1: The income profile without insurance

[6]In our setting, an effort of secondary prevention would reduce $q_2^l$ and $q_2^h$, and corresponds to self-protection; whereas effort of tertiary prevention reduces $L$ and corresponds to self-insurance

During the first period, the utility function is supposed to be separable in wealth and effort and the utility-cost of exerting high effort of primary prevention is noted $\psi \equiv \psi(\overline{e}) - \psi(\underline{e})$. We moreover assume time separability of preference (to distinguish saving and insurance behavior)[7], and for the sake of simplicity, that utility is linear in wealth during the first period.

Let us note $u(.)$ (with $u'(.) > 0$ and $u''(.) < 0$) the utility function of both types of old agents. We assume that there is no direct utility loss due to health status. This would anyway worsen the welfare of high risk agents and therefore advocate for a lower classification risk.

To be insured against this two-period risk, a (benevolent) mutual insurer offers to young agents a dynamic insurance contract, that is a contract specifying premia and coverage for both periods and depending on risk status in second period[8]. The timing of the game is described in Figure 2

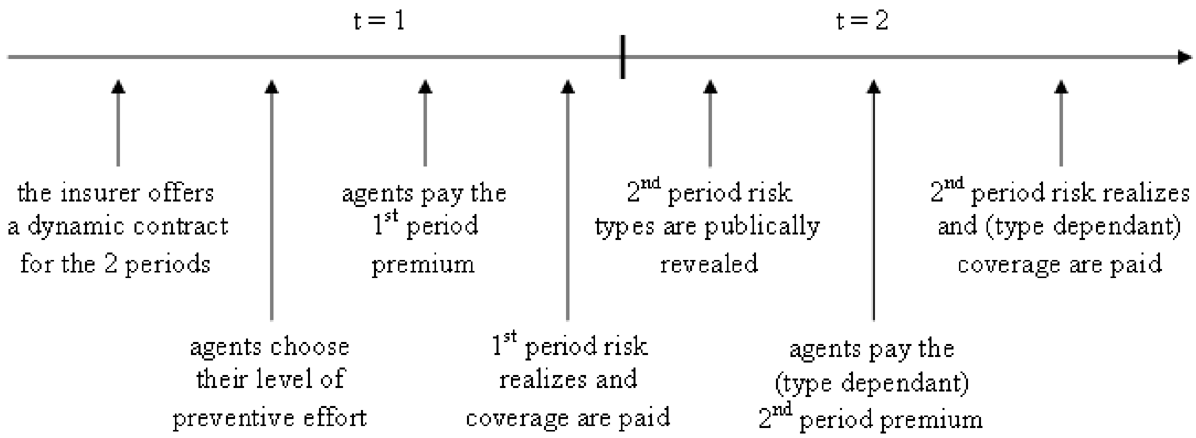


Figure 2: The timing of the game

We moreover assume that there is no market insuring against the classification risk as proposed by Tabarrok (1999) when studying the issue of genetic testing (he calls it "genetic insurance"). Here, dynamic insurance contracts allow the insurer to use prepayment of premia in first period to decrease the premium offered to high risk type agents when old (as shown in Hendel Lizzeri 2003).

[7]The use of incentive constraints prevent us to model Kreps-Porteus preferences. This would allow us to fully disentangle risk and time effects. However, the use of such non-expected utility makes the problem untractable as it greatly complicates the writing of the incentive compatible constraint

[8]The reader should note that the following problem also fits in the case of competing insurance companies that do not seek to propose profitable one-period contract. We briefly discuss this alternative interpretation in Section 6.

However, when effort is unobservable and not contractible upon, this may lower the incentive to exert high preventive effort. The aim of this paper is thus to analyze the trade-off resulting from a decrease in premium of high risk in second period, between an increase in insurance and a decrease in the incentive for primary preventive effort. This then allows us to analyze under which condition the optimal contract resulting from this trade-off is sustainable when we introduce the possibility of one-period (spot) insurance.

Note here that prepayment of premia can be related to precautionary saving as it corresponds to an intertemporal transfer of wealth used to deal with future uncertainty. These two mechanisms are however different in fundamental respect that plays a important role in our setting. Indeed, whereas savings have the same return in every future state (and therefore don't have much impact on classification risk), the insurance company can choose to reallocate the prepaid part of premia differently across the states (what can have an important impact on classification risk).

# 4 The optimal dynamic contract

## 4.1 The benchmark case of observable effort

It is first easy to show, using the concavity of the utility function, that the dynamic insurance contract necessarily specify complete insurance (in the sense that it provides an agent with the same wealth whether she suffers the damage or not) once risk types are known [9]. A dynamic contract is therefore fully defined by a triplet $(\Pi_1, \Pi_2^l, \Pi_2^h)$ of premia corresponding respectively to the expected costs $K_1$, $K_2^l$ and $K_2^h$, and the coverage is in any cases equal to the amount of the loss $L$. Under this dynamic insurance contract, the income profile of a newborn agent can then be schematized as follows:

[9] This issue is more problematic in Hendel and Lizzeri (2003) and Nishimura (2004) that model life insurance and therefore specify state (alive/dead) dependent utility function

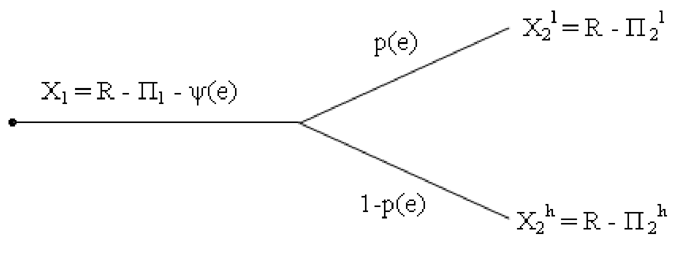


Figure 3: The income profile under the insurance contract

The risk of being classified high risk is then measured by the difference between the second period premia.

**Definition 2** *The classification risk corresponds to the risk of being classified high risk by his insurer and therefore to paid a high premium. In our two-type model with complete insurance in each state, this risk is simply measured by the spread between the premia paid by each type in second period:* $\Pi_2^h - \Pi_2^l$

The mutual insurance firm, as non profit organization, then seek to maximize the expected utility of a young individual that exert an effort e:

$$(R - \Pi_1) - \psi(e) + p(e)u\left(R - \Pi_2^l\right) + (1 - p(e))u\left(R - \Pi_2^h\right) \tag{4.1}$$

By definition the use of external capital is excluded in a mutual organization. However, if the mutual insurer is large enough, it can rely on the law of large numbers, and the zero profit condition writes

$$\Pi_1 + p(e)\Pi_2^l + (1 - p(e))\Pi_2^h = K_1 + p(e)K_2^l + (1 - p(e))K_2^h \equiv E(K|e) \tag{4.2}$$

This states that the sum of premia collected (from young and old agents) allows (in expectation) for the reimbursement of heath costs. Recall here that we consider an overlapping generation model with identical agents (that therefore exert the same prevention effort) and generations of the same size.

With observable effort it is then optimal to set ${\Pi_2^l}^* = {\Pi_2^h}^* = \Pi_2^*$ such that $u'(R - \Pi_2^*) = 1$ and $\Pi_1^* = E(K|e) - \Pi_2^*$. Therefore, the optimal premia in second stage are independent of the level of preventive effort and there is no classification risk at the optimum. However, the premium paid at first stage is decreasing with the level of effort as $p(\overline{e}) > p(\underline{e}) \Rightarrow \overline{K} \equiv E(K|e = \overline{e}) < E(K|e = \underline{e}) \equiv \underline{K}$.

Therefore, without moral hazard, assuming $\underline{K} - \overline{K} > \psi$ the optimal contract specifies:

$$\begin{cases} e = \overline{e} \\ {\Pi_2^l}^* = {\Pi_2^h}^* \equiv \Pi_2^* \text{ with } u'(R - \Pi_2^*) = 1 \\ \Pi_1^* = \overline{K} - \Pi_2^* \end{cases} \tag{4.3}$$

## 4.2 The optimal dynamic contract under moral hazard

Now, if efforts of primary prevention are not observable, that is under moral hazard, agents have an incentive the exert the maximal level of effort if the insurance contract satisfies

$$u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \frac{\psi}{\Delta p} \tag{4.4}$$

Therefore, the optimal contract that gives the incentive to exert the high level of effort[10] is solution of:

$$\max_{\Pi_1, \Pi_2^l, \Pi_2^h} \quad (R - \Pi_1) - \overline{\psi} + \overline{p} u\left(R - \Pi_2^l\right) + (1 - \overline{p}) u\left(R - \Pi_2^h\right) \tag{4.5}$$

$$\text{s.t.} \quad \begin{cases} \Pi_1 + \overline{p}\Pi_2^l + (1 - \overline{p})\Pi_2^h \geq \overline{K} \\ u(R - \Pi_2^l) - u(R - \Pi_2^h) \geq \dfrac{\psi}{\Delta p} \end{cases}$$

The contract solution of this program then represents the overall optimum if it provides agents with more expected utility that the contract $(\underline{K} - \Pi_2^*, \Pi_2^*, \Pi_2^*)$, the optimal contract with low effort. We only focus in the following on the optimal incentive compatible contract. We don't discuss the issue of the optimal level of effort and rather assume that it is optimal for all agents to exert the maximal level of effort[11].

[10] The generalization to continuous effort seems difficult as it would introduce marginal utility in the program through a two-step optimization. However, our model seems to be easy generalizable to a finite number of effort levels. As we focus on the incentive to exert the maximal level of effort, the contract would be incentive compatible if for each level of effort, the benefit of exerting the highest effort outweighs the cost. The binding incentive constraint would then correspond to the level of effort that has the highest cost-benefit ratio.

[11] In a static model, Jullien et al. (1999) gives condition under which more risk-averse agents optimally exert more effort of secondary and tertiary prevention (self-insurance and self-protection)

The experienced reader should note that the program (4.5) – that correspond to a dynamic extension of the classical moral hazard problem – can be easy related to a principal-agent model with outside option effect that can be formulated (in a two-state, two-level of effort case) as:

$$\max_{X_1, X_2^l, X_2^h} \quad X_1 \tag{4.6}$$

$$\text{s.t.} \quad \begin{cases} X_1 + \overline{p} X_2^l + (1-\overline{p}) X_2^h \leq \overline{W} \\ u(X_2^l) - u(X_2^h) \geq \dfrac{\psi}{\Delta p} \\ \overline{p} u(X_2^l) + (1-\overline{p}) u(X_2^h) \geq u(Y) \end{cases}$$

where $X_1$ represent the principal payoff; $X_2^l$ and $X_2^h$ the revenues of the agent in two states that occur with respective probability $\overline{p}$ and $(1-\overline{p})$ when the agent exerts the effort at a cost of utility $\psi$; $\overline{W}$ the total wealth and $Y$ the outside option of the agent. The result of Thiele and Wambach (1999), induces in the setting that if $P > 3A$ - and consequently if the agent is protective - under unobservable efforts makes the incentive scheme less expensive for the principal $\Big(\dfrac{\partial X_1}{\partial Y}$ is higher under moral hazard than if effort is observable$\Big)$.

