When to Allow Buyers to Sell? - Bundling in Mixed Two-Sided Markets*

Ming Gao†

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Abstract

We define mixed two-sided markets as two-sided markets where users of a platform can appear on different sides of the market in different transactions. That is, sellers can also be buyers and vice versa, such as in online trading and stock exchanges. We provide a model to capture the dynamics in such markets. The mixedness of the two market sides makes it possible for the platform to bundle its services originally intended for different sides. We study a new and somewhat hybrid kind of bundling strategies with two-part tariffs which is widely used by platforms in such markets, and show that bundling is more likely to dominate separate sales when the market has a higher degree of mixedness. This result remains robust under full and bounded user rationality.

1 Introduction

This paper defines a new kind of two-sided markets - mixed two-sided markets - and studies bundling by a monopoly platform in such markets. It links the theoretical literatures on two-sided markets and bundling. In particular, we provide a model that captures the dynamics in such markets and find sufficient conditions for bundling to be more profitable than unbundled sales by a monopolist.

A two-sided market is defined as **mixed** if a user of the two-sided platform can appear on different sides of the market in different transactions; otherwise, the two-sided market is called **standard**. The existing theoretical literature of two-sided markets has focused on modelling the standard two-sided markets (see, for instance, Armstrong (2004 and 2005) and Rochet and Tirole (2003 and 2006)), where the agents on each side of the market are implicitly restricted to having non-positive

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†Economics Department, London Business School, Regent's Park, London NW1 4SA, UK. Email: `mgao.phd2005@london.edu`.

valuation for the service that the platform provides to the opposite side of the market. In this paper, however, we relax this restriction. We allow any agent to have positive valuation for the services that the platform provides to both sides. At her own discretion, any agent can be a buyer, a seller, or both a buyer and a seller (in different transactions). Figure 1 illustrates the difference between a mixed and a standard two-sided market.

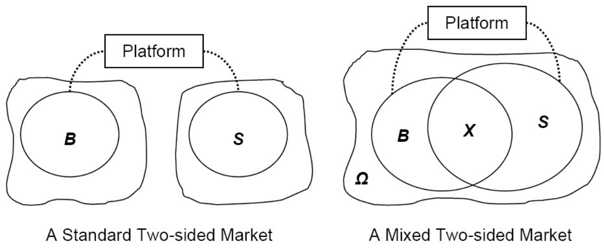

Figure 1: Standard And Mixed Two-Sided Markets

Examples of mixed two-sided markets are quite pervasive. The online trading market is one such example, where people can quite freely buy and sell as they please through a platform like eBay. Our model setting is best interpreted in the online-trading context. The telecommunications market can also be thought of as a mixed two-sided market when we consider the phone calls as goods transferred between the calling and answering parties via the mobile network platform. Indeed, many if not most kinds of financial intermediation where traders are allowed to both buy and sell products also have the defining characteristics of mixed two-sided markets. Such markets include social lending, securities brokerage and stock exchange.

For a mixed two-sided platform, the market still has two sides - the seller side and the buyer side - although they "overlap" with each other. Like in standard two-sided markets, there exist indirect network effects between the two sides - the value that a potential seller expects from using the platform depends on the size of the buyer side, and vice versa. In this paper we use a set-up that is a variation of the general framework by Rochet and Tirole (2006) to capture the indirect network effects in a mixed two-sided market. In this sense, our paper extends the literature on two-sided markets to the mixed case.

In general, a two-sided platform provides two kinds of services, each to one side of the market. For simplicity we call them buying and selling services, respectively. The key distinction between mixed two-sided markets and standard two-sided markets is that we allow all users in mixed two-sided markets to value positively the use of both kinds of services, while all users in standard two-sided markets value only one kind of service. An important implication of this distinction is that the mixed two-sided platform can consider combining the two kinds of services that it normally offers to different sides of the market, and provide them in bundles to all potential users on both sides.[1] Each user, by paying the appropriate fees, can use the platform for

[1] In mobile telecommunication markets where RPP (Receiving Party Pays) principle is used

selling, buying or both. This is exactly the kind of bundling that we study in this paper.[2] Note that such bundling is not relevant in standard two-sided markets.

The literature on bundling distinguishes between pure bundling (only selling bundles of two goods) and mixed bundling (providing two goods both separately and in bundles) (see, for instance, McAfee, McMillan and Whinston (1989), Fang and Norman (2006) and Banal-Estanol and Ottaviani (2007)). What we study in this paper is a new and somewhat hybrid kind of bundling with two-part tariffs (consisting of access fees and transaction fees). That is, if an agent wants to use the platform, she first needs to pay the relevant one-off access fee to obtain a certain kind of membership (e.g. seller membership) , then she needs to pay an additional transaction fee applicable to her role in the transaction (e.g. as a seller) each time she uses the relevant service. The platform chooses whether to offer seller membership and buyer membership separately (by charging two fees) or to bundle them together (by charging one combined access fee), but it does not offer both options together. The transaction fees are always charged separately in our model. If we think of membership and transaction as different kinds of services provided by the platform to both sides of the market, the pricing strategy studied here is essentially pure bundling of memberships and unbundled transactions.[3]

We show that there exists a bundling effect by using this hybrid bundling strategy, and its appeal to the mixed two-sided platform is directly related to the **degree of mixedness** of the market (measured by the proportion of the users who are both sellers and buyers, i.e. the **seller-buyers**, amongst all users[4]). The prevalence of seller-buyers on the platform represents the very nature of mixed two-sided markets. We show that the higher the degree of mixedness, the more likely that bundling dominates unbundled sales.

While a variety of other kinds of pricing strategies are possible and sometimes observed, we believe the particular kind described above is most prevalent in real-life mixed two-sided markets. Aside from eBay, mobile network operators often charge subscribers a membership fee for connection (with both calling and answering capacities) plus different per-minute usage fees (at least to pay-as-you-go users) for making and answering calls. Similar fee structures are also widely used by financial itermediaries, such as stock exchanges and security brokers. In this paper we provide insight for the popularity of such pricing strategies. In another paper (Gao (2009b), *Bundling with Network Effects*) we analyze in detail pure bundling, mixed bundling and some other variations of bundling strategies by mixed two-sided platforms, which provides more general and "complete" results.

We use a three-stage game to capture the dynamics in the market. Whether

(e.g. China), for instance, mobile carriers sometimes offer bundles of talking minutes including both calling and answering times.

[2]We do not consider, for instance, an eBay seller bundling two products and selling them on the platform, or eBay bundling two kinds of selling services (that are both designed for sellers but not buyers).

[3]Note that the value of the platform to any user, whether as a seller or a buyer, can only be realised when both membership and transaction of the same side are used.

[4]See section 4.1 for a detailed discussion of this measure.

agents choose to join the platform (by paying the relevant membership fee(s)) depends on how much they expect to gain from future transactions on the platform. The expected surplus from future transactions can be affected by whether agents fully foresee and take into account all possible future benefits as a seller, a buyer and a seller-buyer, or only focus on their current "urgent" needs (e.g. purchasing a newly released CD on eBay) while ignoring the other potential benefits (e.g. selling some old CDs on eBay in the future).[5] We consider these two cases one by one, by making different assumptions about agents' rationality. It is worth noting that when agents are boundedly rational and ignores parts of the potential benefits from future transactions, the demand for the platform's bundled services will in general be lower, *ceteris paribus*. However, we will show that all our results on the desirability of bundling under full agent rationality remain unchanged under bounded agent rationality. This "robustness check" confirms that the results come purely from bundling and mixedness.

Besides the pricing strategies, we also discuss briefly in the end of the paper a situation where the platform introduces an activation procedure in order to change its cost structure, which is sometimes observed in reality. We show that such procedures will further enhance the desirability of bundling vis-à-vis separate sales.

The remainder of this paper is structured as follows: Section 2 describes our model settings and basic assumptions. In section 3 we discuss the separate-sales strategy as a benchmark. Section 4 is dedicated for the model of bundling, where we first set up a model under full agent rationality and provide results of the bundling effect, then test them under bounded agent rationality; in the last part of section 4 we discuss bundling with activation costs. Section 5 concludes and discusses possible future extensions.

2 Modeling Set-Up[6]

We assume there is a market with one monopoly platform and a continuum $[0, 1]$ of agents. The agents may want to trade a certain kind of good with one another. The only possible means of trading between any two agents is to make transactions through the platform, which provides services of buying and selling, at some charges.

The market works in three stages:

Stage 1 - The platform chooses a system design option (separate sales or bundling) and announces relevant prices;

Stage 2 - Each agent observes the prices and decides at her own discretion, whether to join the platform as a buyer, a seller, both a buyer and a seller (by paying the relevant membership fees), or not to use the platform;

Stage 3 - Users who decide to join the platform trade.

