Patents in a Model of Growth with Persistent Leadership

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Abstract

This paper analyzes the effects of patent policies in a quality - ladder model of growth where incumbent firms preemptively innovate in order to keep their position of leadership. The amount of R&D undertaken by leaders increases if an innovation becomes more valuable to an entrant and policies that make it easier to replace incumbents and to obtain considerable market power right upon entry increase growth. I show that making patent policies conditional on whether an innovation is made by an entrant or an incumbent can increase growth and also analyze the effects of conditioning the strength of patent protecion on the size of the lead. In certain cases, an intermediate probabiltiy of patent enforcement leads to the highest average rate of growth.

1 Introduction

In many industries innovation continuously improves the quality or reduces the costs of existing goods, implying a process of creative destruction where old innovations are displaced by new ones. Given that patents are used to stimulate innovation, the specific design of the patent system can have important consequences for the incentives to innovate in such a context. O ´Donoghue and Zweimüller

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(2004) study the effects of patent policies in a quality - ladder model of growth, assuming that innovations are always carried out by entrants. In reality, innovations are however often carried out by incumbent firms implying that these incumbents stay the industry leaders for a sustained period of time[1]. In this paper I analyze the effects of patent policies in a quality - ladder model with persistent leadership where incumbents innovate preemptively. The basic model that I use is a simplified version of Denicolo (2001) who again builds on the seminal work of Gilbert and Newbery (1982) but doesn ´t analyze patent policy. I assume that a firm that has done two successive innovations (is two steps ahead) can earn larger profits than an entrant who has to compete with the previous incumbent in the product market. However, increasing the lead beyond two steps doesn ´t lead to larger profits so that a monopolistic R&D firm doesn´t have any incentives to continue innovating after it has reached a two - step lead. All firms have access to the same technology, which is characterized by decreasing returns at the industry level. There is free entry into the R&D sector and the condition that expected profits of entrants must be zero pins down the equilibrium rate of innovation. As entrants value entry (which brings a one - step lead) less than leaders value not being replaced and keeping their (two - step) lead, all R&D is carried out by leaders (who are assumed to move first). However, the amount of R&D that leaders undertake to prevent replacement depends on the value of an innovation for an entrant expecting to become the new leader upon entry. Because of that, the effects of patent policy on growth mainly stem from the effect they have on the value of an innovation for an entrant (which is hypothetical, as entry doesn ´t occur in equilibrium).

If entrants have to pay licensing fees in order to be allowed to access the knowledge stock of previous innovators, this reduces the value of an innovation for them and therefore entry pressure and growth as the R&D effort that the incumbent needs to exert in order to prevent entry is reduced. On the contrary, allowing entrants to collude with previous incumbents in order to keep prices high (or to sell their patents to these incumbents so that they can keep their 2 - step lead)

[1] Malerba and Orsenigo (1999) find that between 1978 and 1991, the percentage of patents granted to firms that had already innovated within their sector was 70% in Germany, 60% in France, 57% in the UK, 39% in Italy and 68% in the USA.

increases the value of an innovation for entrants and equilibrium growth. Equilibrium growth is maximized if a "patent transfer scheme" is implemented which forces incumbents to hand all their patents over to an entrant who has improved upon their technology. While all these results also hold in a context where innovations are carried out by entrants and where there is leapfrogging, there are also differences between the cases of persistent leadership and leapfrogging:

O ´Donoghue and Zweimüller (2004) show that making new innovations infringe on the patents of previous innovators ("leading breadth") can stimulate growth by allowing firms to consolidate market power (if that is not possible without leading breadth) in the case of leapfrogging. In the case of persistent leadership the incumbent however already enjoys the maximal possible market power so that leading breadth mainly acts as a barrier to entry unless it allows both firms to reduce total R&D costs. In the leapfrogging model, introducing a patentability requirement (a minimal inventive step) can increase growth and welfare by avoiding an excessive rate of turnover, low markups and inefficiently small innovative steps. In the case of persistent leadership a patentability requirement might still be useful if is only imposed on the incumbent, but if it is imposed on entrants it decreases entry pressure and the amount of R&D undertaken by incumbents.