Protectiveness will also play a preponderant role in our setting when analyzing the impact of moral hazard. The solution of program (4.5) is defined by[12]:

$$\begin{cases} u(R - \Pi_2^{l\,**}) - u(R - \Pi_2^{h\,**}) = \dfrac{\psi}{\Delta p} \\ \dfrac{\overline{p}}{u'\left(R - \Pi_2^{l\,**}\right)} + \dfrac{1-\overline{p}}{u'\left(R - \Pi_2^{h\,**}\right)} = 1 \\ \Pi_1^{**} = \overline{K} - \overline{p}\Pi_2^{l\,**} - (1-\overline{p})\Pi_2^{h\,**} = \overline{K} - E(\Pi_2^{**}) \end{cases} \tag{4.7}$$

Therefore, if $(1/u')$ is concave (resp. convex), the optimal incentive contract under moral hazard satisfies $\dfrac{1}{u'\left(R - \overline{K} + \Pi_1^{**}\right)} = \dfrac{1}{u'\left(R - E\left(\Pi_2^{**}\right)\right)} > \dfrac{\overline{\pi}}{u'\left(R - \Pi_2^{l\,**}\right)} + \dfrac{1-\overline{\pi}}{u'\left(R - \Pi_2^{h\,**}\right)} = 1$ $\Big($ resp. $\dfrac{1}{u'\left(R - \overline{K} + \Pi_1^{**}\right)} < 1\Big)$.

As, under observable effort, $\dfrac{1}{u'(R - \overline{K} + \Pi_1^{*})} = \dfrac{1}{u'(R - \Pi_2^{*})} = 1$, the next proposition hold.

[12] Note here that the first order condition of the program (4.5) is the same than in the standard moral hazard model (see Laffont and Martimort 2002, Chapter 4)

**Proposition 1** *If agents are protective (resp. not protective) in second period, moral hazard enhances (resp. reduces) prepayment of premia* $\left(\text{ as then } \Pi_1^{**} > \Pi_1^*, \text{ resp. } \Pi_1^{**} < \Pi_1^* \right)$.

Proposition 1 can be linked with the finding of Gollier et al. (2000). They state that if $P \geq 2A$, better information structure reduces the efficient level of consumption in first period. They define a "better information structure" as a more dispersed probability distribution. Therefore, it corresponds to more uncertainty. In our setting, moral hazard also correspond to more uncertainty as under observable effort, agents have the same level of wealth in second period whatever their type. As in Gollier et al. (2000) if $P \geq 2A$, this leads to a decrease in first period consumption through an increase in the premium paid in first stage. Gollier et al. (2000) interpret this condition as coming from two conflicting effects that arises from an increase in uncertainty. First, the increase in uncertainty decreases first period consumption because of 'precautionary motives' (pain disaggregation). This effect is more important the larger the index of absolute produce $P$ introduced by Kimball (1990). However, as agents are risk adverse, the increase in uncertainty reduces expected wealth in second period. Therefore, it increases the marginal value on first period revenue and thus tends to reduce prepayment. The intensity of this effect is reflected by the index of absolute risk aversion $A$. They state that the first effect dominates if the index of absolute prudence is larger than twice the index of absolute risk aversion, that is if agents are protective.

In our context, the condition $P \geq 2A$ can also be related to the moral hazard concerns. First, the uncertainty during second period leads to prepayment of premia (that may be linked with precautionary saving) if prudence is high. In our setting, through the zero profit condition, this increases average wealth in second period. Then, because of the concavity of the utility function, the optimal contract has to exhibit a higher classification risk (a higher spread between second period premia) to remain incentive compatible. This last effect goes against an increase in first period premium, and dominates if agents are "too risk adverse" relatively to their prudence. Proposition 1 states that the will be the case if $\frac{A}{P} \geq \frac{1}{2}$.

Now turn to second period premia. The first order condition gives $\frac{\overline{p}}{u'\left(R-\Pi_2^{l\,**}\right)} + \frac{1-\overline{p}}{u'\left(R-\Pi_2^{h\,**}\right)} = \frac{1}{u'\left(R-\Pi_2^*\right)}$. Moreover, the incentive constraint implies $\Pi_2^l < \Pi_2^h$. This leads to the following proposition.

**Proposition 2** *Whatever the extent of prepayment of premia when young, the unobservability of effort improves the welfare of low risk agents and worsens the welfare of high risk agents when old. Therefore, it increases classification risk.*

This result is mainly driven by the incentive scheme. The first order condition leads to decreasing relationship between $\Pi_2^l$ and $\Pi_2^h$ and is satisfied at the first best contract $\left(\Pi_2^l = \Pi_2^h = \Pi_2^*\right)$. Now, to be incentive compatible, the optimal contract necessarily specifies $\Pi_2^l < \Pi_2^h$. Therefore, at the optimum $\Pi_2^l < \Pi_2^* < \Pi_2^h$.

It is worthwhile to note that second period premia are fully determined by the first order condition and the incentive constraint. The feasibility constraint then determines the premium paid when young depending on expected second period premia. This allows analyzing the solution graphically. To do so let us recall $X_1 \equiv R - \Pi_1 - \overline{\psi}$, $X_2^l \equiv R - \Pi_2^l$, $X_2^h \equiv R - \Pi_2^h$ and study the optimal premia in the plan $(X_2^l, X_2^h)$.

Consider first the incentive constraint $u\left(X_2^l\right) - u\left(X_2^h\right) = \frac{\psi}{\Delta p}$. In the plan $(X_2^l, X_2^h)$ it defines an increasing and concave curve below the 45-degree line (labeled IC in Figure 4). Moreover, the distance between the incentive constraint and the 45-degree line is increasing in $X_2^l$ as the function $f\left(X_2^l\right) = X_2^l - u^{-1}\left(u\left(X_2^l\right) - \frac{\psi}{\Delta p}\right)$ is increasing in $X_2^l$ when $u'\left(X_2^l\right) < u'\left[u^{-1}\left(u\left(X_2^l\right) - \frac{\psi}{\Delta p}\right)\right] = u'\left(X_2^h\right)$.

In the plan $\left(X_2^l, X_2^h\right)$ the first order condition $\dfrac{\overline{p}}{u'\left(R - \Pi_2^{l**}\right)} + \dfrac{1-\overline{p}}{u'\left(R - \Pi_2^{h**}\right)} = 1$ (labeled FOC in Figure 4), corresponds to a decreasing curve going through point $(X_2^*, X_2^*)$. It is moreover tangent to the line $pX_2^l + (1-p)X_2^h = X_2^*$ at $(X_2^*, X_2^*)$, convex is agents are protective and concave otherwise.

On the basis of those two curves, we can then infer the first period wealth with the zero-profit condition that can be written as $E(X_2) = -X_1 + 2R - (\overline{K} + \overline{\psi})$. This effect is represented in Figure 4 through the line $E(X_2) = c$.

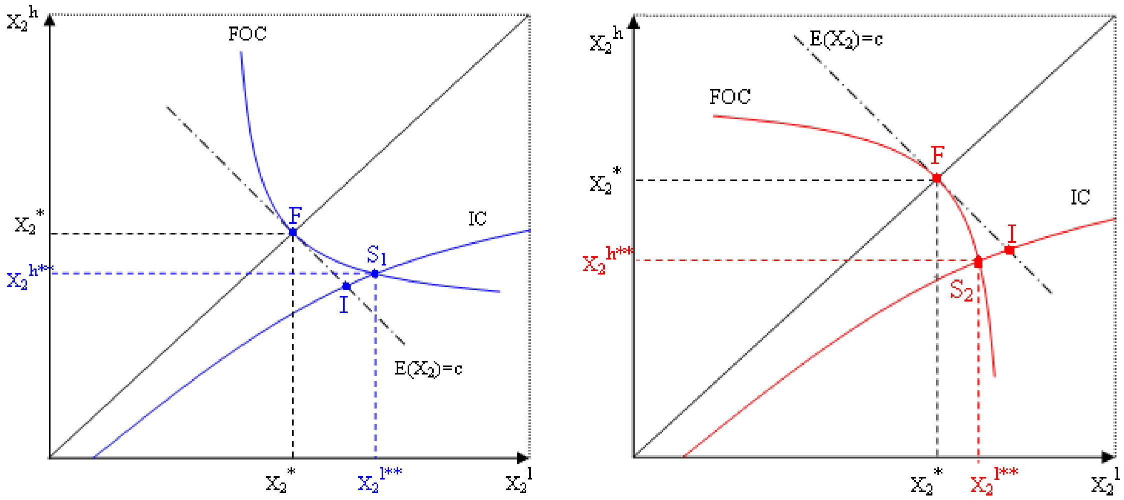


Figure 4: The impact of protectiveness on second period premia

This graphical analysis highlights that, depending on protectiveness, moral hazard has different effects on second period premia. It first appears that whatever the concavity of $1/u'$, an "incentive" effect leads - relative to the complete information benchmark - to an increase in wealth for low risk agents (decrease in $\Pi_2^l$) and a decrease in wealth for high risk agents. Graphically this corresponds to a move along the line $E(X_2) = c$ from point $F$ to point $I$. This effect is combined with a "protectiveness" effect that depends on the concavity of $1/u'$. Indeed, if $1/u'$ is concave (left hand side figure) the "incentive" effect is coupled with a move to the north-east along the incentive constraint from point $I$ to $S_1$. Therefore, when agents are protective, the "protectiveness" effect corresponds to a decrease in both second period premia. This last effect moreover leads to the increase in prepayment described in proposition 1, as it move the line $E(X_2) = c$ upward.

The reverse effect (represented by a move from $I$ to $S_2$) holds when $1/u'$ is convex (right hand side figure). The "protectiveness" effect then corresponds to a decrease in both $X_2^l$ and $X_2^h$ (that leads to the increase in $X_1$ found in Proposition 1). However, as the first order condition is decreasing, the "incentive" effect always dominates and Proposition 2 holds.