[5]See section 4.2 for a detailed explanation of an motivating example of bounded agent rationality.

[6]The set-up we use is very general in the two-sided markets literature, especially following that of Rochet and Tirole (2006).

2.1 The platform

The platform has two options of system design and tariff plans:

Separate sales - The platform separates the services provided to different sides of the market, and sets different two-part tariffs for different sides.

We denote the separate-sales price vector $\mathbf{P}_s \equiv (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$, where A^S and A^B are respective membership fees for each seller and buyer, and a_s^S and a_s^B are respective marginal usage fees per transaction borne by the seller and buyer involved.

Only those who have paid the relevant fees are provided with the appropriate functionalities. Thus anyone who wishes to use the services of both sides must pay both membership fees. In this strategy the two sides in the market are treated as if they were the two distinctive customer groups in a standard two-sided market.

Bundling - The platform offers a bundle of both selling and buying services to any potential user.

The bundling price vector is denoted $\mathbf{P}_m \equiv (a_m^S, a_m^B, A) \in \mathbb{R}^{+3}$, where A is the membership fee for each user, and a_m^S and a_m^B are marginal usage fees per transaction borne by the seller and the buyer involved, respectively.[7]

All members of the platform who have paid A are provided with both buying and selling functionalities on the platform. They then only need to pay marginal fee a_m^B in transactions where they buy, and a_m^S in transactions where they sell.

These two system design options involve different cost structures for the platform. With separate sales, the platform incurs fixed costs F^S per seller and F^B per buyer, and a marginal cost c per transaction. With bundling, it incurs a fixed cost F per user and the same marginal cost c per transaction[8]. We assume $\max(F^S, F^B) \leq F \leq F^S + F^B$ throughout this paper, i.e. it is possible for the platform to have economies of scope.

Between these two design options, the platform chooses the one that yields higher profits.

Notice that neither of these design strategies can exactly replicate the other unless all three membership fees (A^S, A^B and A) are set to be zero. At any positive membership fees, the allocation of agents in different market segments (sellers, buyers, etc.) will not be the same under different designs. This means demand will in general be different under different designs. This point becomes clearer after we specify all the settings, and will be addressed in detail in section 4.1.

2.2 The agents

The whole set $[0, 1]$ of agents is denoted Ω. An agent $\omega \in \Omega$, by using the platform, gets a constant benefit v_ω^S from each sale, and a constant benefit v_ω^B from each

[7]As discussed earlier, this is essentially pure bundling of memberships and unbundled transactions.

[8]This cost structure applies in the whole paper, except for section 4.3 where the platform's activation procedure changes some of the specifications here.

purchase, which are only known to the agent herself.[9] From the platform's point of view, the marginal benefits that an agent gets from each sale and purchase are random variables, which we denote by v^S and v^B, respectively, and we assume the platform knows their distribution.[10]

Assumption 1 *v^S and v^B are drawn independently from respective cumulative distributions $G^S(\cdot)$ and $G^B(\cdot)$, with respective density functions $g^S(\cdot)$ and $g^B(\cdot)$.*

In stage 2 of the game, each agent makes her own decision whether to use the platform's services or not. We call this the agent's **membership decision**. We say an agent becomes a user (or member) of the platform by paying the relevant membership fee(s). The set of all users is denoted Y.

In stage 3 of the game, each user makes her own decision whether to sell, to buy, or to both sell and buy, by paying the relevant transaction fee(s).[11] We call this the agent's **trading decision**. Any agent who uses the selling (respectively, buying) service is called a **seller** (respectively, **buyer**).

We denote the set of all sellers S (also referred to as the seller side) and the set of all buyers B (buyer side). If a user is both a seller and a buyer, she's called a **seller-buyer**, otherwise she's either a **pure seller** or a **pure buyer**. We denote the set of pure sellers PS, the set of pure buyers PB, and the set of seller-buyers X.

The relationships among the above sets are summarized as follows:

$$\begin{aligned} &S = PS \cup X; \quad B = PB \cup X; \\ &X = S \cap B; \quad\;\; Y = S \cup B = PS \cup PB \cup X. \end{aligned}$$

Ex post, a mixed two-sided market is one where $X \neq \varnothing$.

Now we define the number of agents in these sets as the probability measures of the relevant sets, among which the most important ones are:

The number of sellers: $N^S \equiv \Pr[\omega \in S]$.

The number of buyers: $N^B \equiv \Pr[\omega \in B]$.

The number of seller-buyers: $N^X \equiv \Pr[\omega \in X]$.

The number of users: $N \equiv \Pr[\omega \in Y] = N^S + N^B - N^X$.

To the platform, these numbers are demand from different market segments.

[9]This assumption is also used in Rochet and Tirole (2006). Notice v^S_ω (resp. v^B_ω) is the "net" benefit from a sale (resp. purchase) for seller (resp. buyer) ω, including any payment transfer from the buyer (resp. seller). Only the fees charged by the platforms are not included in v^S_ω and v^B_ω.

[10]In addition to v^S and v^B, Rochet and Tirole (2006) also used "fixed" benefits from being a member of the platform - B^S and B^B. Our implicit assumption here is that B^S and B^B are the same for every agent and are normalized to zero.

[11]Any user must trade, because having paid a non-negative membership fee, no-trading would give her non-positive net payoff which is weakly worse than her outside option (0 payoff). In this case we assume she should not have joined.

2.3 The trade

We assume each buyer makes exactly N^S transactions in total, and each seller makes exactly N^B transactions in total.[12] The total volume of transactions is then $N^S \cdot N^B$.

If the platform charges transaction fees a^S and a^B, then the expected surplus a seller with v^S gets from all transactions that she makes is (gross of membership fee):

$$u^S \equiv (v^S - a^S)N^B$$

And a the expected surplus of a buyer with v^B is (gross of membership fee):

$$u^B \equiv (v^B - a^B)N^S$$

An agent's net surplus from being a user is then the total surplus she gets from trading minus the membership fee she pays. For example, a seller-buyer's net expected surplus is $u^S + u^B - A^S - A^B$ under separate sales, while it is $u^S + u^B - A$ under bundling.

2.4 Agents' decisions

Given the platform's choice of design and the relevant prices announced, each agent makes her membership decision by comparing her net expected surplus as a user with the outside option, which is equivalent to comparing her gross expected surplus from trading with the membership fee. We make the following assumptions:

- *Any agent's outside option (i.e. not joining the platform) gives zero surplus.*
- *Whenever an agent is indifferent between joining and not joining, she doesn't join.*

With separate sales, an agent makes two separate membership decisions, each regarding one side of the platform. With bundling, a user will have three potential options of trading - being a seller, a buyer, or a seller-buyer. We start with separate sales.

3 Separate Sales[13]

With separate sales, the platform separates the services provided to different market sides, and announces charges $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B)$. Because we have assumed the

[12] This assumption is used in Rochet and Tirole (2006) and we also use it here to faciliate a direct comparison between the results.

Concerns might arise regarding self-trade of the same seller-buyer. However this is not a problem in our set-up, since each individual agent is infinitesimal and her self-trade has zero measure in the accounting of the volume of transactions based on probablity measures.

[13] The analyses and results under separate sales are very similar to those of the model in Rochet and Tirole (2006). This is no surprise because by our formulation, a mixed two-sided market with separate sales is "equivalent" to a standard two-sided market.

benefits from buying and selling to be independent for the same agent, each agent will make separate decisions about joining the buyer side and the seller side. This means we have the following demand at $\mathbf{P}_s$:[14]

$$\begin{aligned} N_s^S &= \Pr[u^S - A^S > 0] = 1 - G^S(\tfrac{A^S}{N_s^B} + a_s^S) \\ N_s^B &= \Pr[u^B - A^B > 0] = 1 - G^B(\tfrac{A^B}{N_s^S} + a_s^B) \end{aligned} \quad (1)$$

N_s^S and N_s^B are simultaneously determined by system (1) at $\mathbf{P}_s$.