In the case where in order to make an innovation two R&D stages have to be completed where the first consists in discovering an intermediate R&D input growth is maximal if entrants are allowed to patent the intermediate R&D input while incumbents are not. The reason for this is that such a policy prevents incumbents from blocking follow - on R&D by entrants but still makes them race to invent the R&D input in order to prevent that an entrant patents it.

If there are fixed costs of entering the R&D sector, no entrant ever enters as they expect that upon entry the incumbent will do so much R&D and therefore increase their R&D costs that they cannot earn any profits anymore by doing R&D themselves. In this case, equilibrium growth is zero unless patents expire with a positive probability so that incumbents are replaced from time to time and entrants have again incentives to innovate in order to obtain a lead over their rivals. In this case average growth is maximized for an intermediate probability of patent protection. This is also the case in other scenarios where incumbents can prevent entry without innovating themselves, like for example if they can make ex ante agreements with potential entrants in order to reduce R&D spending or if they can increase the R&D costs of entrants by hiring a certain number of researchers but make these researchers do other things than R&D instead.

While in the main case the level of preemptive R&D exerted by leaders doesn ´t depend on their lead, this might not be the case if the probability of patent protection is state dependent. If patents expire sufficiently more quickly in the case of a one - step than in the case of a two - step lead firms with a one - step lead do more R&D than firms with a two - step lead as being two steps ahead becomes relatively more attractive. While increasing the probability of patent expiration reduces growth in the main case, increasing the probability of expiration for firms with a one - step lead can increase average growth for a certain range of parameters. However, given firms with a one - step lead do more R&D than firms with a two - step lead, increasing the rate of patent expiration for firms with a two - step lead cannot increase average growth by making a one - step lead more likely. The reason for this is that increasing the probability of expiration decreases the value of an innovation for an entrant and therefore entry pressure and the incentives for firms with a two (or zero –) step lead to innovate and this negative effect on R&D overcompensates the positive effect stemming from a higher probability of being in the state with a one - step lead.

The analysis is similar in the case of a fixed entry fee: while increasing such a fee (that has to be paid by entrants once they enter the product market) decreases entry pressure and R&D effort of firms with a two - step lead, it can induce firms with a one - step lead to race faster as obtaining a two - step lead becomes relatively more attractive.

1.1 Related literature

In most quality - ladder growth models (like Aghion and Howitt (1992)) the case of leapfrogging is analyzed although in the case of competitive R&D with Walrasian markets incumbents are actually indifferent about their share in total R&D so that there might as well be (some) persistence in leadership (see Cozzi (2007)). In many continuous time patent race models (like Reinganum (1983 and 1985)) where innovation occurs with a Poisson arrival rate so that there is no duplication and where marginal R&D costs are increasing at the firm - but not at the industry level preemption is not possible and in the standard case with simultaneous moves and drastic innovations, incumbents invest less in R&D than challengers. In a similar setup with fixed costs of entering the R&D sector, Etro (2004) finds that in the case where there is free entry and industry leaders move first in the R&D game (are Stackelberg leaders) they do more R&D than entrants so that there is some persistence in leadership. Denicolo (2001) analyzes the case where preemption is possible due to decreasing R&D productivity at the industry level and finds that there is persistent leadership if innovations are nondrastic and incumbents move first. Fudenberg, Gilbert, Stiglitz and Tirole (1983) analyze under which conditions preemption is possible and when there can be competition in patent races.

The question of how antitrust policies should be designed in innovative industries where entrants expect to become the next incumbents is analyzed by Segal and Whinston (2007) who also find that entrants should in most cases be well protected against incumbents in order to guarantee that profit flows for successful innovators don ´t become too backloaded. They also study the case of innovation by leaders but don´t analyze patent policies in this context.

State - dependent intellectual property rights have first been introduced by Acemoglu and Akcigit (2008) who argue that patent protection should be stronger for firms that have a larger technological lead over their rivals. They analyze a model of step - by - step innovation in which there is a race between two firms in each sector and where the laggard first has to catch up through duplicative (but noninfringing) R&D before he can undertake frontier R&D (unless there is compulsory licensing of the leading edge technology). R&D productivity is assumed to be decreasing at the firm - but not at the industry level so that patent policy can also have an effect on innovation by affecting the division of total R&D between firms. An important difference in my model is therefore that there is free entry (without the need of catchup - R&D) and that there is the possibility to preempt entry. In Acemoglu and Akcigit (2008) innovation incentives are largest if firms are in the neck - and - neck state and decrease if the lead of one firm over the other becomes larger while in my model they are either the same in all states or larger in the case of a one - step lead than in the case of a two - step lead and the neck - and - neck case. In order to induce firms that are one step ahead to do more R&D than needed to prevent entry, expiration rates have to be sufficiently state dependent in my model, so that there are threshold effects that don ´t exist in Acemoglu and Akcigit (2008).