Propositions 1 and 2 confirm our interpretation in term of protectiveness of the condition $P \geq 2A$. As shown in Crainich and Eeckhoudt (2005), prudence, as well as risk aversion, can be interpreted as a preference toward risk. Here we show how these two kinds of preferences can be linked. In our setting, agents can use two mechanisms to reduce classification risk in second period. Either they can exert a preventive effort that reduces the probability of becoming high risk, or they can transfer wealth from period 1 to period 2 though prepayment of premia. Being linked with pain disaggregation, this last mechanism is related with the notion of prudence. However, through the incentive constraint and the zero profit condition, prepayment also influence classification risk and is therefore related by risk aversion. Similarly, as shown by Chiu (2000) and Eeckhoudt and Gollier (2005), preventive effort is related to both prudence and risk aversion.

Here we show that the preference for one mechanism to another is driven by the ratio absolute prudence to absolute risk aversion and more precisely, by the position of the ratio with respect to a critical level equal to 2.

As in the literature we discussed in section 3.2, the condition $P - 2A \geq 0$ therefore comes from a trade-off between a decrease in present consumption and an increase in preventive effort, that arises from second period uncertainty (here moral hazard generates classification risk). If agents are protective (that is if $P \geq 2A$), they prefer to transfer wealth from period 1 to period 2 rather than to exert effort, when they face classification risk. In this sense they rather protect themselves than prevent. In our setting this materialized therefore in an increase in front-loading ($\Pi_1^{**} > \Pi_1^*$) and a more difficult incentive scheme. It is then necessary to specify a large spread between second period premia to make sure that protective agents exert the effort (see Figure 4). The reverse effort holds for non protective agents that rather exert preventive effort than transfer wealth. As transferring wealth correspond in our setting to an increase in insurance against second period health cost, non protective agents' preferences correspond to the adage "an once of prevention is worth than a pound of cure". In this case, lower classification risk is therefore incentive compatible and uncertainty reduces prepayment of premia.

It seems important to note here that the reluctance of protective agents to exert effort, does not come from time inconsistency (as it can for example arise from beta-delta preferences a la Laibson 1997). Indeed, protective agents are perfectly time consistent but do not choose preventive effort (but rather wealth transfers) to face future uncertainty. This behavior may be caused by the uncertain nature of prevention relative to the predetermined (by the insurance contract) returns of prepayment.

# 5 Comparative Statics: How to Reduce Classification Risk?

In this section, we analyze the effect of the different parameters of the model on optimal premia. By defining which variables affect classification risk, we are then able to formulate some policy recommendations on the ways to reduce this kind of risk that produces inequalities. Our model contains three classes of variables: the variables regarding the income process (the sure revenue $R$ and the expected health cost $K_i^j$), those regarding the preventive effort (the cost of effort $\psi$ and the probabilities of being low type for both level of effort $\overline{p}$ and $\underline{p}$) and the variables relative to the behavior toward risk (the degrees of protectiveness, prudence and risk aversion) included in the utility function.

It is firstly worthwhile to note that – mainly because of the hypothesis of linear first period utility – the variables relating to the income process play a minor role in the determination of optimal premia. There is indeed no wealth effect in our model in the sense that all premia are proportional to sure revenue $\left(\frac{d\Pi_2^l}{dR} = \frac{d\Pi_2^h}{dR} = -\frac{d\Pi_1}{dR} = 1\right)$. Moreover, the expected costs of health only impact positively the first period premium (through the zero profit condition) and have no influence on reclassification risk.

## 5.1 Reduce the Cost of Primary Preventive Effort

The cost of primary preventive effort appears to be a first tool on which policymaker (using subsidizes) or insurer (as in the example presented in the introduction) may act. Through the system (4.7), our model allows to analyze of the impact of this cost $\psi$ on optimal premia and especially on reclassification risk.

To do so let us differentiate the solution system with respect to $\Pi_1$, $\Pi_2^l$, $\Pi_2^h$ and $\psi$. This leads to:

$$\begin{cases} -u'(R-\Pi_2^l)d\Pi_2^l + u'(R-\Pi_2^h)d\Pi_2^h = \frac{d\psi}{\Delta p} \\ \overline{p}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^2}d\Pi_2^l + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^2}d\Pi_2^h = 0 \\ d\Pi_1 + \overline{p}d\Pi_2^l - (1-\overline{p})d\Pi_2^h = 0 \end{cases}$$

With the first two equations one gets

$$\begin{cases} \dfrac{d\Pi_2^h}{d\psi} = \Delta p \dfrac{\overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + \overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3} > 0 \\ \dfrac{d\Pi_2^l}{d\psi} = -\Delta p \dfrac{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + \overline{p}u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3} < 0 \end{cases}$$

and therefore

$$\frac{d\Pi_1}{d\psi} = \overline{p}\,(1-\overline{p})\,\Delta p \frac{u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^2 - u''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^2}{(1-\overline{p})\,u''\left(R-\Pi_2^h\right)\left[u'\left(R-\Pi_2^l\right)\right]^3 + pu''\left(R-\Pi_2^l\right)\left[u'\left(R-\Pi_2^h\right)\right]^3}$$

This allows us to formulate the following proposition on the impact of a decrease in $\psi$

**Proposition 3** *A decrease in the cost of primary prevention*

- *decreases classification risk (as it increases the premium paid by low risk and decreases the premium paid by high risk agents)*
- *decreases prepayment if agents are protective (increases it otherwise)*

The effect on classification risk is quite straightforward. A decrease in the cost of prevention enhances the incentive to exert the effort. Therefore, the insurance contract can exhibit a lower classification risk and remain incentive compatible. Therefore, if a policymaker wants to reduce the inequality resulting from classification risk he should work on reducing the cost of prevention. By the same mechanism as for Proposition 1, this will then decrease (resp. increase) prepayment if $P > 2A$ (resp. $P < 2A$). These effects can also be displayed graphically as an increase in $\psi$ corresponds to a downward shit of the incentive curve.

Moreover, the same effects can be induced by a decrease in the probability of being low risk when not exerting the effort: $\underline{p}$. These changes in $\underline{p}$ are however difficult to interpret as it corresponds to a change in the probability for the agents that don't exert the effort, keeping the probability for those that exert the effort constant.

## 5.2 Increase the Effectiveness of Primary Prevention

The role of the probability of being low risk when exerting the preventive effort is more easily understandable. Indeed, an increase in $\overline{p}$, keeping $\underline{p}$ constant, can be interpreted as an improvement of the effectiveness of prevention. For example, investing in research on primary prevention, can increase the probability for the effort to lead to low risk type. It therefore opens a new spectrum for public policy.

Using (4.7), comparative statics on changes in $\overline{p}$ gives

$$\begin{cases} \dfrac{d\Pi_2^l}{d\overline{p}} = -\dfrac{\dfrac{1}{u'\left(R-\Pi_2^l\right)} - \dfrac{1}{u'\left(R-\Pi_2^h\right)} - (1-\overline{p})\,\dfrac{\psi}{(\Delta p)^2}\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}}{u'\left(R-\Pi_2^l\right)\left[\overline{p}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3} + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}\right]} > 0 \\ \dfrac{d\Pi_2^l}{d\overline{p}} = -\dfrac{\dfrac{1}{u'\left(R-\Pi_2^l\right)} - \dfrac{1}{u'\left(R-\Pi_2^h\right)} + \overline{p}\dfrac{\psi}{(\Delta p)^2}\dfrac{u''\left(R-\Pi_2^l\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3}}{u'\left(R-\Pi_2^l\right)\left[\overline{p}\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^l\right)\right]^3} + (1-\overline{p})\dfrac{u''\left(R-\Pi_2^h\right)}{\left[u'\left(R-\Pi_2^h\right)\right]^3}\right]} \end{cases}$$

This leads to

**Proposition 4** *An increase in the probability of being low risk type in second period when exerting the preventive effort*

- *increases the optimal premium paid by low risk agents in second stage*
- *decreases classification risk if agents are protective*

First, an increase in the probability of being low risk for agents that exert the preventive effort (keeping constant this probability for agents that don't exert effort) increases the benefit of effort. It is then easier to provide the incentive to exert effort. The optimal contract can therefore lead to a lower welfare in good state of nature and still be incentive compatible.

Through the incentive constraint, this implies a decrease in second period premium for risky agents. However, the increases in $\overline{p}$ also decreases the weight attached to this bad state in the objective function. This leads to a decrease in optimal wealth of high risk. The combination of these two effects is ambiguous. Therefore, the effect of an increase in $\overline{p}$ on the expected wealth in second period is also ambiguous and we can not conclude on the impact of this increase on first period wealth.

However, it is possible to state that an increase in the probability of being low risk decreases the risk of classification when agents are protective:

$$\frac{d\left(X_2^l-X_2^h\right)}{d\overline{p}}=\frac{\left[\frac{1}{u'\left(X_2^l\right)}-\frac{1}{u'\left(X_2^h\right)}\right]^2+\frac{\overline{p}}{u'\left(X_2^l\right)u'\left(X_2^h\right)}\frac{\psi}{(\Delta p)^2}\left[\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3}-\frac{u''\left(X_2^l\right)}{\left[u'\left(X_2^l\right)\right]^3}\right]-\frac{\psi}{\Delta p}\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3u'\left(X_2^l\right)}}{\overline{p}\frac{u''\left(X_2^l\right)}{\left[u'\left(X_2^l\right)\right]^3}+(1-\overline{p})\frac{u''\left(X_2^h\right)}{\left[u'\left(X_2^h\right)\right]^3}}$$

Therefore, if agents are protective, the policymaker can reduce the classification risk by improving the effectiveness of primary prevention, for example through investments in medical research.

### 5.3 Comparative protectiveness

In the previous sections we have highlighted the important role of protectiveness on the determination of optimal premia. Whether agents are protective or not, widely impacts the consequences moral hazard has on premia. This raises the question of the influence of changes in the degree of protectiveness, that is in the concavity of $1/u'$. To study this issue let us first define formally the degree of protectiveness.