The allocation of agents in different market segments at $\mathbf{P}_s$ is illustrated in Figure 2.[15]

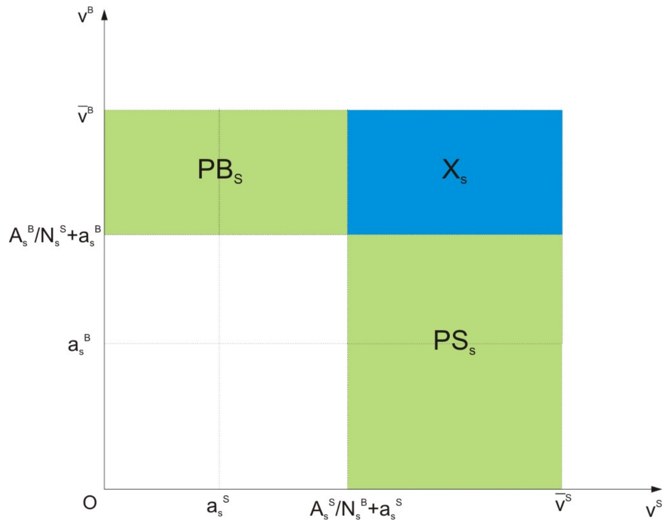

Figure 2: Separate Sales

The platform's profit at the separate-sales price $\mathbf{P}_s$ is then:

$$\Pi_s(\mathbf{P}_s) \equiv \underbrace{(A^S - F^S)N_s^S}_{\text{profit from sellers' mem fee}} + \underbrace{(A^B - F^B)N_s^B}_{\text{profit from buyers' mem fee}} + \underbrace{(a_s^S + a_s^B - c)N_s^S N_s^B}_{\text{profit from transactions}} \quad (2)$$

Lemma 1 *(Redundancy of separate-sales prices) For any $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$, there exists a degenerate price vector $\mathbf{P}_{s0} = (a_s^{S\prime}, a_s^{B\prime}, 0, 0) \in \mathbb{R}^{+4}$, such that $\mathbf{P}_{s0}$ exactly replicates the demands and profit under $\mathbf{P}_s$.*

Proof. For any $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B) \in \mathbb{R}^{+4}$, let N_s^S and N_s^B denote the demands

[14] We use subscript s to represent separate sales. Notice by our assumptions above, whenever one of N_s^S and N_s^B is zero, the other must also be zero. In the following discussions we only consider positive numbers of agents.

[15] In Figure 2 we assume v^S has suppport $[0, \bar{v}^S]$ and v^B has suppport $[0, \bar{v}^B]$, respectively. The same assumption is also used in Figures 3, 4 and 5.

induced by $\mathbf{P}_s$ which must solve (1) above. Now let

$$\begin{aligned} a_s^{S\prime} &\equiv a_s^S + \frac{A^S}{N_s^B} \\ a_s^{B\prime} &\equiv a_s^B + \frac{A^B}{N_s^S} \end{aligned} \tag{3}$$

Then $\mathbf{P}_{s0} = (a_s^{S\prime}, a_s^{B\prime}, 0, 0)$ is in $\mathbb{R}^{+4}$ and N_s^S and N_s^B also solves (1) at $\mathbf{P}_{s0}$. It is easy to check that $\Pi_s(\mathbf{P}_s) = \Pi_s(\mathbf{P}_{s0})$. ■

The redundancy in the platform's separate-sales pricing strategy $\mathbf{P}_s$ can be seen directly in Figure 2, where we only need one price from each side of the market to determine all the demand segments.[16]

From now on, we focus without loss of generality on the degenerate vector $\mathbf{P}_{s0}$ as the relevant separate-sales strategy. And we assume there exists an optimal degenerate vector denoted by $\mathbf{P}_{s0}^* \equiv (a_s^{S*}, a_s^{B*}, 0, 0)$ at which the separate-sales profit is maximized, i.e. $\mathbf{P}_{s0}^* \in \arg\max_{\mathbf{P}_s} \Pi_s(\mathbf{P}_s)$.

4 Bundling

In this section we propose a model for the bundling strategy. Suppose the platform announces price $\mathbf{P}_m = (a_m^S, a_m^B, A)$. In an agent's membership decision, she needs to compare her gross expected surplus from trading with the bundled membership fee A. What she takes into consideration in the gross expected surplus from trading depends on what assumptions we make about her rationality. We first study the situation where agents are fully rational, and then discuss what happens when they are not. In general, the demand for bundled services is higher under full rationality than under bounded rationality. Our results, however, remain robust in both cases.

In the end of this section we discuss the situation where the platform uses activation procedures to reduce costs.

4.1 Bundling with Full Agent Rationality

Here we assume agents are fully rational and when faced with the bundling strategy $\mathbf{P}_m$ they use backward induction when making their membership decisions in stage 2 and trading decisions in stage 3. This assumption may sometimes be referred to as M1 in our discussion later.[17]

Starting backwards from trading decisions, agents need to choose the highest among u^S, u^B and $u^S + u^B$ given bundling price vector $\mathbf{P}_m$. In effect, each agent's membership decision at $\mathbf{P}_m$ is based on a comparison between the expected surplus from her trading decision, $\max(u^S, u^B, u^S + u^B)$, and the membership fee A. This means the set of users at $\mathbf{P}_m$ is the following:

$$Y_{m1} \equiv \{\omega \mid \max(u^S, u^B, u^S + u^B) > A\}$$

[16] Lemma 1 is also a result in Rochet and Tirole (2006).

[17] We use subscript $m1$ for all results derived under bundling with full rationality.

And the group of seller-buyers at $\mathbf{P}_m$ is:

$$\begin{aligned} X_{m1} &\equiv \{\omega \in Y_{m1} \mid \max(u^S, u^B, u^S + u^B) = u^S + u^B\} \\ &= \{\omega \mid u^S > 0, u^B > 0, \text{ and } u^S + u^B > A\} \end{aligned}$$

We can then find the demand in different market segments using the distributions $G^S(\cdot)$ and $G^B(\cdot)$, which in general will depend on all three prices - a_m^S, a_m^B and A. The allocation of agents at $\mathbf{P}_m$ is shown in Figure 3.

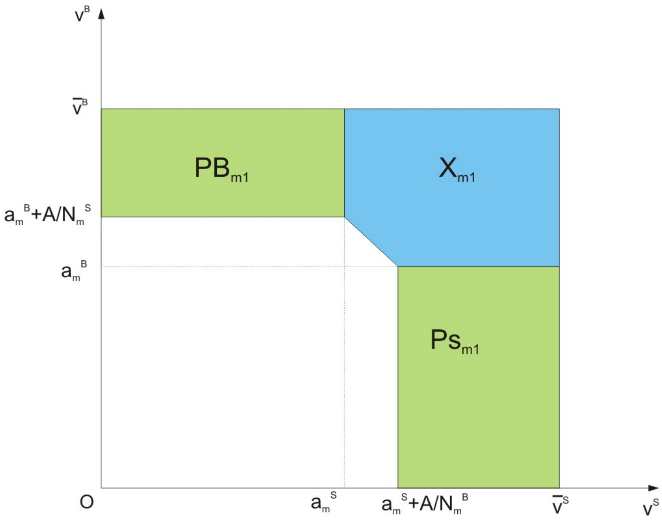

Figure 3: Bundling with Full Rationality

We denote the demand at bundling price $\mathbf{P}_m = (a_m^S, a_m^B, A)$ with full agent rationality as:[18]

$$\begin{aligned} N_{m1}^S &= n_{m1}^S(a_m^S, a_m^B, A) \\ N_{m1}^B &= n_{m1}^B(a_m^S, a_m^B, A) \\ N_{m1} &= n_{m1}(a_m^S, a_m^B, A) \end{aligned}$$

As shown in Figure 3, all three price instruments - a_m^S, a_m^B and A - are necessary to characterize the demand, thus there is no redundancy in the bundling pricing strategy. Later in this section we will see this is crucial to the advantage of the bundling strategy.

The platform's profit can also be represented as a function of $\mathbf{P}_m$:

$$\Pi_{m1}(\mathbf{P}_m) = \underbrace{(A - F)N_{m1}}_{\text{profit from membership}} + \underbrace{(a_m^S + a_m^B - c)N_{m1}^S N_{m1}^B}_{\text{profit from transactions}} \quad (4)$$

We now compare bundling with separate sales.

[18] The specific demand formulae are shown in the appendix, where N_{m1}^S and N_{m1}^B are determined at price $\mathbf{P}_m$ by a simultaneous system of two equations (6). In case the solutions to the system are correspondences, we take the suprema of them. This is feasible because N_{m1}^S and N_{m1}^B are both bounded ($N_{m1}^S, N_{m1}^B \leq 1$).

Lemma 2 *At any variable fees* $(a^S, a^B) \in \mathbb{R}^{+2}$*, the degenerate separate-sales strategy* $\mathbf{P}_{s0} = (a^S, a^B, 0, 0)$ *and the degenerate bundling strategy* $\mathbf{P}_{m0} = (a^S, a^B, 0)$ *produce exactly the same demand in each market segment.*

Proof. See appendix. ■

This lemma is quite intuitive. When all membership fees are set to zero, a separate-sales price and a bundle price with exactly the same usage fees will look exactly the same to agents. Thus each agent will make the same membership and trading decisions under these prices, resulting in the same demand in each market segment.