Bessen and Maskin (forthcoming) analyze the role of patents in a model of sequential innovation with imperfect (inefficient) licensing markets and find that innovation and welfare (even for innovators) might be larger in the case without than in the case with patent protection if firms can appropriate some surplus even without patents. They assume that the same two firms stay in the market forever and that there is no entry and that even imitating firms can make positive profits. A firm that suffers from not having patent protection today might therefore gain from it tomorrow. In my model innovators cannot make any profits without patent protection and due to the assumption of free entry, entrants and imitators earn zero profits so that an innovator who ´s patent expires cannot benefit from having free access to the innovations of others in the future. But even under these assumptions that might be expected to favour stronger patent protection and without assuming inefficient licensing I also find that innovation incentives and (average) growth can in certain cases decrease if patent protection is increased beyond a certain level. While I distinguish between forward protection and the protection against imitation Bessen and Maskin analyze a specific patent policy which protects a certain innovation against imitation and at the same time gives the inventor blocking power (forward protection) over follow - on innovations. And it is mainly this forward protection in combination with the malfunctioning licensing markets that makes it possible that patents inhibit innovation in their model.

Chu (2009) studies a generalized version of the model of O ´Donoghue and Zweimüller (2004) and quantitatively estimates the effect of blocking patents on R&D using US data. He looks at the case where profit flows are backloaded due to the granting of forward protection which allows previous innovators to block follow

- on innovations and compares it to the case where profit flows are completely frontloaded. He finds that eliminating blocking patents can increase R&D by about two to six times. While Chu (2009) doesn´t describe how exactly blocking patents can be eliminated, that means how in the case with forward protection the bargaining power of previous innovators can be reduced so that profit flows for new innovators become more frontloaded, I show that complete frontloading can be obtained through a patent transfer scheme which forces previous innovators to freely hand their patents over to entrants who improve upon their innovations. Such a patent transfer scheme therefore seems to be an effective way to eliminate blocking patents and might encourage R&D substantially.

Using a dataset and a survey from the German manufacturing sector Czarnitzki, Etro and Kraft (2008) find empirical support for the basic claim of this paper that incumbents invest more in R&D if entry pressure increases.

2 The model setup

2.1 Technology and preferences

There is a continuum of individuals of mass one that are all endowed with L units of labour and derive utility from the consumption of a perfectly divisible homogenous good x and of a quality good q of which either one or zero units can be consumed. Intertemporal utility of each agent is given by

$$U(\tau) = \int_{t=\tau}^{\infty} (x(t) + q(t))\, e^{-\rho(t-\tau)} dt.$$

Producing one unit of the homogenous good requires a labour input of one and for simplicity it is assumed that production costs of the quality good are zero. There exist generations $i \in \{1, ..., k\}$ of the quality good where $q_{i+1} = q_i + \mu$ so that quality increases by the same absolute amount μ from generation to generation. In a given period, the utility derived from the consumption of quality goods $q(t)$ is equal to the quality of the newest (highest i) generation that is consumed. Consuming an older generation in addition to a newer one therefore doesn ´t create any additional utility. The next generation $k+1$ of the quality good can be invented if R&D is undertaken. Using the amount $n(t)$ of labour generates the instantaneous Poisson arrival rate $\phi(t) = \min\left\{\left(\frac{n(t)}{c}\right)^{\frac{1}{1+\epsilon}}, \phi_m\right\}$ of an innovation. This technology is the same for all firms and if $\epsilon > 0$, marginal and average R&D productivity decrease in the total (industry wide) number of R&D workers that are hired[2]. The arrival rate cannot surpass the level ϕ_m due to technological reasons. As will become clear later on this assumption is made in order to keep the analysis (especially of "state dependent intellectual property rights") simple.