**Definition 3** *An agent $v$ with second period utility function $v(.)$, will be said to have a higher degree of protectiveness (or to be more protective) than agent $u$ with second period utility function $u(.)$ if $1/v'(.)$ is a concave transformation of $1/u'$.*

**Remark 1** *The equivalent to state that $v$ is more protective than $u$ if $P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x) \forall x$*

**Proof:**

$$\frac{1}{v'}\left(\left(\frac{1}{u'}\right)^{-1}(t)\right) = g(t) \text{ with } g \text{ increasing and concave}$$

$$\Rightarrow \quad g'(t) = \frac{v''\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}{\left[v'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)\right]^2} \cdot \frac{\left[u'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)\right]^2}{u''\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)} = \frac{A_v\left(\left(\frac{1}{u_1'}\right)^{-1}(t)\right)u'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}{A_u\left(\left(\frac{1}{u_1'}\right)^{-1}(t)\right)v'\left(\left(\frac{1}{u'}\right)^{-1}(t)\right)}$$

$$\Rightarrow \quad \text{g is concave if and only if } \frac{d}{dx}\log\left(\frac{A_v(x)u'(x)}{A_u(x)v'(x)}\right) \leq 0$$

$$\Leftrightarrow \quad \frac{A_v'(x)}{A_v(x)} + \frac{u''(x)}{u'(x)} - \frac{A_u'(x)}{A_u(x)} - \frac{v''(x)}{v'(x)} \leq 0$$

$$\Leftrightarrow \quad P_v(x) - 2A_v(x) \geq P_u(x) - 2A_u(x)$$

Comparative statics on protectiveness then comes to compare utility functions. The effect of an increase in protectiveness is therefore hard to grasp as it also implies changes in incentive constraint and in first best contract. Therefore, to explicit results on the effect of protectiveness on optimal dynamic contract, it is necessary to impose further assumptions on preferences.

# 6 Explicit Examples

## 6.1 The case of Constant Relative Risk Aversion

The first convenient way to specify the utility function is to assume that agents' preferences are represented by CRRA (Constant Relative Risk Aversion) utility functions. CRRA utility functions indeed exhibit some interesting properties. If

$$u(x) = \begin{cases} \frac{x^{1-\gamma}}{1-\gamma} & \text{for } \gamma > 0, \gamma \neq 1 \\ ln(x) & \text{for } \gamma = 1 \end{cases}$$

$A(x) = \frac{\gamma}{x}$ and $P(x) = \frac{\gamma+1}{x}$. Therefore, if agents $v$ and $u$ are both characterized by CRRA utility function with respective parameters $\gamma_v$ and $\gamma_u$, from remark 1, $v$ is more protective than $u$ for every level of wealth if $\gamma_v < \gamma_u$. CRRA utility functions also exhibit the convenient feature of leading to the same first best contract $\left(\overline{K} - R + 1, R - 1, R - 1\right)$ whatever the parameter of risk aversion $(u'(X_2^*) = 1 \Leftrightarrow X_2^* = 1 \forall \gamma > 0)$.

Consider two types of agents characterized by two CRRA utility functions $v(.)$ and $u(.)$ such that $1/v' = \phi \circ (1/u')$ with $\phi(.)$ increasing and concave; that is agents of type $v$ are more protective than agents of type $u$.

If policyholders are of type $v$ the optimal contract $\left(\Pi_{1v}^{**}, {\Pi_{2v}^{l}}^{**}, {\Pi_{2v}^{h}}^{**}\right)$ has to satisfy

$$\begin{cases} \dfrac{\overline{p}}{v'\left(R - {\Pi_{2v}^{l}}^{**}\right)} + \dfrac{1-\overline{p}}{v'\left(R - {\Pi_{2v}^{h}}^{**}\right)} = 1 \\ v(R - {\Pi_{2v}^{l}}^{**}) - v(R - {\Pi_{2v}^{h}}^{**}) = \dfrac{\psi}{\Delta p} \\ \Pi_{1v}^{**} = \overline{K} - \overline{p}{\Pi_{2v}^{l}}^{**} - (1-\overline{p}){\Pi_{2v}^{h}}^{**} \end{cases}$$

whereas if agents are characterized by the utility function $u(.)$ the optimal incentive compatible contract is the triplet $\left(\Pi_{1u}^{**}, {\Pi_{2u}^{l}}^{**}, {\Pi_{2u}^{h}}^{**}\right)$ such that

$$\begin{cases} \dfrac{\overline{p}}{u'\left(R - {\Pi_{2u}^{l}}^{**}\right)} + \dfrac{1-\overline{p}}{u'\left(R - {\Pi_{2u}^{h}}^{**}\right)} = 1 \\ u(R - {\Pi_{2u}^{l}}^{**}) - u(R - {\Pi_{2u}^{h}}^{**}) = \dfrac{\psi}{\Delta p} \\ \Pi_{1u}^{**} = \overline{K} - \overline{p}{\Pi_{2u}^{l}}^{**} - (1-\overline{p}){\Pi_{2u}^{h}}^{**} \end{cases}$$

Consider the relative position of the curve characterized by the first two equations of each system in the plan $\left(X_2^l, X_2^h\right)$.

Regarding the first order conditions, one of the properties of CRRA utility functions turns out to be crucial. All CRRA utility functions leading to the same first best contract we necessarily have $\frac{1}{v'(1)} = \phi\left(\frac{1}{u'(1)}\right) = \frac{1}{u'(1)} = 1$ that is $\phi(1) = 1$. Therefore

$$\begin{aligned} & \tfrac{\overline{p}}{v'\left(R-{\Pi_{2v}^{l}}^{**}\right)} + \tfrac{1-\overline{p}}{v'\left(R-{\Pi_{2v}^{h}}^{**}\right)} = \overline{p}\phi\left(\tfrac{1}{v'\left(R-{\Pi_{2v}^{l}}^{**}\right)}\right) + (1-\overline{p})\,\phi\left(\tfrac{1}{v'\left(R-{\Pi_{2v}^{h}}^{**}\right)}\right) = 1 \\ \Rightarrow\quad & \tfrac{\overline{p}}{v'\left(R-{\Pi_{2v}^{l}}^{**}\right)} + \tfrac{1-\overline{p}}{v'\left(R-{\Pi_{2v}^{h}}^{**}\right)} > \phi^{-1}(1) = 1 = \tfrac{\overline{p}}{u'\left(R-{\Pi_{2u}^{l}}^{**}\right)} + \tfrac{1-\overline{p}}{u'\left(R-{\Pi_{2u}^{h}}^{**}\right)} \end{aligned}$$

as $\phi$ is increasing and concave. The function $1/u'$ being increasing, this implies that in the plan $\left(X_2^l, X_2^h\right)$, the first order condition of agents $v$ (labeled $FOC_v$ in Figure 5) is higher than the one of agents $u$ (labeled $FOC_u$), although they both have the same tangency at $(X_2^*, X_2^*)$.

As stated in previous section this effect is however coupled with an effect on the incentive constraint. In the case of CRRA utility function, the incentive constraint becomes $\frac{(X_2^l)^{1-\gamma}}{1-\gamma} - \frac{(X_2^h)^{1-\gamma}}{1-\gamma} = \frac{\psi}{\Delta p}$. In the plan $\left(X_2^l, X_2^h\right)$, the slope of the incentive constraint writes $\frac{dX_2^h}{dX_2^l} = \left(\frac{X_2^h}{X_2^l}\right)^{\gamma} < 1$ and is decreasing in $\gamma$. Therefore, an increase in protectiveness makes the slope of the incentive constraint turn counter-clockwise (from $IC_u$ to $IC_v$ in Figure 5). We are however unable to define when the incentive constraints of both individuals cross and the graphical analysis is not sufficient to conclude in this case.

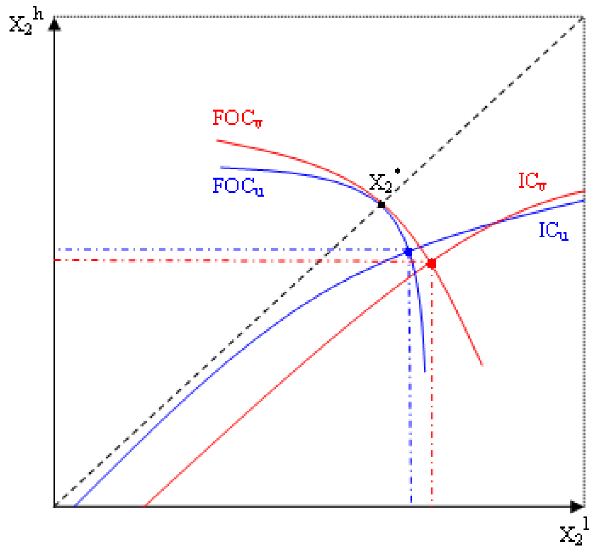


Figure 5: Comparative protectiveness in the case of CRRA preferences

Let us therefore turn to an analytical analysis.

With CRRA preferences, the optimal second period contract is described by the system

$$\begin{cases} \frac{\left(X_2^l\right)^{1-\gamma}}{1-\gamma} - \frac{\left(X_2^h\right)^{1-\gamma}}{1-\gamma} = \frac{\psi}{\Delta p} \\ \overline{p}\left(X_2^l\right)^{\gamma} + (1-\overline{p})\left(X_2^h\right)^{\gamma} = 1 \end{cases} \tag{6.1}$$

Therefore,

$$\begin{cases} \dfrac{dX_2^l}{d\gamma} = -\dfrac{\dfrac{\psi}{\Delta p(1-\gamma)} - \dfrac{1}{1-\gamma}\ln\left(X_2^l\right)\left(X_2^l\right)^{1-\gamma} + \dfrac{1}{\gamma(1-\gamma)}\ln\left(X_2^h\right)\left(X_2^h\right)^{1-\gamma} + \dfrac{\overline{p}}{1-\overline{p}}\dfrac{1}{\gamma}\ln\left(X_2^l\right)\left(X_2^l\right)^{\gamma}\left(X_2^h\right)^{1-2\gamma}}{\dfrac{1}{(1-\overline{p})}\left(X_2^l\right)^{\gamma-1}\left[(1-\overline{p})\left(X_2^l\right)^{1-2\gamma} + \overline{p}\left(X_2^h\right)^{1-2\gamma}\right]} \\ \dfrac{dX_2^h}{d\gamma} = \dfrac{\dfrac{\psi}{\Delta p(1-\gamma)} - \dfrac{1}{\gamma(1-\gamma)}\ln\left(X_2^l\right)\left(X_2^l\right)^{1-\gamma} + \dfrac{1}{1-\gamma}\ln\left(X_2^h\right)\left(X_2^h\right)^{1-\gamma} - \dfrac{1-\overline{p}}{\overline{p}}\dfrac{1}{\gamma}\ln\left(X_2^h\right)\left(X_2^h\right)^{\gamma}\left(X_2^l\right)^{1-2\gamma}}{\dfrac{1}{\overline{p}}\left(X_2^h\right)^{\gamma-1}\left[(1-\overline{p})\left(X_2^l\right)^{1-2\gamma} + \overline{p}\left(X_2^h\right)^{1-2\gamma}\right]} \end{cases}$$

As $X_2^* = 1 \forall \gamma > 0$, from Proposition 2, $\ln\left(X_2^l\right) > 0$ and $\ln\left(X_2^h\right) < 0$. Therefore, if $\gamma > 1$, the three last terms of both numerators are positive and next proposition holds

**Proposition 5** *If agents' preferences in second period are represented by a Constant Relative Risk Aversion utility function with risk coefficient larger than one, an increase in the degree of protectiveness*

- *decreases the premium paid by low risk agents*
- *increases the premium paid by high risk agents*
- *and thus increases classification risk*

*in second period, provided the cost of effort is low enough (relative to its benefits)*

Indeed, when $(1-\gamma) < 0$, provided the first term of the numerators is low enough relative to the other terms, the optimal wealth in second stage decreases with the coefficient of risk aversion $\gamma$ when the agent turns out to low risk $\left(\frac{dX_2^l}{d\gamma} > 0\right)$ and increases with $\gamma$ for high risk agents $\left(\frac{dX_2^h}{d\gamma} < 0\right)$. In the case of CRRA utility functions, the degree of protectiveness increases when $\gamma$ decreases and Proposition 5 follows.