When membership fees are positive, however, the same agent may make different trading decisions and hence different membership decisions under different designs. This will in general lead to different demand under different designs.

Suppose we set the same a^S, a^B in both strategies and $A^S = A^B = A > 0$. It is easy to see the difference in demand from Figures 2 and 3. In particular, there exist non-users, pure sellers, and pure buyers in the separate-sales system who become seller-buyers under bundling. Their surplus from either buying or selling alone is not high enough to compensate for the membership fee, but their combined surplus is. They value the savings from a lower combined membership fee for both services under bundling.[19]

If we set $A^S = A^B = \frac{1}{2}A > 0$, still there exist pure sellers and pure buyers (among others) under separate sales who become seller-buyers under bundling. Take an agent α, for example, whose $v_\alpha^S \in (a^S, \frac{A^S}{N^B} + a^S)$ and $v_\alpha^B > \frac{A}{N^S} + a^B$.[20] She will be a pure buyer under separate sales because $0 < u_\alpha^S < A^S$ and $u_\alpha^B > A > A^B$; but she will be a seller-buyer under bundling because $u_\alpha^S + u_\alpha^B > A$.

In both cases above, there may also be agents dropping out during the transition from separate sales to bundling. In general, separate sales and bundling will yield different agent allocation (depending on value distributions) and hence different demand, unless all membership fees are zero.

The Bundling Effect

Now we make the comparison between separate sales and bundling by using a thought experiment. We start from a situation where the platform has achieved the highest profit under separate sales, $\Pi_s(\mathbf{P}_{s0}^*)$, with strategy $\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$. Now consider a switch from separate sales to bundling, while keeping all the fees unchanged. We denote the resulting bundling strategy $\mathbf{P}'_{m0} \equiv (a_s^{S*}, a_s^{B*}, 0)$, and profit $\Pi_{m1}(\mathbf{P}'_{m0})$.

By Lemma 2 we immediately know that the demand in each market segment remains unchanged. Thus whether profit changes depends solely on how costs are

[19]Notice there are other differences in demand that are not represented in the figures. They are caused by the network effects which can be seen from the simultaneous system (6) in appendix from which demand functions are derived. In these equations, the demand of either side depends also on the size of the other side.

[20]Suppose she expects the platform to have N^S sellers and N^B buyers.

changed. We postpone the discussion of the effect of cost changes and focus here on another question: Starting from $\mathbf{P}'_{m0}$, can the platform manipulate the bundled membership fee to get higher bundling profits than $\Pi_{m1}(\mathbf{P}'_{m0})$? The answer is yes.

Proposition 1 (**The Bundling Effect**) *If separate sales price* $\mathbf{P}^*_{s0} = (a_s^{S*}, a_s^{B*}, 0, 0)$ *is optimal, i.e.* $\mathbf{P}^*_{s0} \in \arg\max_{\mathbf{P}_s} \Pi_s(\mathbf{P}_s)$, *then at bundling price* $\mathbf{P}'_{m0} = (a_s^{S*}, a_s^{B*}, 0)$, *we have* $\frac{\partial \Pi_{m1}}{\partial A}(\mathbf{P}'_{m0}) > 0$. *That is, at* $\mathbf{P}'_{m0}$ *raising the bundled membership fee from zero strictly increases bundling profit.*

Proof. See appendix. ■

Proposition 1 shows the effect of bundling in mixed two-sided markets. It is important to point out first that Proposition 1 does not compare profits under bundling and separate sales. It only says that at *after* switching to bundling, at price $\mathbf{P}'_{m0}$ the platform is able to increase bundling profit by raising A.[21] We discuss its intuition with the help of Figure 4.

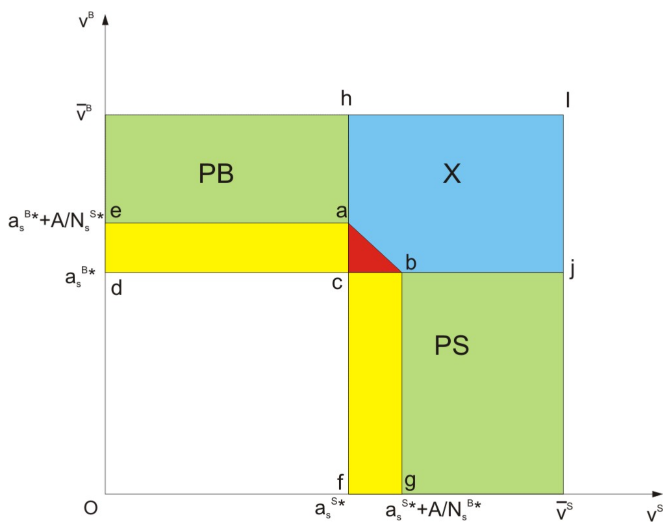

Figure 4: The Bundling Effect

For pure sellers (PS in Figure 4), their demand at $\mathbf{P}'_{m0} = (a_s^{S*}, a_s^{B*}, A = 0)$ is fully determined by a_s^{S*} and a_s^{B*} (the area $jcf\bar{v}^S$). A rise in the bundled membership fee A shrinks their demand by the area $bcfg$, resulting in a new demand of the area $jbg\bar{v}^S$. Similarly, a rise in A shrinks pure buyers' demand by the area $acde$; and reduces seller-buyers' demand by the area abc. Thus raising A directly reduces both N^S and N^B.

The decrease in N^S and N^B has two effects. First, it directly reduces the total volume of transactions which is $N^S \cdot N^B$ and thus reduces the platform's revenues

[21] The direct comparison with the profit under separate sales will be discussed later in Proposition 2.

from transactions. Second, the platform saves a fixed cost F for each user who drops out due to the membership fee rise.

In addition to the decrease in demand, raising A has a third direct effect - the platform now earns more membership revenue from each remaining user, since A has increased from zero.

Now consider only the two market segments PS and PB. Raising A has a similar effect on their demand as raising a_s^{S*} or a_s^{B*}, respectively, only now adjusted by a factor $\frac{1-N_s^{B*}}{N_s^{B*}}$ or $\frac{1-N_s^{S*}}{N_s^{S*}}$, respectively. Had the fee raise happened in the separate-sales case, the optimality of the transaction prices a_s^{S*} and a_s^{B*} would have implied that, for PS and PB, all the three effects mentioned above would completely cancel out one another, leaving profit unchanged. Under bundling, however, the net effect on profit turns out to be positive, because the cost-saving effect is stronger here. Under separate sales, a decrease in demand would have only saved F^S per pure seller and F^B per pure buyer, while here it saves F ($\geq \max[F^S, F^B]$) per user. Thus the combined effect on PS and PB is an increase in bundling profit.

Consider now only the seller-buyers. The decrease of their demand due to raising A turns out to be second order to the increase in their membership payment (note the area *abc* has a probability measure proportional to A^2) and thus the platform ends up also making a profit (exactly of the size N_s^{X*}) from all seller-buyers by raising A.

Comparison with McAfee, McMillan and Whinston (1989)

The bundling effect shown above appears analogous to the mixed bundling effect in McAfee, McMillan and Whinston (1989) (henceforth MMW (1989)), in that the conclusion shows a positive marginal effect on bundling profit by a price change from optimal separate sales prices. The analogy is somewhat counter-intuitive since the bundling we study here is essentially *pure* rather than *mixed* bundling. In the pure bundling literature, conditions of this form are not common since the demand and cost structure is completely (instead of "marginally") different under pure bundling than under separate sales. Here the cost structure also changes completely (from F^S and F^B under separate sales to F under bundling), but demand changes are continuous (or marginal). The latter is due to the three-stage structure and two-part tariffs used in our setting. Because of the redundancy in the two-part tariffs under separate sales, we can focus without loss of generality on the transaction fees only, and fix them at the optimal separate sales level. This allows us to fix agents' gross expected surplus from transactions, which is one of two factors that determine demand. The only other factor is the membership fee(s), which is exactly where the bundling happens in our setting. Thus by increasing the bundled membership fee, we can study the marginal effect of bundling, without changing the demand dramatically. This is why our result resembles that of mixed bundling, although our setting resembles that of pure bundling.