2.2 First best

In each period labour has to be allocated between the production of the homogenous good and R&D so that the resource constraint $L = x(t) + n(t)$ is satisfied. An innovation increases the utility derived from consuming the newest version of the quality good by μ and as this increase is permanent (as it is incorporated in the quality of future generations) the increase in intertemporal utility resulting from an innovation is given by $\frac{\mu}{\rho}$. Maximizing intertemporal utility subject to the resource constraints therefore amounts to maximizing the expected value of undertaking R&D $\phi(t)\frac{\mu}{\rho}$ minus its opportunity cost in terms of the homogenous good $n(t) = c\phi(t)^{1+\epsilon}$. This gives the optimal innovation arrival rate as $\phi^* = \min\left\{\left(\frac{\mu}{\rho c(1+\epsilon)}\right)^{\frac{1}{\epsilon}}; \phi_m; \left(\frac{L}{c}\right)^{\frac{1}{1+\epsilon}}\right\}$. In the case where $\epsilon \to 0$ and $\frac{\mu}{\rho c} > 1$ it is therefore optimal to do the maximal possible amount of R&D which either leads to the maximal arrival rate ϕ_m or the rate $\left(\frac{L}{c}\right)^{\frac{1}{1+\epsilon}}$ that results if the whole labour force does R&D. In the following it will be assumed that L is large relative to ϕ_m so that the maximal instantaneous arrival rate of an innovation is given by ϕ_m. Under these conditions, increasing the rate of innovation ϕ if it is below the maximal rate in a decentralized equilibrium is therefore welfare increasing. Because of that, I only analyze the effect of policies on the rate of innovation in the following sections (where $\epsilon \to 0$ will be assumed) knowing that a policy that increases innovation also increases welfare given that it doesn ´t imply any other costs.

[2] A reason for this might be that it is impossible to perfectly coordinate the R&D activities of all researchers so that the probability of duplicative research increases in the number of workers hired

2.3 Market allocation

The price of the homogenous good is normalized to one and both the sector producing the homogenous good and the R&D sector are assumed to be perfectly competitive so that the wage per unit of labour is equal to one. An innovator of a new generation of the quality good gets a patent which allows him to exclude others from producing his generation of the good. However, the patent expires once two more innovations have occurred so that goods that are two or more quality steps below the newest generation are in the public domain and are supplied at marginal cost of zero. There is Bertrand competition between firms producing quality goods so that the maximal price that can be charged by the leading firm is given by the difference between the quality of it ´s good compared to that of the closest competitor. A firm with a one - step lead can therefore charge a price equal to μ and firms with a lead of two or more steps a price equal to 2μ and earn per period profits equal to this price. In the following I denote flow profits[3] of a firm with a one - step lead by π_1 and those of a firm with a two - or more step lead by π_2. While the relation $\pi_2 > \pi_1 = \mu$ always holds, the analysis is not restricted to the special case where $\pi_2 = 2\pi_1$ and also the case where $\pi_2 < 2\pi_1$ - which could for example result from a price cap - is allowed for. Such a price cap might be imposed in order to reduce static inefficiencies (deadweight losses) arising from monopoly pricing (which are however not present in the simple model considered here). The somewhat artificial assumption that patents expire once two more innovations have occurred and that imposes an upper bound on prices and profits could also be replaced by assuming that pirating firms can produce the newest generation of the quality good at a marginal cost (which might include fines) equal to π_2. Another interpretation of the feature that profits cannot be increased if the lead of a firm exceeds two steps would be that firms with a two - step lead can charge the unconstrained monopoly price and make profits π_2 while profits of firms with a one - step lead (π_1) are lower because these firms have to engage in limit pricing to keep the closest competitor out of the market. However, such an analysis would require a richer model like the one studied by Denicolo (2001). In

[3] R&D costs are not taken into account here and would reduce net profits below the value labeled flow profits here.

the appendix (TO BE ADDED) I look at such a more realistic but also somewhat more complicated model and obtain similar results as in the simple model. The feature that an innovation increases profits for entrants more than for incumbents (that means that $\Delta\pi_1 = \pi_1 > \Delta\pi_2 = \pi_2 - \pi_2 = 0$ if the incumbent has a two - step lead and $\pi_1 > \pi_2 - \pi_1$ in the case of a one - step lead) is commonly called the "Arrow replacement effect" and can be obtained in many settings[4].