This result can be explained by the two effects induced by the increase in protectiveness already highlighted. First, the pure protectiveness effect, represented by the move of the first order condition in the plan $(X_2^l, X_2^h)$ presented above, leads to a decrease in both second period premia (and an increase in first period premium). Proposition 5 states that the incentive effect highly depends on the cost of effort.

This mainly comes from the influence of moves in $\gamma$ on the level of utility reached for low level of wealth. Indeed, an increase in the coefficient of relative risk aversion has a large impact on low levels of utility, whereas the change in utility for high level of consumption is relatively small (cf. Gollier 2001 Figure 2.2). This impacts the intensity of the counter-clockwise move of the incentive constraint. When the needed spread between wealth is low (low level of $\frac{\psi}{\Delta p}$), the incentive constraints of agents having different CRRA utility functions cross for high level of wealth. Therefore, the first order condition of more protective agents crosses their incentive constraint when it is below the incentive constraint of less protective agents (as in the case represented in Figure 5).

Then, the combination of the two effects leads to an increase in wealth of low risk agents and a decrease in wealth of high risk. However, when $\frac{\psi}{\Delta p}$ is too high, the incentive constraints of two agents may cross before crossing the first order condition, leading to the reverse effects (a decrease in $X_2^l$ and an increase in $X_2^h$).

The restriction to non-protective agents ($\gamma > 1$) may seem awkward, but is pretty standard in the case of CRRA utility function. For example, Gollier (2001) argues that this condition holds for most households in real economy. This moreover corresponds to the necessary condition for utility to be unbounded below, which is a standard assumption in principal-agents models (see for example Grossman and Hart 1983). It notably ensures that no nonnegativity constraints on income bind at the optimum.

Proposition 5 moreover have implications on first period premium. From the zero profit condition, $\frac{d\Pi_1}{d\gamma} = -\overline{p}\frac{d\Pi_2^l}{d\gamma} - (1-\overline{p})\,\frac{d\Pi_2^l}{d\gamma}$. Therefore, for high value of $\overline{p}$, the effect of low risk premium dominates. Then, in the configuration of proposition 5, the first stage premium increases in protectiveness. This is confirmed and generalized by the study of the system

$$\begin{cases} \dfrac{\left(X_2^l\right)^{1-\gamma}}{1-\gamma} - \dfrac{\left(X_2^h\right)^{1-\gamma}}{1-\gamma} = \dfrac{\psi}{\Delta p} \\ \overline{p}\left(X_2^l\right)^{\gamma} + (1-\overline{p})\left(X_2^h\right)^{\gamma} = 1 \\ X_1 + \overline{p}X_2^l + (1-\overline{p})\,X_2^h = 2R - \overline{K} \end{cases}$$

that gives

$$\begin{aligned}\frac{dX_1}{d\gamma} &= \Bigg\{ (1-\overline{p})^2 \left(X_2^h\right)^{2\gamma} ln\left(X_2^h\right) + \overline{p}^2 \left(X_2^l\right)^{2\gamma} ln\left(X_2^l\right) + \overline{p}\,(1-\overline{p})\,\frac{\gamma}{(1-\gamma)^2}\Big[ \left(X_2^h\right)^{\gamma-\frac{1}{2}} \left(X_2^l\right)^{\frac{1}{2}} \\ &\quad - \left(X_2^l\right)^{\gamma-\frac{1}{2}} \left(X_2^h\right)^{\frac{1}{2}} \Big]^2 + \frac{\overline{p}\,(1-\overline{p})}{1-\gamma}\Big( \ln\left(X_2^h\right) X_2^h \left(X_2^l\right)^{\gamma} \left[\left(X_2^h\right)^{\gamma-1} - \left(X_2^l\right)^{\gamma-1}\right] \\ &\quad + \ln\left(X_2^l\right) X_2^l \left(X_2^h\right)^{\gamma} \left[\left(X_2^l\right)^{\gamma-1} - \left(X_2^h\right)^{\gamma-1}\right] \Big) \Bigg\} \left[\gamma\left(\overline{p}\left(X_2^l\right)^{2\gamma-1} + (1-\overline{p})\left(X_2^h\right)^{2\gamma-1}\right)\right]^{-1}\end{aligned}$$

The first term being the only negative one, it turns out that, when the probability of being high risk type $(1-\overline{p})$ is low enough, an increase in protectiveness increases the first period premium. Simulations with CRRA preferences (cf. Appendix) moreover points out that this seems to be the case whatever the probability of being high risk. The intuitive effects driving Proposition 1 therefore seems to be generalizable to changes in the degree of protectiveness.

We are unable to determine analytically the limit cost of effort in Proposition 5. However, this can be done when analyzing the welfare in the good state of nature. To do so let us consider the following change of variables. If we define $Y_2^l \equiv u(X_2^l)$ and $f \equiv u^{-1}$, we have $u(X_2^h) = Y_2^l - \frac{\psi}{\Delta p}$ and the system defining the second period premia simplifies in

$$H(Y_2^l) \equiv \overline{p} f'(Y_2^l) + (1-\overline{p})\, f'\left(Y_2^l - \frac{\psi}{\Delta p}\right) = 1 \tag{6.2}$$

In the case of CRRA preferences,

$$f\,(Y) = \exp\left[\frac{1}{1-\gamma}\ln\left((1-\gamma)Y\right)\right] \text{ and } f'\,(Y) = \exp\left[\frac{\gamma}{1-\gamma}\ln\left((1-\gamma)Y\right)\right]$$

$(f')$ being increasing, it follows from (6.2) that

$$\text{sgn}\left(\frac{\partial Y_2^l}{\partial\gamma}\right) = -\text{sgn}\left(\overline{p}\frac{\partial f'}{\partial\gamma}(Y_2^l) + (1-\overline{p})\,\frac{\partial f'}{\partial\gamma}\left(Y_2^l - \frac{\psi}{\Delta p}\right)\right)$$

Then, as $\text{sgn}\left(\frac{\partial f'}{\partial\gamma}(Y)\right) = \text{sgn}\left(\ln\left((1-\gamma)\,Y\right) - \gamma\right)$, one gets

$$\frac{\partial f'}{\partial\gamma}(Y) \geq 0 \Leftrightarrow Y \leq Y_\gamma \equiv \frac{e^{\gamma}}{1-\gamma}\text{when}\gamma > 1$$

Therefore, a sufficient condition for the welfare in low state to be increasing with protectiveness $\left(\frac{\partial Y_2^l}{\partial \gamma} \leq 0\right)$, is $Y_2^l$ (and thus $Y_2^l - \frac{\psi}{\Delta p}$) to be lower than $Y_\gamma$. As $H(Y_2^l)$ is increasing, this condition is equivalent to $H(Y_\gamma) \geq 1$, for which a sufficient condition is $f'\left(Y_\gamma - \frac{\psi}{\Delta p}\right) \geq 1$ that is $\dfrac{\psi}{\Delta p} \leq \dfrac{e^\gamma - 1}{1-\gamma}$.

**Proposition 6** *If agents' preferences in second period are represented by a Constant Relative Risk Aversion utility function with risk coefficient larger than one, an increase in protectiveness increases the welfare of low risk agents in second period, if* $\dfrac{\psi}{\Delta p} \leq \dfrac{e^\gamma - 1}{1-\gamma}$.

We however can not infer results on level of wealth (and thus on premia) from this proposition as the coefficient of relative risk aversion doesn't have a monotonic impact on utility derived from a given level of wealth (see Gollier 2001, Figure 2.2).

## 6.2 A suitable utility function with linear reciprocal derivative

To go further in the analysis of the impact of protectiveness on premia and have clearer results, it is accommodating to build a suitable utility function that satisfies additional simplifying properties.

From equation (6.2), it first seems simplifying to specify linear $f'(y)$, that is $f'(y) = \theta y$ where $\theta$ depicts the behavior toward risk. As, $f = u^{-1}$, this corresponds to $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-a)}$ where $a$ represents the constant of integration. Then, to isolate the effect protectiveness, it appears useful to consider a class of utility function for which (as in the CRRA case) the first best premium doesn't depend on the behavior toward risk parameter. To do so let us specify the constant $a$ such that $u'(1) = 1 \forall \theta$ that is $a = 1 - \frac{1}{2\theta}$. Let us therefore consider the class of utility function:

$$u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1} \quad , \quad \theta > 0 \tag{6.3}$$

which is twice continuously differentiable, increasing and concave above some (subsistence) level [13] $\underline{x} \equiv 1 - \frac{1}{2\theta}$ and satisfies the Inada conditions $\lim\limits_{x \to \underline{x}} u'(x) = +\infty$ and $\lim\limits_{x \to +\infty} u'(x) = 0$.

[13] In this sense, this class of utility functions can be related to the Stone-Geary class

An agent whose preferences is described by (6.3) is risk adverse $\left(A = \frac{\theta}{2\theta(x-1)+1} > 0\right)$, prudent $\left(P = \frac{3\theta}{2\theta(x-1)+1} > 0\right)$ and protective $\left(P - 2A = \frac{\theta}{2\theta(x-1)+1} > 0\right)$ for all level of $\theta$ (positive). Moreover, if we consider two agents $u$ and $v$ having such preferences, $v$ is more protective than $u$ ($P_v - 2A_v > P_u - 2A_u \; \forall x > \underline{x}$) if and only if $\theta_v > \theta_u$.