The differences between our result and that of MMW (1989) are also apparent. First, Proposition 1 shows one way to increase the *bundling* profit only, and does not address directly the comparison between separate and bundled sales. This is exactly because the bundling we study here involves *pure* bundling of two membership services, which changes the fixed cost incurred for every user, unlike in the mixed bundling case where the costs due to users of only one product remain unchanged. Since the switch from separate sales to bundling in our setting is not as "smooth" in costs as that in MMW (1989), it does not allow a direct comparison of profits.[22]

A second difference is that Proposition 1 shows that an *increase* in the bundled membership fee from zero is profitable, unlike in MMW (1989) where a *decrease* in the bundle price (from the sum of optimal separate-sales prices) is profitable. Aside from the fact that increasing the bundle price above the sum of separate-sales prices is not feasible under mixed bundling, this difference also has to do with the three-stage structure and two-part tariffs used in our setting. In Figure 4, the boarder between X and PB and that between X and PS are fixed at the optimal separate-sales levels and do not change when we raise A. This makes the change in N^X second order to the rise in membership revenue and thus makes raising A profitable overall. In MMW (1989)'s one-part pricing and one-stage game, however, changes in demand due to changes in the bundle price are never second order to the revenue changes, and thus there does not exist the same effect as in our setting.

In another paper, Gao (2009b), we study mixed bundling with network effects by a mixed two-sided platform which is a direct extension of MMW (1989)'s result.

The Degree of Mixedness

By Proposition 1 alone we still do not know how the profit changes *during* the transition from separate sales to bundling, i.e. whether $\Pi_{m1}(\mathbf{P}'_{m0})$ is larger or smaller than $\Pi_s(\mathbf{P}^*_{s0})$. This comparison depends solely on how the cost structure changes during the transition, including:

i) additional fixed cost of each pure seller (increase by $F - F^S \geq 0$) and each pure buyer (increase by $F - F^B \geq 0$); and

ii) reduction on fixed cost of each seller–buyer (decrease by $F^S + F^B - F \geq 0$).

As is common in the bundling literature, the cost structure matters in the condition for bundling to yield a higher profit than separate sales. We provide such a condition for mixed two-sided markets in Proposition 2.

Proposition 2 *Suppose* $\min(F^S, F^B) > 0$, *then bundling strictly dominates separate sales if*

$$\frac{N_s^{X*}}{N_s^*} \geq \frac{F - \min(F^S, F^B)}{\min(F^S, F^B)}$$

where $\frac{N_s^{X*}}{N_s^*}$ *is the proportion of seller-buyers in all users at the optimal separate sales prices* $\mathbf{P}^*_{s0}$, *which measures the degree of mixedness, and*

$\frac{F-\min(F^S,F^B)}{\min(F^S,F^B)}$ *is the percentage increase in the lower-cost group's per-user fixed cost due to bundling.*

[22] In section 4.3 we change the assumptions about the cost structure and a direct profit comparison becomes feasible.

Proof. See appendix. ■

Proposition 2 directly connects the bundling effect to the mixedness of the two-sided market. Its intuition is two-fold.

First and foremost, the higher the degree of mixedness of the market, the more likely that bundling will dominate separate sales. The degree of mixedness of the market is measured by the proportion of seller-buyers in all users at the optimal separate sales prices, who are the most active traders on the platform. Although endogenous, the degree of mixedness is essentially a measure of the concentration of high-valued agents in the population. Seller-buyers have high values for both sides of the market. They act as sellers and buyers in different transactions and bring double revenues to the platform. Bundling can capture a higher demand in this "special" market segment than separate sales. The condition in Proposition 2 ensures that there are sufficiently many of seller-buyers amongst users such that the losses and savings due to cost changes are balanced out during the transition from separate sales to bundling and that the transition between strategies is "smooth" in profit.

Second, the bundling strategy has one more "degree of freedom" than separate sales as implied by Lemmas 1 and 2. The separate-sales demand is determined by two price instruments (a_s^S and a_s^B, for example), while the bundling demand is determined by three price instruments (a_m^S, a_m^B and A). Even though the separate-sales strategy (a_s^{S*}, a_s^{B*}) was optimal, after switching to bundling the platform can use the third price, access fee A, to further manipulate demand. And this manipulation turns out to be profitable as shown in Proposition 1.

The weakness of proposition 2 is that it does not provide a necessary condition for bundling to dominate. But its strength lies in the fact that it puts no (direct) constraints on the distributions of users' values. Instead, it intuitively conditions on the comparison between the degree of mixedness and the change in costs, which gives a more general and straightforward expression than conditioning on the specific forms of the distributions.

Proposition 2 may be used to explain why most mixed two-sided platforms, such as landline and mobile telecom operators and financial intermediaries, choose to use the bundling strategy. Even with separate sales, in equilibrium in these markets there will likely be a large proportion of users using services of both sides, which reflects the nature of mixedness in these markets. Moreover, in these markets there does not appear to be significant changes in fixed cost per user when one-way service is "upgraded" to two-way. Thus by Proposition 2 the monopoly platforms can achieve higher profits by choosing bundling instead of separate sales.

4.2 Bundling with Bounded Agent Rationality

The assumption in the previous section, that agents are fully rational and they take into consideration (by backward induction) all future benefits from trading (as a seller, a buyer, and a seller-buyer) in their membership decisions, seems to work in favor of bundling vis-à-vis separate sales. If agents consider only part of all the benefits from stage 3, in general the demand for the platform's services under bundling would be lower than it is with full rationality. In this section we test the results on bundling strategy under an assumption of bounded agent rationality. We show that even when each agent in her membership decision only considers the benefit from trading on one side of the market, the previous results still hold.

We continue to use bundling price $\mathbf{P}_m = (a_m^S, a_m^B, A)$.

A Motivating Example of Bounded Rationality

Sometimes, users may not be able to factor all potential benefits into their membership decisions. Take, for example, an agent who wants to buy a CD on eBay. Before signing up, suppose she has a high expected payoff from this purchase. She also has some idea about the possibility that she could one day sell her old CDs there and expects a positive payoff from selling (which is lower than that from the current purchase). Nonetheless, before joining she is not sure how the system will work and how the payoff from future selling can be realized. Thus at the moment she may have to make the decision based only on her current need. After becoming a user, however, through time she gets familiarized with the services and can finally realize her expected payoff from selling as well. In this example, the complexity of the system, or the agent's lack of information (of how to realize expected payoff), makes it impractical to take account of the "less urgent" need when making the decision. The agent here is not fully rational when making her choices - she cannot consider all potential needs *ex ante*. It is this particular kind of bounded rationality that we will focus on in this section, and it is summarized in the following assumption:

Assumption M2 *Given bundling price $\mathbf{P}_m$, each agent makes her membership decision based only on her more urgent need - the higher of u^S and u^B. The less urgent need only affects her trading decision after becoming a member.*

We believe for an online trading market M2 is more realistic than the assumption of full rationality. Note that M2 does not apply in the case of separate sales. Under separate sales, the agent is "forced" to make the fully rational membership decision because the membership fees are separated for buying and selling. Thus, compared with M1 (full rationality), M2 simply works against the appeal of the bundling strategy vis-à-vis separate sales and hence can serve as a "test" of the robustness of our previous results under full rationality.

By M2, the group of all users at bundling price $\mathbf{P}_m$ is the following set:[23]

$$Y_{m2} \equiv \{\omega \mid \max(u^S, u^B) > A\}$$

From an *ex post* view, the set of seller-buyers under M2 at $\mathbf{P}_m$ takes the following form:

$$\begin{aligned} X_{m2} &\equiv \{\omega \in Y_{m2} \mid \min(u^S, u^B) > 0\} \\ &= \{\omega \mid \max(u^S, u^B) > A, \text{ and } \min(u^S, u^B) > 0\} \end{aligned}$$

[23] We use subscript $m2$ for all results derived under assumption M2.

Each seller-buyer, in the first instance of using the platform, is only interested in either buying or selling. The first requirement $\max(u^S, u^B) > A$ makes sure she is willing to sign up in the first instance. Nonetheless, after becoming a member she is able to use the service of the opposite side without any further fixed fees. The second requirement $\min(u^S, u^B) > 0$ is to guarantee she will also use the service of the other side later on. Thus it is exactly the agents in set X_{m2} that will end up using the services of both sides.

With the distributions of values, we can find the demand in different market segments at price $\mathbf{P}_m$ under M2, which in general will also depend on all three prices - a^S, a^B and A.[24] This means in general bundling under M2 will also produce different market demand than separate sales, unless all membership fees are zero. The allocation of agents in different market segments at $\mathbf{P}_m$ under M2 is shown in Figure 5.