In order to guarantee that a positive amount of the homogenous good is consumed in equilibrium the relation $L > \pi_2$ must be satisfied. Due to the linearity of the intertemporal utility function the rate of interest must be equal to the rate of time preference in equilibrium: $r(t) = r = \rho$.

2.4 Equilibrium

Given an innovation has the value V_E for a newly entering firm, entry occurs until the average cost of innovating is equal to this value, that means until the zero profit condition $V_E \leq c\phi(t)^{\epsilon}$ is satisfied[5]. This condition pins down a lower bound for the equilibrium rate of innovation as an increasing function of V_E. But the fact that this condition depends on the value of an innovation for an entrant doesn´t imply that in equilibrium R&D is actually carried out by entrants. Take the case of an incumbent firm with a two - step lead. Without any threat of entry this firm does not do any R&D as it cannot increase profits above the current level π_2. However, if there is free entry the incumbent knows that if she doesn ´t do any R&D the instantaneous probability of replacement is given by the zero

[4] Another way to introduce this effect into this model is to assume that utility is given by: $U(\tau) = \int\limits_{t=\tau}^{\infty} \left(x(t)^{\nu} + q(t)\right) e^{-\rho(t-\tau)} dt$. For $\nu < 1$ incremental profits fall with the size of the lead so that entrants have larger stand alone innovation incentives than incumbents. If $\nu > 1$ the Arrow replacement effect is reversed and monopolists have larger (stand alone) incentives to innovate. The case where $\nu > 1$ is however less relevant from an economic point of view as it seems more reasonable to assume that marginal utility from consuming the homogenous good is decreasing.

[5] Take the case where R&D productivity is decreasing at the industry level due to duplication and where if two firms are successful at the same time each gets the patent with probability one half. Then, an entering firm that contributes to an overall increase in R&D effort doesn´t take into account that it increases the risk of duplication for all other firms as well and therefore still finds it profitable to enter if the average costs are lower than V_E even if the marginal costs are higher.

profit condition as $\phi(t) = \left(\frac{V_E}{c}\right)^{\frac{1}{\epsilon}}$. Moreover, given that entrants can adjust their amount of R&D after observing the level of R&D undertaken by the incumbent (who moves first) this probability is independent of whether the incumbent does part of the R&D herself as all what matters for entry are the average costs of R&D that are increasing in the overall level of R&D so that one unit of R&D undertaken by the incumbent crowds out one unit undertaken by entrants. As successfully innovating entrants have to compete with the previous incumbents and only get profits π_1 upon entry (and maybe later on π_2 if they do a follow - on innovation) the value of an innovation V_E for an entrant is lower than the willingness to pay of an incumbent firm for keeping its two - step lead. Therefore, an incumbent with a two - step lead preempts entry by doing exactly as much R&D as needed to push average R&D costs up to (or slightly above) the value of an innovation for an entrant (so that the zero profit condition is satisfied with equality)[6]. This is exactly the argument first brought forward by Gilbert and Newbery (1982). The same reasoning can be applied to an incumbent with a one - step lead who is willing to pay more for the next innovation (which guarantees a two - step lead) than an entering firm (which only gets a one - step lead). However - as it will become clear later on - an incumbent with a one - step lead might under certain circumstances even do more R&D than necessary to prevent entry.

We can therefore conclude that there is persistent leadership and that the rate of innovation depends (through the zero profit condition) positively on the value of an innovation V_E of an entrant. To solve for the equilibrium we therefore need to determine V_E given that an entering firm expects to expand its lead to two steps in the future and then to stay the leader and to do only as much R&D as

[6] The equilibrium analyzed here can either be seen as one where the incumbent is a Stackelberg leader in the R&D game or as a Walrasian equilibrium where the total demand for R&D labour (which is a decreasing function of the wage) is equal to the supply (which is perfectly elastic for the wage of one) and in which the auctioneer allocates all R&D labour to the incumbent who is willing to pay at least as much for it as the entrants. As the entrants get zero profits in equilibrium, they are indifferent about the quantity of R&D that they undertake and, in case an incumbent would do less R&D than the equilibrium level the Walrasian auctioneer would just assign a larger amount of R&D to entrants in order to obtain the equilibrium (see Cozzi (2008) for a more detailed discussion).