The levels of wealth reached under the optimal incentive contract $\left(\text{that exists only if } \frac{\psi}{\Delta p} < \frac{1}{\overline{p}\theta}\right)$ then writes:

$$\left\{\begin{array}{rcl} X_2^{l**} & = & 1 + (1-\overline{p})\frac{\psi}{\Delta p} + \frac{1}{2}\left((1-\overline{p})\frac{\psi}{\Delta p}\right)^2\theta \\ X_2^{h**} & = & 1 - \overline{p}\frac{\psi}{\Delta p} + \frac{1}{2}\left(\overline{p}\frac{\psi}{\Delta p}\right)^2\theta \\ X_1^{**} - X_1^{*} & = & 1 - E\left(X_2^{**}\right) = -\frac{1}{2}\overline{p}\left(1-\overline{p}\right)\left(\frac{\psi}{\Delta p}\right)^2\theta \end{array}\right.$$

Therefore,

**Proposition 7** *If agents' preferences are defined by* $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$, *an increase in the degree of protectiveness*

*(i) increases first period premium*

*(ii) decreases second period premia*

*(iii) increase classification risk if the good state of health is the more likely* $(p > 1/2)$

The use of a utility function whose reciprocal has a linear derivative allows us to better isolate the impact of protectiveness. We can indeed relate the results of Proposition 7 with the effect highlighted in Proposition 1. An increase in protectiveness (that is in $P - 2A$) increases the first period premium for pain disaggregation motive (increase in $P$) and/or because agents are less sensitive to the increase in classification risk it may cause (decrease in $A$). The incentive effect of such an increase does not prevent - for this class of utility function - from a decrease in both second period premia.

Moreover, the increase in wealth is higher in the most probable state and therefore this increase in the first period may not be coupled with an increase in classification risk if the bad state is highly probable. The optimal incentive contract moreover confirms the result of Proposition 3 and 4 as here (i) an increase in the cost of effort increases the first period premium and the classification risk (when the optimal contract exists, that is when $\frac{\psi}{\Delta p} < \frac{1}{\overline{p}\theta}$) and (ii) an increase in the probability of being low risk when exerting the effort decreases the classification risk and the optimal wealth in the good state.

## 7 The introduction of short-term (spot) insurance

Hendel and Lizzeri (2003) have pointed out that, in the absence of moral hazard, dynamic insurance contracts are subject to lapsation in second period. Taking the feature into account, the timing our model becomes

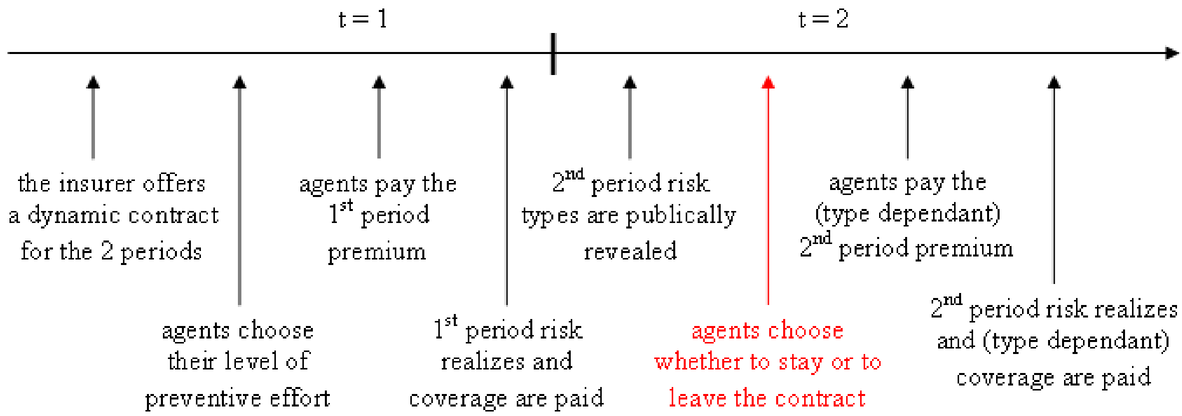


Figure 6: The timing of the game including interim participation choice

In the absence of severance payment (proposed by Cochrane 1995), healthier agents may then leave the dynamic contract to go to a competing short-term (spot) insurer that offers actuarially fair premia. This will be the case if the optimal incentive premium for low risk agent is higher than their expected health cost $\left(\Pi_2^{l\,**} > K_2^l\right)$. The second period optimal contract presented previously however does not depend on expected costs (the expected health costs over the two periods only influences the first period premium). We can still infer that for a given expected health costs schedule, the lower the premium offered by the mutual dynamic insurer, the lower the incentive for healthy agents to lapse (and go to the spot insurer).

Therefore, from Proposition 2, the mutual insurer described here suffers from less lapsation if effort is unobservable. Moreover, from Proposition 5 and 7:

**Proposition 8** *A mutual insurer that offers long-term contracts is more likely to be sustainable to the competition of companies offering spot (short-term) contracts if it insures more protective agents, when agents preferences*

- *are described by* $u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$
- *are CRRA, provided the cost of primary preventive effort is low enough*

Our work also offers an alternative explanation to the various degrees of front-loading and of lapsation observed in dynamic insurance contracts. In the empirical part of their work, Hendel and Lizzeri (2003) show that in life insurance, more front-loading is associated with lower lapsation. They then argue that this phenomenon can be explained by heterogeneity in agents' income growth (that is in the cost of front-loading). It appears from our model that this can also be explained by heterogeneity in protectiveness. Program (4.5) indeed also fits with the problem of a competitive insurance company that does not discount the future (in the zero profit condition (4.2)). In this case, it appears from our study that the co-existence of dynamic contract with spot contract, and the various degree of front-loading may be explained by moral hazard and heterogeneous protectiveness.

Moreover, contracts of more protective agents may exhibits more front loading and less lapsation if the reciprocal of agents' utility function has a linear derivative ($u(x) = \frac{1}{\theta}\sqrt{2\theta(x-1)+1}$, $\theta > 0$). Simulations with CRRA utility function (cf. Appendix) seems to indicate that this would also be the case when agents have CRRA preferences if the cost of primary preventive effort is low enough.

# 8 Allowing for Cross Subsidization:
# A Step Toward Adverse Selection

A natural extension of our model consists in allowing for cross subsidization between agents with heterogeneous preferences. Let us therefore study a situation where a mutual insurer dynamically insures a population composed of two type of agents with respective utility functions $u$ and $v$ and respective proportion $\lambda$ and $1-\lambda$. Assuming the insurer is unable to distinguish between types and can only offer a single (pooling) contract, the optimal incentive compatible contract is solution of

$$\begin{aligned}
\max_{\Pi_1,\Pi_2^l,\Pi_2^h} \quad & (R-\Pi_1) - \overline{\psi} + \lambda\left[\overline{p}u\left(R-\Pi_2^l\right) + (1-\overline{p})u\left(R-\Pi_2^h\right)\right] \\
& + (1-\lambda)\left[\overline{p}v\left(R-\Pi_2^l\right) + (1-\overline{p})v\left(R-\Pi_2^h\right)\right] \\
\text{s.t.} \quad & \begin{cases} \Pi_1 + \overline{p}\Pi_2^l + (1-\overline{p})\Pi_2^h \geq \overline{K} \\ u(R-\Pi_2^l) - u(R-\Pi_2^h) \geq \dfrac{\psi}{\Delta\pi} \\ v(R-\Pi_2^l) - v(R-\Pi_2^h) \geq \dfrac{\psi}{\Delta\pi} \end{cases}
\end{aligned}$$

To guarantee that only one incentive constraint will bind at the optimum, let us moreover assume that agents of type $v$ have a strictly lower preference for future, that is $v'(x) < u'(x)\ \forall x$. This guarantees that the incentive constraint of agents with preferences $u$ doesn't bind at the optimum ($u(R-\Pi_2^l) - u(R-\Pi_2^h) > v(R-\Pi_2^l) - v(R-\Pi_2^h)\ \forall(\Pi_2^l, \Pi_2^h)$).

Then, the first order conditions of the program can be written as:

$$\frac{\overline{p}}{v'\left(R-\Pi_2^l\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_2^h\right)}=\lambda+(1-\lambda)\left(\overline{p}\frac{u'\left(R-\Pi_2^l\right)}{v'\left(R-\Pi_2^l\right)}+(1-\overline{p})\frac{u'\left(R-\Pi_2^h\right)}{v'\left(R-\Pi_2^h\right)}\right)$$

Reminding that we assume $v'(x) < u'(x)$, this gives $\frac{\overline{p}}{v'\left(R-\Pi_2^l\right)}+\frac{1-\overline{p}}{v'\left(R-\Pi_2^h\right)} > 1$. As the incentive constraint of agents of type $v$ always binds at the equilibrium, the following proposition hold

**Proposition 9** *Allowing for cross subsidization between heterogeneous agents*

- *increases prepayment*
- *decreases second period premia*
- *increases classification risk*

*of agents with the lowest preference for future*

By definition, the pooling contract provides arbitration between the preferences of the two types of agents. In our setting, the agents with the lower preference for future are the most difficult to incite and therefore drive the incentive compatibility of the contract. Reminding that in the plan $(X_2^l, X_2^h)$ the incentive compatible constraint defines an increase curve that goes away from the 45° line as $X_2^l$ increase, Proposition 9 holds. Then, cross-subsidization has two main implications for the agents with the lower preference for future. First, as it increases their second period wealth (in both states), cross-subsidization lowers their incentive to leave the contract when they turn out to be low risk. It therefore stabilizes the dynamic contract. This is however done at the cost of an increase in classification risk, for this type of agents at least. Without further assumptions, we are unable to infer the impact of cross-subsidization on the welfare of agents of type $u$ (with a higher preference for future), as it would highly depend on the relative degree of protectiveness of both type. The introduction of adverse selection in our setting remains therefore an open issue that may call for some refinement in the model, as the non-linearity of first period preferences or the full specification of the second period utility function.

# 9 Applications to other insurance markets

The model presented above in the case of health insurance seems to be applicable to other insurance markets with slight modifications.