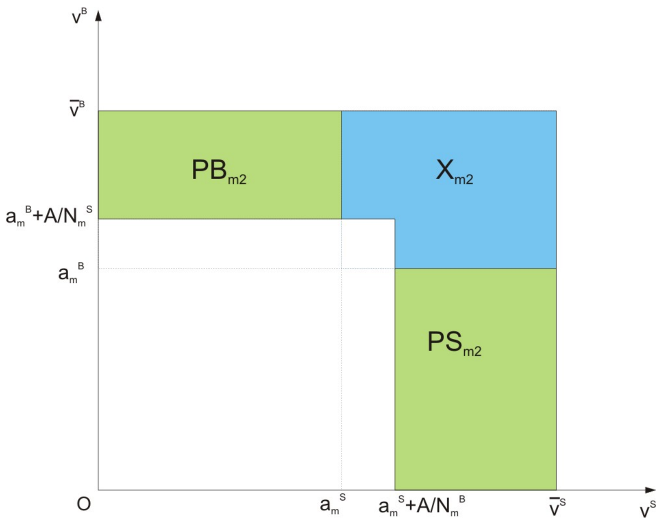

Figure 5: Bundling with Bounded Rationality

We denote the demand at bundling price $\mathbf{P}_m = (a^S, a^B, A)$ under M2 as:[25]

$$N^S_{m2} = n^S_{m2}(a^S, a^B, A)$$
$$N^B_{m2} = n^B_{m2}(a^S, a^B, A)$$
$$N_{m2} = n_{m2}(a^S, a^B, A)$$

[24] There is no redundancy in the bundling pricing strategy under M2. This can be seen from Figure 5 where all three prices - a^S, a^B and A - are necessary to characterize the demand.

[25] The specific demand formulae under M2 are provided in the appendix, where, like in the case with full rationality, N^S_{m2} and N^B_{m2} at price $\mathbf{P}_m$ are also determined by a simultaneous system of two equations (12). In case the solutions to the system are correspondences, we still take their suprema.

The platform has the following profit formula at bundling price $\mathbf{P}_m$ under M2:

$$\Pi_{m2}(\mathbf{P}_m) = (A - F)N_{m2} + (a^S + a^B - c)N^S_{m2}N^B_{m2} \tag{5}$$

The Bundling Effect under Bounded Agent Rationality

Without the need of further specification of distributions, we have the following general conclusion.

Proposition 3 *Propositions 1 and 2 also hold with bounded agent rationality.*

Proof. See appendix. ■

The intuition of Proposition 3 is: Although in general the demand under bounded rationality is lower than the demand under full rationality at the same bundling price, when that price includes a zero membership fee, the different demand converges to the same level. This can be seen by setting zero membership fees in the definitions of X_{m1} and X_{m2}, which makes them coincide, and so will the demand in any other market segment. Since under full rationality the conditions in Propositions 1 and 2 hold at degenerate bundling prices, they will continue to hold at the same degenerate bundling prices under bounded rationality.

Proposition 3 implies that the strength of the bundling strategy doesn't depend on agents' rationality of being able to foresee and take into account other potential surplus from trading on the platform besides the "most urgently needed" service. Rather, the strength of bundling lies in its consistency with the mixed nature of the market. The results shown in Propositions 1, 2 and 3 are only due to the bundling effect and the mixedness of the market.

4.3 Bundling with Activation Cost

We sometimes observe activation procedures required by platforms in real life. Mobile network subscribers usually need to call the network operator to activate the SIM card by providing some user information before being connected. eBay also requires users to choose the service functionalities before they can use them. While the platform may have many reasons to introduce this kind of procedures, we interpret its incentive in this section as imposing a small but positive **activation cost** on the users' part in order to separate different user types which in turn allows it to save fixed costs under bundling. We make the following new assumption:[26]

Assumption AC *The platform requires that each user need to activate each kind of service individually after becoming a member. An activation procedure will involve an arbitrarily small but positive cost $\gamma > 0$ for the agent. The platform does not incur the fixed cost of the relevant service until an agent activates it.*

[26] Note that in section 4.3, in addition to assumption AC, we also implicitly revert to the full rationality assumption M1, althogh the results shown here also hold under the bounded rationality assumption M2.

Under both separate sales and bundling, if an agent only activates the buying service (resp. selling service), the platform only incurs F^B (resp. F^S) for her; if an agent activates both services, the platform incurs F for her. $F \in [\max(F^S, F^B), F^S + F^B]$.

The activation cost is not a transfer to the platform. It may correspond to the short moments an agent spends providing user information or the little effort she makes to click on some buttons online. Because it is arbitrarily small, it has negligible effect on agents' membership and trading decisions and hence we consider the demand in all the market segments unchanged. However the activation procedure is useful for (among other things) separating the pure buyers and pure sellers from the seller-buyers. Before introducing the activation procedure, the platform cannot distinguish user type and thus have to incur F ($\geq \max(F^S, F^B)$) for any user. Under assumption AC, pure buyers and pure sellers will only activate one service since the positive activation cost will deter them from activating the other service from which they have non-positive gross expected surplus, and thus the platform can distinguish them and only need to incur the relevant lower cost (F^S or F^B) for each of them. Therefore the platform saves fixed costs under bundling.

In summary, the two changes in the cost structure compared to our earlier setting by introducing the activation procedure are:

i) under separate sales, an agent who activates both kinds of services (by incurring 2γ) will now cost the platform F instead of $F^S + F^B$;

ii) under bundling, an agent who activates only the buying service (by incurring γ) now costs the platform F^B instead of F, an agent who activates only the selling service (by incurring γ) now costs the platform F^S instead of F, and only those who activates both (by incurring 2γ) will cost the platform F.

In the discussion of Proposition 2 in section 4.1 we explained that the transition from separate sales to bundling is not smooth in costs - the structure of fixed costs changes completely from F^S per seller and F^B per buyer to F for every user. This caused a problem for a direct profit comparison. Under assumption AC, however, as discussed above there is no longer any change in the cost structure during the transition from separate sales to bundling, and thus the transition becomes smooth in profit. Therefore we immediately have the following new result.

Proposition 4 *When there is activation cost $\gamma > 0$, bundling strictly dominates separate sales whenever $X^* \neq \varnothing$.*

Proof. See appendix. ■

Proposition 4 is essentially Proposition 1 under assumption AC, where the new cost structure allows a smooth transition from separate sales to bundling that leaves profit unchanged, and thus there is no need for the condition in Proposition 2 for bundling to dominate. As long as there are seller-buyers at the optimal separate-sales price, Proposition 1 ensures that bundling is strictly more profitable.

Since the cost structure no longer play a role in the condition in Proposition 4, this result is purely due to the bundling effect, i.e. the fact that the bundling strategy has one more price instrument - the bundled membership fee - which can capture a higher demand of seller-buyers without affecting the demand in other market segments.

5 Conclusion

In real life, many two-sided markets are mixed - sellers may also buy and buyers can also sell. To the best of our knowledge, this particular kind of two-sided markets have not been formally modeled in the theoretical literature on two-sided markets.

Because an agent in a mixed two-sided market may have positive valuation for the services that the platform provides to both sides, bundling (of different services intended for different sides) becomes a relevant strategy for mixed two-sided platforms. A hybrid kind of bundling strategies with two-part tariffs, i.e. pure bundling of memberships and unbundled transactions, have become prevalent in real-life mixed two-sided markets, such as telecommunications and financial intermediation, which have not been formally modeled in the bundling literature either.

In this paper we provide a model that captures all these dynamics.

We show that there exists a bundling effect when the mixed platform uses this hybrid bundling strategy, because it has one more price instrument - the bundled membership fee - than the separate-sales strategy. We also show that the bundling effect is closely related to the degree of mixedness of the market, measured by the proportion of the users who are both sellers and buyers amongst all users. Such users have high valuation for both kinds of services and are most active on the platform. Their existense and prevalence on the platform represents the very nature of mixed two-sided markets. We show that the higher the degree of mixedness, the more likely that bundling dominates unbundled sales.

These results remain robust independently of whether agents are able to foresee and take into account all potential surplus from trading on the platform or only consider the surplus from their "most urgently needed" services. This robustness check shows that our results come purely from bundling and the mixedness of the market.

We also discuss a situation where the platform introduces an activation procedure which allows it to separate user types and save costs. We show that such procedures will relax the condition of the degree of mixedness and further enhance the desirability of bundling vis-à-vis separate sales.

The model we have presented in this paper can be viewed from two different perspectives.

First, it extends the theoretical literature on two-sided markets to the mixed case, where sellers may also buy and buyers can also sell; and

Second, it extends the bundling literature to the case where there exist network effects between the demands of the two products being bundled, and the pricing strategies used are two-part tariffs.

There are far more work to be done in either of these two directions and this paper is simply the first step. In another paper, Gao (2009b) - *Bundling with*

Network Effects, we analyze in detail pure bundling, mixed bundling and some other variations of bundling strategies by mixed two-sided platforms, which provides some more general and complete results. Another very interesting extension would be to conduct some empirical studies of the relationship between particular pricing strategies (the level of membership fees, for example) and the degree of mixedness in particular markets to test the theory presented here.