A similar equilibrium might also be obtained as a the outcome of a second price auction in which incumbents and entrants simultaneously bid for R&D labour.

needed to prevent entry. Denoting the value of being two (one) steps ahead by V_2 (V_1) and the R&D effort undertaken by a firm with a two - step (one - step) lead by ϕ_2 (ϕ_1) the following arbitrage conditions must be satisfied:

$$rV_2 = \pi_2 - c\phi_2^{1+\epsilon} \quad \text{and} \quad rV_1 = \pi_1 - c\phi_1^{1+\epsilon} - \phi_1 V_1 + \phi_1 V_2.$$

The right hand side in the first equation indicates the per period profits derived from selling the quality good minus the costs of conducting the level of R&D ϕ_2 that is needed to prevent entry. In the second equation there are two additional terms because in the case of an innovation (which occurs with arrival rate ϕ_1) the firm gains a two - step lead and looses its one - step lead. As the value of an innovation for an entrant is exactly the value of getting a one - step lead, we can solve the two equations for V_E:

$$V_E = V_1 = \frac{\pi_1 - c\phi_1^{1+\epsilon}}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2^{1+\epsilon}}{r}$$

The zero profit condition therefore becomes:

$$\frac{\pi_1 - c\phi_1^{1+\epsilon}}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2^{1+\epsilon}}{r} = c{\phi_2}^{\epsilon} \leq c{\phi_1}^{\epsilon}.$$

While the zero profit condition is for sure binding in the case where the incumbent firm has a two - step lead it might be the case that a firm with a one - step lead wants to get a two - step lead so quickly that it does more R&D than required to increase average R&D costs to a level that prevents entry.

In order to simplify the analysis the case where $\epsilon \to 0$ is considered in the following. As the value of an innovation for an entrant (V_E) depends on the amount of R&D ϕ_2 that entrants expect to do in the future to prevent entry (and not on current R&D), in the case where $\epsilon = 0$ and for V_E given, the equilibrium amount of R&D undertaken by the incumbent or by entrants is undetermined if the ZP condition is satisfied with equality ($V_E = c$). However, for ϵ slightly positive the preemption equilibrium results and the R&D rate is well specified in each period and equal to the one that entrants expect to choose themselves once they are leaders. In the following it is therefore assumed that in the case where $\epsilon = 0$ the equilibrium is selected that results as the limit if $\epsilon \to 0$. For $\epsilon = 0$ marginal and average R&D costs are equal to c and the zero profit condition is given by:

$$V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} = c$$

In order to determine the equilibrium value of ϕ_2 we need to know the level of

ϕ_1 chosen by a firm with a one - step lead. Such a firm takes V_2 and therefore ϕ_2 as given and maximizes V_1 $(= V_E)$ with respect to ϕ_1. We get: $\frac{\partial V_E}{\partial \phi_1} = \frac{\pi_2 - \pi_1 - cr - c\phi_2}{(r+\phi_1)^2}$ so that $\phi_1 = \phi_m$ if $\pi_2 - \pi_1 - cr - c\phi_2 > 0$ and $\phi_1 = \phi_2$ if $\pi_2 - \pi_1 - cr - c\phi_2 \leq 0$. If ϕ_2 is low enough, firms with a one - step lead chose the maximal possible R&D effort ϕ_m while, if ϕ_2 is large, firms with a one - step lead have no incentives to conduct more R&D than needed to preempt entry and they set $\phi_1 = \phi_2$[7]. As $V_2 = \frac{\pi_2 - c\phi_2}{r}$ decreases in ϕ_2, V_E also decreases in ϕ_2 if ϕ_1 is set to maximize V_E given ϕ_2[8]. By implicitly differentiating the zero profit condition, we can state the following:

Proposition 1 *The equilibrium growth rate ϕ_2 is increasing in π_1 and π_2 and decreasing in r $(= \rho)$ and c.*

The intuition for these results is straightforward: as all the R&D is carried out by the leading firm, there is persistent leadership and given the leading firm already has a lead of two steps, it does just as much R&D as needed to preempt entry. And this preemptive R&D level depends positively on the value of an innovation for an entrant who anticipates to become the next leader[9] which again increases in the profit flows π_1 and π_2 and decreases in the rate of interest r due to discounting. Entry pressure also increases if R&D costs are lower, that means if c decreases.