## 9.1 Life insurance

A first application concerns life insurance. The considered insurer then offers a protection against the risk of death, and agents can reduce the probability of having a high probability of death in second period by exerting preventive effort. If we assume ad-hoc altruism (in the sense that the indemnity paid to the beneficiary directly enters in the insured's utility function), optimal insurance is complete in each state and is solution of a program similar to 4.5. We however need to introduce in this extension the fact that agents can die in first period (that is the survival probability). As $q_1$ represents here the risk of death in period 1, only a portion $(1-q_1)$ of a generation is still alive in period 2. This effect enters the objective, the incentive constraint and the zero profit condition such that the program becomes

$$\max_{\Pi_1,\Pi_2^l,\Pi_2^h} \quad (R-\Pi_1)-\overline{\psi}+(1-q_1)\left[\overline{p}u\left(R-\Pi_2^l\right)+(1-\overline{p})u\left(R-\Pi_2^h\right)\right] \tag{9.1}$$

$$\text{s.t.} \quad \begin{cases} \Pi_1+(1-q_1)\left[\overline{p}\Pi_2^l+(1-\overline{p})\Pi_2^h\right]\geq\overline{K}=q_1R+(1-q_1)\left[\overline{p}q_2^lR+(1-\overline{p})q_2^hR\right] \\ u(R-\Pi_2^l)-u(R-\Pi_2^h)\geq\dfrac{\psi}{(1-q_1)\Delta p} \end{cases}$$

The solution of this new program, very similar to original one, is given by:

$$\begin{cases} u(R-\Pi_2^{l\,**})-u(R-\Pi_2^{h\,**})=\dfrac{\psi}{(1-q_1)\Delta p} \\ \dfrac{\overline{p}}{u'\left(R-\Pi_2^{l\,**}\right)}+\dfrac{1-\overline{p}}{u'\left(R-\Pi_2^{h\,**}\right)}=\dfrac{1}{1-q_1} \\ \Pi_1^{**}=\overline{K}-\overline{p}\Pi_2^{l\,**}-(1-\overline{p})\Pi_2^{h\,**}=\overline{K}-E(\Pi_2^{**}) \end{cases} \tag{9.2}$$

Then, all the above properties hold in the case of life insurance with (ad-hoc) altruistic agents. Particularly, long term life insurance appears more stable relative to spot insurance as it insures more protective agents. Moreover, the effect of $q_1$ turns out to be ambiguous. If the probability of dying during the first period decreases the relative valuation of the second period (and therefore tends to decrease inter-generational insurance), it also reduces the proportion of agents that shares the prepaid premia in second period (and then leads to an increase in second period wealth).

### 9.2 Unemployment insurance

With more amendments, our model also seems to be applicable to unemployment insurance.

Consider a social unemployment insurance in our simple overlapping generation model. In their early part of life, all agents face the same probability of being unemployed (or have the same expected length of unemployment) and can invest in training effort $e$. Partially based on this effort, agents can then either be employed as a "skilled" (executive) or "unskilled" (non executive) worker in second period. We moreover assume (as it seems to be the case in real economies) that the risk of unemployment is higher among unskilled workers than among skilled ones. Modeling the fact that the three types of agents also differ in wages, the income profile without insurance can be summarized as follows:

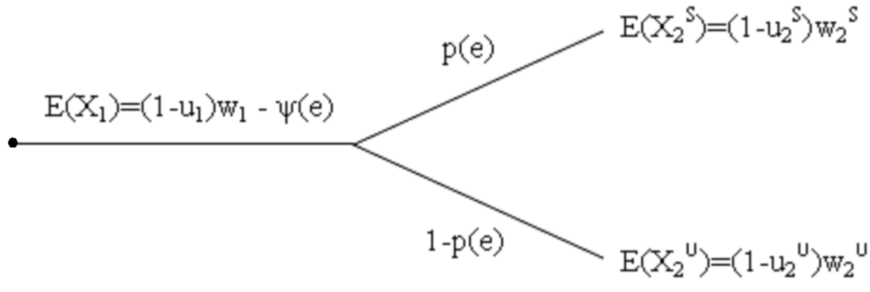


Figure 7: The income profile without unemployment insurance

with $u_1$, $u_2^S$ and $u_2^U$ the respective probabilities of being unemployed for youth, skilled and unskilled workers ; and $w_1$, $w_2^S$ and $w_2^U$ the respective wages of youth, skilled and unskilled workers.

As in our baseline model, it is then optimal for the mutual (social) insurer to provide risk-adverse agents with a complete insurance in each state that is with a triplet of sure consumption profiles $C_1$, $C_2^S$ et $C_2^U$ solution of

$$\max_{C_1, C_2^S, C_2^U} \quad (R - C_1) - \overline{\psi} + \overline{p}u\left(C_2^S\right) + (1-\overline{p})u\left(C_2^U\right) \tag{9.3}$$

$$\text{s.t.} \quad \begin{cases} C_1 + \overline{p}C_2^S + (1-\overline{p})C_2^U \geq (1-u_1)w_1 + \overline{p}(1-u_2^S)w_2^S + (1-\overline{p})(1-u_2^U)w_2^U \\ u(C_2^S) - u(C_2^U) \geq \dfrac{\psi}{(1-q_1)\Delta p} \end{cases}$$

where $\overline{\psi}$ represents the cost of exerting high effort, $\overline{p}$ and $\underline{p}$ being the respective probability of becoming a skilled worker when exerting and not exerting the training effort. $\Delta p \equiv \overline{p} - \underline{p}$.

Such a model allows studying the extent of intra- and inter-generational employment insurance. This seems to be empirically relevant as evidences for both type of insurance can be found in France for example. Indeed it appears that the employment benefit are, in France, equal to 75% of the last gross wage for the lowest wage bracket and about 57% for the highest one. This is in step with intragenerational insurance between skilled and unskilled workers. As these unemployment benefits are larger for workers above 50 years old, this intragenerational insurance moreover seems to combined with an intergenerational one.

In this setting, our model highlights a tradeoff between training effort and intergenerational insurance. Protective agents then prefer to rely on intergenerational insurance rather than on training to deal with the risk of having longer unemployment duration in second period. This application therefore seems to be linked to the actual debate on unemployment insurance (in particular in France) about the tradeoff between unemployment benefits generosity and training subsidization. Moreover, it appears through the preceding analysis that an increase in the efficiency of training or a decrease in its cost enhances the optimal incentive compatible intragenerational insurance (by decreasing the spread between $C_2^S$ and $C_2^U$). Finally, our work suggests that an increase in agents' degree of protectiveness decreases the redistributive pattern of unemployment insurance between skilled and unskilled workers.

## 10 Conclusion

We highlight in this paper the role of prudence and risk aversion on optimal dynamic insurance contracts. To do so we define the notion of "protectiveness" as the difference between the index of absolute prudence and twice the index of absolute risk aversion. Adding to usual models an effort of primary prevention, we show that this notion plays a central role in defining the optimal level of prepayment of premia and the optimal incentive compatible classification risk. First, our analysis states that moral hazard always increases classification risk (relative to the complete information benchmark) and increases first period premium (and thus may lead to more prepayment of premia) when agents are protective. This reveals the tradeoff between primary prevention and (intergenerational) insurance that arises from future uncertainty.

It moreover appears that the classification risk can be reduced by decreasing the cost of prevention or by increasing the effectiveness of prevention (when agents are protective). Therefore, if it aims at making insurance more affordable to high risk agents, the policy maker should seek at reducing the cost of primary prevention and at increasing its efficiency.

Specifying CRRA (Constant Relative Risk Aversion) preferences we moreover show that an increase in agents' degree of protectiveness decreases the premium offered to low risk agents in second period, if the cost of preventive effort is low enough. Then, the more protective its policyholders, the more stable the mutual insurer when confronted to competing companies that offer short term contracts. To go further in the analysis of comparative protectiveness, we specify a utility function that exhibits the suitable property of having a linear reciprocal derivative. With such preferences, it appears that an increase in agents' degree of protectiveness optimally increases first period premium and decreases second period premia. Heterogeneity in behavior toward risk therefore appears as an alternative explanation of the properties of dynamic insurance contract observed by Hendel and Lizzeri (2003). We indeed find that such heterogeneity can explain the different level of prepayment and the fact that contracts with higher levels of prepayment is associated with lower lapsation. Whether this explanation is more relevant that the heterogeneity in income growth put forward by Hendel and Lizzeri (2003) remains an open issue.

Is left for future research to analyze the impact of heterogeneous protectiveness on adverse selection. Preliminary work on this refinement highlights the importance of the relative time preferences. It indeed appears that cross subsidization increases prepayment for the agents with the lower preference for future and is therefore likely to stabilize the dynamic contract. In future work, it would then be worthwhile to analyze the impact of heterogeneous degrees of protectiveness on this cross-subsidization. Our analysis suggests that agents with a low degree of protectiveness would subsidy more protective agents. In this case, the coexistence of mutual dynamic insurance with spot market (in first period) would be explained by heterogeneous behaviors toward risk, the dynamic form being designed to insure the most protective policyholders. However, this effect being coupled with the time preference effect we just discussed, the effect of cross-subsidization is ambiguous. It therefore seams that the study of adverse selection in dynamic contract call further assumptions and probably for the full specify of the utility functions.

It also seems interesting to study the role of protectiveness on the optimal level of effort. In future work it would indeed be worthwhile to analyze if more protective agents exert more primary preventive in line with the work of Jullien et al. (1999) on the link between risk aversion and (secondary and tertiary) prevention. Our work also seems to opens perspectives on the role of information. The information on risk type takes an essential role in the writing of dynamic insurance contracts. It therefore seems natural to study the impact of genetic testing and medical checkups in this area. Following Barigozzi and Henriet (2008), policyholders would then choose on the one hand to undertake or not the tests and on the other hand to reveal or not the results. The objective of future work would then be to study the impact of these choices on the optimal level of effort and on the optimal dynamic contract. It indeed seems interesting to study the value of information through the trade-off between the resulting increases in effort and classification risk.

It would lastly be interesting to infer to what extend economic agents are protective. The specification of usual utility functions indeed leads to conflicting statement regarding the index of protectiveness. Whereas agents with CRRA preferences are non-protective for realistic values of parameters ($\gamma > 1$), individual whose preferences exhibit Constant Absolute Risk Aversion (CARA) always show a positive level of protectiveness. The use of simple lotteries or the resort to experiments therefore seems to be necessary to determine whether/when agents are protective or not. Applying the notion of "risk apportionment" (used in the case of additive risks by Eeckhoudt, L. and Schlesinger 2006) to multiplicative risks in simple lotteries, Eeckhoudt et al. (2008) provide a sufficient condition for agents to be protective. They indeed offer simple conditions on preferences among lotteries for which the index of relative prudence is larger than 2 and the index of relative risk aversion is lower than 1.