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6 Appendix - Proofs

Lemma 2

Demand under separate-sales price $\mathbf{P}_s = (a^S, a^B, A^S, A^B)$:

$$\begin{aligned} N_s^S &= 1 - G^S(\tfrac{A^S}{N_s^B} + a^S) \\ N_s^B &= 1 - G^B(\tfrac{A^B}{N_s^S} + a^B) \\ N_s^X &= N_s^S \cdot N_s^B \\ N_s &= N_s^S + N_s^B - N_s^X \end{aligned}$$

Demand N_{m1}^S and N_{m1}^B under bundling price $\mathbf{P}_m = (a^S, a^B, A)$ (under M1) are determined by the following two simultaneous equations:

$$\begin{aligned} N_{m1}^S &= G^B(a^B)[1 - G^S(\tfrac{A}{N_{m1}^B} + a^S)] + N_{m1}^X \\ N_{m1}^B &= G^S(a^S)[1 - G^B(\tfrac{A}{N_{m1}^S} + a^B)] + N_{m1}^X \end{aligned} \tag{6}$$

where

$$N_{m1}^X = [1 - G^S(a^S)][1 - G^B(a^B)] - \int_0^A [G^B(\frac{A - z}{N_{m1}^S} + a^B) - G^B(a^B)] dG^S(\frac{z}{N_{m1}^B} + a^S) \tag{7}$$

And thus

$N_{m1} = N_{m1}^S + N_{m1}^B - N_{m1}^X = G^B(a^B)[1 - G^S(\frac{A}{N_{m1}^B} + a^S)] + G^S(a^S)[1 - G^B(\frac{A}{N_{m1}^S} + a^B)] + N_{m1}^X$

At $\mathbf{P}_{s0} = (a^S, a^B, 0, 0)$ and $\mathbf{P}_{m0} = (a^S, a^B, 0)$ we have:

$$\begin{aligned} N_s^X &= N_{m1}^X = [1 - G^S(a^S)][1 - G^B(a^B)] \\ N_s^S &= N_{m1}^S = 1 - G^S(a^S) \\ N_s^B &= N_{m1}^B = 1 - G^B(a^B) \\ N_s &= N_{m1} = 1 - G^S(a^S)G^B(a^B). \blacksquare \end{aligned}$$

Proposition 1

Step 1:

Under separate sales, we only need to consider degenerate price $\mathbf{P}_{s0} = (a_s^S, a_s^B, 0, 0) \in \mathbb{R}^{+4}$, where

$N_s^S = 1 - G^S(a_s^S)$

$N_s^B = 1 - G^B(a_s^B)$

$\Pi_s(\mathbf{P}_{s0}) = (a_s^S + a_s^B - c) N_s^S N_s^B - F^S N_s^S - F^B N_s^B$

Thus at $\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$, denote

$N_s^{S*} = 1 - G^S(a_s^{S*})$

$N_s^{B*} = 1 - G^B(a_s^{B*})$

$\frac{\partial N_s^S}{\partial a_s^S}(\mathbf{P}_{s0}^*) = -g^S(a_s^{S*})(< 0)$

$\frac{\partial N_s^B}{\partial a_s^B}(\mathbf{P}_{s0}^*) = -g^B(a_s^{B*})(< 0)$

F.O.C. for the optimality of a_s^{S*} and a_s^{B*} requires
$\frac{\partial \Pi_s}{\partial a_s^S}(\mathbf{P}_{s0}^*) = N_s^{S*}N_s^{B*} + [(a_s^{S*}+a_s^{B*}-c)N_s^{B*} - F^S]\cdot \frac{\partial N_s^S}{\partial a_s^S}(\mathbf{P}_{s0}^*) = 0$
$\frac{\partial \Pi_s}{\partial a_s^B}(\mathbf{P}_{s0}^*) = N_s^{S*}N_s^{B*} + [(a_s^{S*}+a_s^{B*}-c)N_s^{S*} - F^B]\cdot \frac{\partial N_s^B}{\partial a_s^B}(\mathbf{P}_{s0}^*) = 0 \Rightarrow$

$$(a_s^{S*}+a_s^{B*}-c)N_s^{B*} - F^S = \frac{N_s^{S*}N_s^{B*}}{g^S(a_s^{S*})}(>0) \tag{8}$$

$$(a_s^{S*}+a_s^{B*}-c)N_s^{S*} - F^B = \frac{N_s^{S*}N_s^{B*}}{g^B(a_s^{B*})}(>0) \tag{9}$$

Step 2:
Demand under bundling price $\mathbf{P}_m = (a^S, a^B, A)$:

$N_{m1}^X = [1-G^S(a^S)][1-G^B(a^B)] - \int_0^A [G^B(\frac{A-z}{N_{m1}^S}+a^B) - G^B(a^B)]dG^S(\frac{z}{N_{m1}^B}+a^S)$
$N_{m1}^{PS} = G^B(a^B)[1-G^S(\frac{A}{N_{m1}^B}+a^S)]$
$N_{m1}^{PB} = G^S(a^S)[1-G^B(\frac{A}{N_{m1}^S}+a^B)]$
$N_{m1}^S = N_{m1}^{PS}+N_{m1}^X$
$N_{m1}^B = N_{m1}^{PB}+N_{m1}^X$
$N_{m1} = N_{m1}^{PS}+N_{m1}^{PB}+N_{m1}^X$

Thus at $\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, 0)$ we have
$N_{m1}^S = N_s^{S*}$
$N_{m1}^B = N_s^{B*}$
$N_{m1}^X = N_s^{S*}\cdot N_s^{B*}$
$\frac{\partial N_{m1}^X}{\partial A}(\mathbf{P'}_{m0}) = 0$ (i.e. the effect of raising A on N_{m1}^X is second order)
$\frac{\partial N_{m1}^S}{\partial A}(\mathbf{P'}_{m0}) = \frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0}) = -\frac{G^B(a_s^{B*})g^S(a_s^{S*})}{N_s^{B*}}(<0)$
$\frac{\partial N_{m1}^B}{\partial A}(\mathbf{P'}_{m0}) = \frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0}) = -\frac{G^S(a_s^{S*})g^B(a_s^{B*})}{N_s^{S*}}(<0)$
$\Pi_{m1}(\mathbf{P}_m) = (A-F)N_{m1} + (a_m^S + a_m^B - c)N_{m1}^S N_{m1}^B$

$$\begin{aligned}\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) &= N_{m1} + \frac{\partial N_{m1}^B}{\partial A}(\mathbf{P'}_{m0})\cdot[(a_s^{S*}+a_s^{B*}-c)N_s^{S*} - F] \\ &\quad + \frac{\partial N_{m1}^S}{\partial A}(\mathbf{P'}_{m0})\cdot[(a_s^{S*}+a_s^{B*}-c)N_s^{B*} - F]\end{aligned} \tag{10}$$

Substituting (8) and (9) into (10), we have

$$\begin{aligned}\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) &= N_s^{S*}\cdot N_s^{B*} + \frac{G^S(a_s^{S*})g^B(a_s^{B*})}{N_s^{S*}}\cdot(F-F^B) + \frac{G^B(a_s^{B*})g^S(a_s^{S*})}{N_s^{B*}}\cdot(F-F^S) \\ &= N_s^{S*}\cdot N_s^{B*} + [-\frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0})\cdot(F-F^B) - \frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0})\cdot(F-F^S)]\end{aligned} \tag{11}$$

Since $\frac{\partial N_{m1}^{PS}}{\partial A}(\mathbf{P'}_{m0}) < 0$, $\frac{\partial N_{m1}^{PB}}{\partial A}(\mathbf{P'}_{m0}) < 0$, and $F \geq \max(F^S, F^B)$, we have $\frac{\partial \Pi_{m1}}{\partial A}(\mathbf{P'}_{m0}) > 0$. ■

Proposition 2

By definition, at $\mathbf{P}^*_{s0} = (a^{S*}_s, a^{B*}_s, 0, 0)$ the platform achieves the highest separate-sales profit, which is

$$\Pi_s(\mathbf{P}^*_{s0}) = (a^{S*}_s + a^{B*}_s - c)N^{S*}_s N^{B*}_s - F^S N^{S*}_s - F^B N^{B*}_s$$

where $\begin{array}{l} N^{S*}_s = 1 - G^S(a^{S*}_s) \\ N^{B*}_s = 1 - G^B(a^{B*}_s) \end{array}$

And the platform's profit at bundling price $\mathbf{P}'_{m0} = (a^{S*}_s, a^{B*}_s, 0)$ is

$$\Pi_{m1}(\mathbf{P}'_{m0}) = (a^{S*}_s + a^{B*}_s - c)N^{S*}_s N^{B*}_s - F(N^{S*}_s + N^{B*}_s - N^{X*}_s)$$

Thus

$$\Pi_{m1}(\mathbf{P}'_{m0}) - \Pi_s(\mathbf{P}^*_{s0}) = FN^{X*}_s - (F - F^S)N^{S*}_s - (F - F^B)N^{B*}_s$$

Since $\min(F^S, F^B) > 0$, without lost of generality, suppose $0 < F^S \leq F^B \leq F$.