In the following sections, the effects of different patent policies are analyzed.

3 Compulsory licensing fees

As each innovation improves upon the last one, innovators use the knowledge accumulated by previous innovators as an R&D input. Let us now assume that

[7] Note that the amount of R&D needed to preempt entry is always given by ϕ_2 as the value of an innovation for an entrant doesn ´t depend on the size of the lead of the current leader.

The conditions under which $\phi_1 = \phi_m$ can actually result in equilibrium are analyzed in sections 13 and 14.

[8] If ϕ_2 is large, $\phi_1 = \phi_m$ which is independent of ϕ_2 so that V_E clearly decreases in ϕ_2. At the critical level $\phi_2 = \frac{\pi_2 - \pi_1}{c} - r$ firms with a one - step lead are indifferent between setting $\phi_1 = \phi_m$ or $\phi_1 = \phi_2$ and if ϕ_2 increases more, we have $\frac{\partial V_E}{\partial \phi_1} < 0$ so that $\frac{\partial V_E}{\partial \phi_2} < 0$ for sure if firms select the preemptive R&D level $\phi_1 = \phi_2$.

[9] Note however that this value is hypothetical as there is no entry in equilibrium

there is a policy requiring an innovator to pay a fixed fee F in order to compensate previous innovators. Even in the case where the entire fee has to be given to the previous incumbent, there is still persistent leadership and preemption. The reason for this is again that the incumbent firm values not being replaced more than the entrant values entering so that there is no value of F for which at the same time the entrant is willing to enter and the incumbent is willing to permit entry by not doing any preemptive R&D. If nevertheless entry occurs (out of equilibrium), entrants therefore expect to become the next leaders and to do all the follow - on R&D themselves and to never receive any licensing fees from others. The value of an innovation for an entrant is therefore given as:

$V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} - F$

The zero profit condition is now $\frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r} = c + F$ so that an increase in F unambiguously reduces equilibrium growth ϕ_2. The intuition for that is simply that any kind of licensing fee or other fixed fee that has to be paid upon entry decreases the value of an innovation for an entrant and therefore the entry pressure and the R&D effort of the incumbent that is required to deter entry.

While the effect of an increase in F on ϕ_2 is clearly negative, increasing F might however induce a higher R&D effort of firms with a one - step lead - as will be discussed later on in section 14.

If it is possible to subsidizing entry (impose a negative F) this increases the rate of R&D ϕ_2 although the subsidy never has to be paid in equilibrium. If the incumbent firm has to pay the entry subsidy, it is even more inclined to preemptively innovate in order to prevent entry.

4 Collusion/ trading patents

In order to reduce the value of an innovation for entrants and entry pressure, incumbents would clearly like to commit to compete with entrants in the product market once entry occurs. However, if they cannot commit to compete, what would be the effect of price collusion or of agreements where one of the firms sells its patent to the other in order to consolidate market power? Given an entrant has made an innovation he has incentives to collude with the incumbent in order to avoid the phase of competition and low profits lasting until he makes the next innovation and gets two steps ahead. Without collusion the value of the entrant firm is given by $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$ while the value of the former incumbent drops to zero. Collusion allows to increase joint profits by charging the higher price π_2 instead of π_1 and also makes it possible to reduce total R&D expenditures in the case where the entrant undertakes R&D effort $\phi_1 = \phi_m > \phi_2$ if there is no collusion. The reason for the latter is that in the case of collusion profit flows are already maximal so that there is no reason to do more R&D than the level ϕ_2 which is necessary to prevent entry. Therefore, the value $V_2 = \frac{\pi_2 - c\phi_2}{r}$ can be shared among the two parties if they collude and each has to get as least as much as the outside option without collusion. The greater the bargaining power of the entrant, the greater is therefore the value of an innovation to an entrant and the more R&D must be undertaken by the incumbent in order to discourage entry ex ante[10]. Allowing collusion therefore increases the equilibrium rate of growth ϕ_2 (unless entrants have no bargaining power at all). Only in the out of equilibrium case of entry where the entrant (who has a one - step lead) chooses $\phi_1 = \phi_m$ if there is no collusion, allowing to collude reduces R&D effort.