## Appendix: Simulations with CRRA preferences - The impact of protectiveness on first period premium

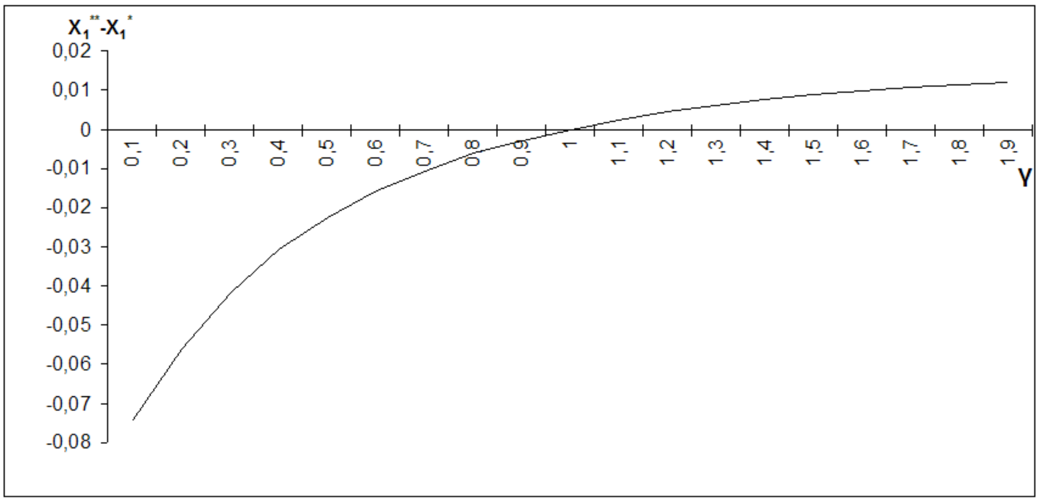


Figure 8: $\frac{\psi}{\Delta p} = 1, \overline{p} = 0,9$

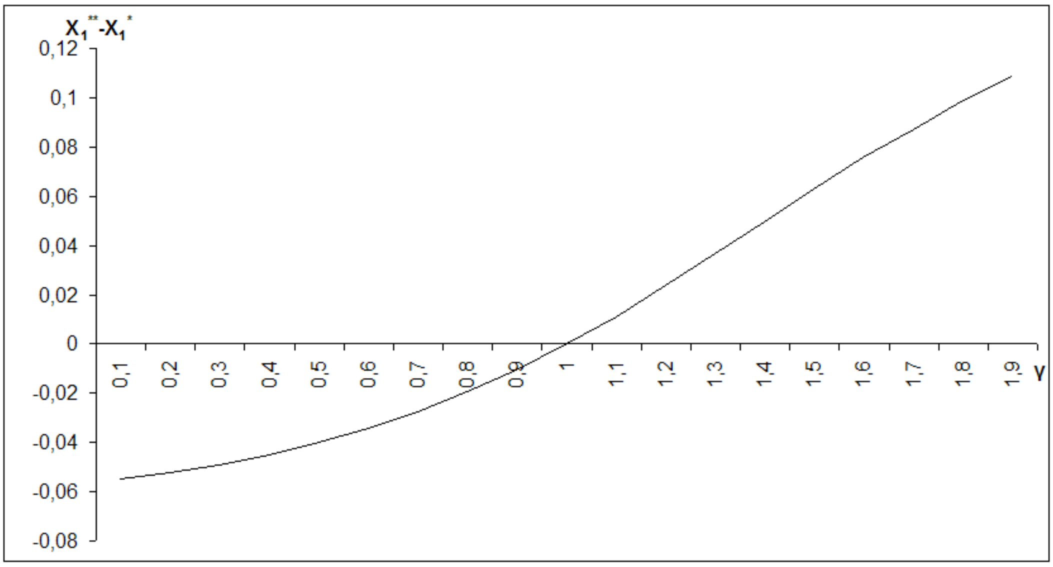


Figure 9: $\frac{\psi}{\Delta p} = 1, \overline{p} = 0,2$

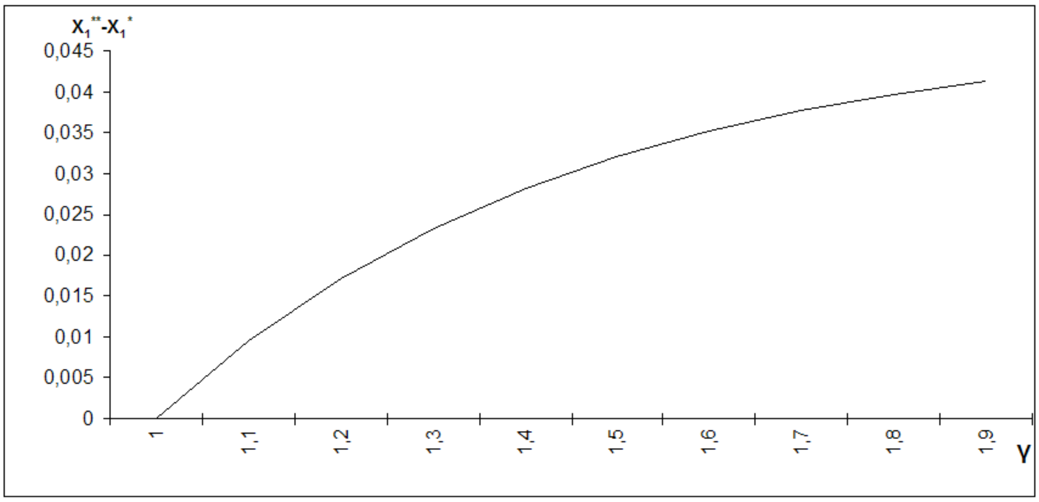


Figure 10: $\frac{\psi}{\Delta p} = 10, \overline{p} = 0,9$

## References


Abbring, J., Heckman, J., Chiappori, P-A. and Pinquet, J., "Adverse Selection and Moral Hazard In Insurance: Can Dynamic Data Help to Distinguish?", *Journal of the European Economic Association*, Vol. 1, No. 2-3, pp. 512-521, 2003.

Amir, R. and Czupryna, M., "On Inverse Utility and Third-Order Effects in the Economics of Uncertainty", *CORE Discussion Paper* No. 2004/45, 2004.

Barigozzi, F. and Henriet, D., "Genetic Information: Comparing Alternative Regulatory Approaches when Prevention Matters", *working paper*, 2008.
disponible at http://www2.toulouse.inra.fr/lerna/congres/egrie/papers/Barigozzi, Francesca - Paper.pdf

Chiu, H., "On the propensity to self-protect", *The Journal of Risk and Insurance*, 67 (4), pp. 555-578, 2000.

Cochrane, J., "Time Consistent Health Insurance," *Journal of Political Economy*, CIII, pp. 445–473, 1995.

Crainich, D. and Eeckhoudt, L., "La notion économique de prudence. Origine et développements récents", *Revue économique*, 56 (5), pp. 1021-1032, 2005.

Eeckhoudt, L., Etner, J. and Schroyen, F., "The values of relative risk aversion and prudence: A context-free interpretation", *Mathematical Social Sciences*, forthcoming, 2008.

Eeckhouldt, L. and Gollier, C., "Are independant optimal risks substitutes?", *Discussion paper, University of Toulouse*, 2000.

Eeckhouldt, L. and Gollier, C., "The impact of prudence on optimal prevention", *Economic Theory*, 26, pp. 989–994, 2005.

Eeckhoudt, L. and Schlesinger, H., "Putting Risk in its Proper Place", American Economic Review, 96, pp. 280-289, 2006.

Ehrlich, I. and Becker, G., "Market insurance, self-insurance and selfprotection", *Journal of Political Economy*, 40, pp. 623-648, 1972.

Frick, K., "Consumer Capital Market Constraints and Guaranteed Renewable Insurance", *Journal of Risk and Uncertainty*, 16, pp. 271–278, 1998.

Gollier, C., "Optimal prevention of unknown risks: A dynamic approach with learning" *Discussion paper, University of Toulouse*, 2002.

Gollier, C., "The Economics of Risk and Time", MIT Press, Cambridge MA, 2001.

Gollier, C., Jullien, B. and Treich, N., "Scientific progress and irreversibility: an economic interpretation of the 'Precautionary Principle'", *Journal of Public Economics* 75, pp. 229–253, 2000.

Grossman, S. and Hart, O., "An Analysis of the Principal-Agent Problem," *Econometrica*, vol. 51(1), pp. 7-45, 1983.

Jullien, B., Salanié, B. and Salanié, F., "Should More Risk-Averse Agents Exert More Effort?", *The Geneva Papers on Risk and Insurance Theory*, 24, pp. 19–28, 1999.

Hendel, I., and Lizzeri, A., "The Role of Commitment in Dynamic Contracts: Evidence from Life Insurance", *The Quarterly Journal of Economics*, February, pp. 299-327, 2003.

Kimball, M., "Precautionary Saving in the Small and in the Large", *Econometrica*, Vol. 58 No. 1, pp. 53-73, 1990.

Laffont, J.-J. and Martimort, D., "The Theory of Incentives: The Principal-Agent Model", Princeton University Press, 2002.

Laibson, D., "Golden Eggs and Hyperbolic Discounting", *The Quarterly Journal of Economics*, CXII (2), pp. 443-477, 1997.

Mehr, R. I. and Commack, E., "Principals of Insurance" 4th Ed., Irwin, Homewood, Il, 1966.

Newman, A., "Risk bearing and Knightian enterpreneurship", *mimeo*, 1995.

Nishimura, Y., "Dynamic Insurance Contracts under Limited Commitment and Moral Hazard" *Working Paper*, Yokohama National University, presented at the 60th Congress of the International Institute of Public Finance, Milan 2004.

Pauly, M., Kunreuther, H. and Hirth, R., "Guaranteed Renewability in Insurance." *Journal of Risk and Uncertainty*, X , pp. 143–156, 1995.

Pauly, M., Nickel, A, and Kunreuther, H., "Guaranteed Renewability with Group Insurance", *Journal of Risk and Uncertainty*, 16, pp. 211–221, 1998.

Tabarrok, A., "Genetic testing: an economic and contractarian analysis", *Journal of Health Economics* 13, pp. 167-207, 1994.

Thiele, H. and Wambach, A., "Wealth Effects in the Principal Agent Model", *Journal of Economic Theory* 89, pp. 247-260, 1999.