Then $\frac{N^{X*}_s}{N^*_s} \geq \frac{F - \min(F^S,F^B)}{\min(F^S,F^B)} = \frac{F-F^S}{F^S} \Rightarrow F^S N^{X*}_s \geq (F - F^S)N^*_s \Rightarrow (F^S + F - F)N^*_s \geq (F - F^S)N^*_s$

$\Rightarrow FN^*_s \geq (F - F^S)(N^*_s + N^X_{s0}) = (F - F^S)(N^{S*}_s + N^{B*}_s) \geq (F - F^S)N^{S*}_s + (F - F^B)N^{B*}_s$

$\Rightarrow \Pi_{m1}(\mathbf{P}'_{m0}) - \Pi_s(\mathbf{P}^*_{s0}) = FN^{X*}_s - (F - F^S)N^{S*}_s - (F - F^B)N^{B*}_s \geq 0$.

If $\Pi_{m1}(\mathbf{P}'_{m0}) - \Pi_s(\mathbf{P}^*_{s0}) > 0$, we are done.

If $\Pi_{m1}(\mathbf{P}'_{m0}) - \Pi_s(\mathbf{P}^*_{s0}) = 0$, by Proposition 1 we know the platform can strictly increase profit by raising A, and we are done.■

Proposition 3

Under M2, demand N^S_{m2} and N^B_{m2} at bundling price $\mathbf{P}_m = (a^S, a^B, A)$ is determined by the following two simultaneous equations:

$$\begin{array}{l} N^S_{m2} = 1 - G^S(a^S) - G^B(\frac{A}{N^S_{m2}} + a^B)[G^S(\frac{A}{N^B_{m2}} + a^S) - G^S(a^S)] \\ N^B_{m2} = 1 - G^B(a^B) - G^S(\frac{A}{N^B_{m2}} + a^S)[G^B(\frac{A}{N^S_{m2}} + a^B) - G^B(a^B)] \end{array} \quad (12)$$

And we also have

$$\begin{array}{l} N^X_{m2} = [1 - G^S(\frac{A}{N^B_{m2}} + a^S)][1 - G^B(a^B)] + [1 - G^B(\frac{A}{N^S_{m2}} + a^B)][1 - G^S(a^S)] \\ \quad -[1 - G^S(\frac{A}{N^B_{m2}} + a^S)][1 - G^B(\frac{A}{N^S_{m2}} + a^B)] \\ N_{m2} = N^S_{m2} + N^B_{m2} - N^X_{m2} = 1 - G^S(\frac{A}{N^B_{m2}} + a^S)G^B(\frac{A}{N^S_{m2}} + a^B) \end{array}$$

Still consider $\mathbf{P}'_{m0} = (a^{S*}_s, a^{B*}_s, 0)$, i.e. a bundling price consisting of the optimal separate-sales marginal fees and a zero membership fee, at which the demand functions under M2 become:

$N_{m2}^S = 1 - G^S(a_s^{S*})$
$N_{m2}^B = 1 - G^B(a_s^{B*})$
$N_{m2}^X = [1 - G^S(a_s^{S*})][1 - G^B(a_s^{B*})]$
$N_{m2} = 1 - G^S(a_s^{S*})G^B(a_s^{B*})$

Again we find they are exactly the same as the demand at the optimal separate-sales price $\mathbf{P}_{s0}^*$ (and also the demand at $\mathbf{P'}_{m0}$ under M1). The rest of the proof follows exactly the same as in the proofs of propositions 1 and 2.■

Proposition 4

Assumption AC does not change the demand functions, but it does change the cost structure and hence the profit functions under both separate sales and bundling. The new profit functions are:

At at separate-sales price $\mathbf{P}_s = (a_s^S, a_s^B, A^S, A^B)$,
$\Pi_s^{AC}(\mathbf{P}_s) = (a_s^S + a_s^B - c)N_s^S N_s^B + (A^S - F^S)N_s^{PS} + (A^B - F^B)N_s^{PB} + (A^S + A^B - F)N_s^X$
And at bundling price $\mathbf{P}_m = (a^S, a^B, A)$,
$\Pi_m^{AC}(\mathbf{P}_m) = (a_m^S + a_m^B - c)N_m^S N_m^B + (A - F^S)N_m^{PS} + (A - F^B)N_m^{PB} + (A - F)N_m^X$
Thus at $\mathbf{P}_{s0}^* = (a_s^{S*}, a_s^{B*}, 0, 0)$ and at $\mathbf{P'}_{m0} = (a_s^{S*}, a_s^{B*}, 0)$, we have
$\Pi_s^{AC}(\mathbf{P}_{s0}^*) = \Pi_m^{AC}(\mathbf{P'}_{m0}) = (a_s^{S*} + a_s^{B*} - c)N_s^{S*}N_s^{B*} - F^S N_s^{PS*} - F^B N_s^{PB*} - F N_s^{X*}$
Thus profit remains unchanged after switching from $\mathbf{P}_{s0}^*$ to $\mathbf{P'}_{m0}$.
F.O.C. for the optimality of a_s^{S*} and a_s^{B*} requires
$\frac{\Pi_s^{AC}}{\partial a_s^S}(\mathbf{P}_{s0}^*) = N_s^{S*}N_s^{B*} + (a_s^{S*} + a_s^{B*} - c) \cdot N_s^{B*} \cdot \frac{\partial N_s^S}{\partial a_s^S}(\mathbf{P}_{s0}^*)$
$-F^S \cdot \frac{\partial N_s^{PS}}{\partial a_s^S}(\mathbf{P}_{s0}^*) - F^B \cdot \frac{\partial N_s^{PB}}{\partial a_s^B}(\mathbf{P}_{s0}^*) - F \cdot \frac{\partial N_s^X}{\partial a_s^B}(\mathbf{P}_{s0}^*) = 0 \Rightarrow$

$$(a_s^{S*} + a_s^{B*} - c) \cdot N_s^{B*} = \frac{N_s^{S*}N_s^{B*}}{g^S(a_s^{S*})} + F^S(1 - N_s^{B*}) + (F - F^B)N_s^{B*} \tag{13}$$

$\frac{\Pi_s^{AC}}{\partial a_s^B}(\mathbf{P}_{s0}^*) = 0 \Rightarrow$

$$(a_s^{S*} + a_s^{B*} - c) \cdot N_s^{S*} = \frac{N_s^{S*}N_s^{B*}}{g^B(a_s^{B*})} + F^B(1 - N_s^{S*}) + (F - F^S)N_s^{S*} \tag{14}$$

Now consider $\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0})$:

$$\begin{aligned}\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0}) &= N_m + \frac{\partial N_m^B}{\partial A}(\mathbf{P'}_{m0}) \cdot [(a_s^{S*} + a_s^{B*} - c)N_s^{S*} - F^B] \\ &\quad + \frac{\partial N_m^S}{\partial A}(\mathbf{P'}_{m0}) \cdot [(a_s^{S*} + a_s^{B*} - c)N_s^{B*} - F^S] \end{aligned} \tag{15}$$

Substituting (13) and (14) into (15), we have

$$\begin{aligned}\frac{\Pi_m^{AC}}{\partial A}(\mathbf{P'}_{m0}) &= N_s^{S*}N_s^{B*} + (F^S + F^B - F) \cdot [G^S(a_s^{S*})g^B(a_s^{B*}) + G^B(a_s^{B*})g^S(a_s^{S*})] \\ &= N_s^{S*}N_s^{B*} + (F^S + F^B - F)[-N_s^{S*}\frac{\partial N_m^{PB}}{\partial A}(\mathbf{P'}_{m0}) - N_s^{B*}\frac{\partial N_m^{PS}}{\partial A}(\mathbf{P'}_{m0})]\end{aligned}$$

Since $\frac{\partial N_m^{PS}}{\partial A}(\mathbf{P'}_{m0}) < 0$, $\frac{\partial N_m^{PB}}{\partial A}(\mathbf{P'}_{m0}) < 0$, and $F \leq F^S + F^B$, so whenever $N_s^{S*} N_s^{B*} \neq 0$, i.e. $X^* \neq \varnothing$, we have $\frac{\partial \Pi_m}{\partial A}(\mathbf{P'}_{m0}) > 0$.■