5 Patent transfer scheme

In the case of collusion equilibrium growth ϕ_2 is maximal if the entrant has all the bargaining power and gets V_2 right upon entry. If entrants tend to be weak bargainers, an interesting question is whether patent policy can be designed in a way to increase the value of an innovation for an entrant and to reduce the bargaining power of incumbents. Incumbents have bargaining power because they can reduce profits of entrants by competing with them in the product market and therefore, the simplest way to reduce their bargaining power is to prevent them from competing once entry has occurred. This can for example be achieved by forcing holders of patents of previous generations of the good to freely give their patents to an entrant firm if it has improved upon their innovations. Such

[10] If the entrant gets the fraction $\alpha\frac{\pi_2 - c\phi_2}{r} > \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$ of the "profit pie" that is shared with the incumbent (where α indicates the bargaining power of the entrant) the zero profit condition is given by $\alpha\frac{\pi_2 - c\phi_2}{r} = c$ so that ϕ_2 increases in α and is larger than in the case without collusion.

a "patent transfer scheme" therefore implies an expropriation of previous patent holders in order to give the maximal (or desired level of) market power to entrants. If entrants get profit streams π_2 right upon entry there can be either persistent leadership or leapfrogging (or a mixture of both) as incumbents and entrants have the same incentives to innovate (the incumbents value not being replaced the same as entrants value entering). The equilibrium rate of growth can now be obtained from the zero profit condition $V_2 = c$ implying $\phi_2 = \frac{\pi_2}{c} - r$.

6 Forward protection

Now, let us assume that the patent of an entrant infringes on the patent of the previous generation of the quality good, so that the newest generation can only be produced with the consent of the holder of the patent of the previous generation. O ´Donoghue and Zweimüller (2004) call such an arrangement "forward protection" and assume that this is the only possibility to consolidate market power between entrants and incumbents so that price collusion is only possible or allowed if the incumbent has blocking power over the innovation of an entrant.

Given there is an incumbent (she) with a two - step lead and an entrant (he) has made an invention and expects to become the new leader and to do all the follow - on R&D in order to preempt further entry. If he doesn ´t reach any agreement with the previous incumbent he won ´t be able to produce his newly invented good and will only start making profits once he has done a second innovation that doesn ´t infringe on the patent of the previous incumbent anymore. The value of the entrant ´s position is then given as $V_E = \frac{-c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r}$. Given entry has occurred, the previous incumbent knows that she will be replaced with probability ϕ_1 and loose all her profits to the entrant who will then be the next leader and get profit flows π_2. The value of being the previous incumbent after entry has occurred and without any collusive agreement is therefore given as $V_I = \frac{\pi_2}{r+\phi_1}$. If an incumbent can commit not to collude after entry occurs she will do so as collusion can only increase the value of an innovation for an entrant and therefore entry pressure and the R&D costs that have to be born by the incumbent to preempt entry ex ante. In the case without collusion forward protection therefore clearly decreases the value of an innovation for an entrant because if prevents him from producing his invention while in the case without forward protection (and without collusion) he can at least earn profits π_1 upon entry.

Given incumbents cannot commit not to collude, is there any gain to be made from colluding? As in the case without collusion the incumbent keeps her two - step lead until she is replaced and the entrant directly gets a two - step lead when replacement occurs, collusion cannot increase joint profits by increasing market power and prices (that are always kept at the maximal level). Therefore, the only case in which collusion can increase the joint surplus is the case where without collusion the entrant chooses the high R&D effort $\phi_1 = \phi_m$ which is larger than the effort $\phi_1 = \phi_2$ required to prevent entry and therefore leads to a lower aggregate payoff. But even in this case the value of an innovation for an entrant is lower than in the case without forward protection (and without collusion) if his bargaining power is low. But it might still be possible that the value of an innovation for an entrant is greater in the case with forward protection and collusion than in the case without forward protection and without collusion if the entrant has a lot of bargaining power and can credibly threaten to set $\phi_1 = \phi_m$ (and therefore to replace the incumbent quickly) if no collusive agreement is reached so that the incumbent is willing to pay a lot to the entrant if he chooses the low R&D effort $\phi_1 = \phi_2$ (TO BE FINISHED...).

The effects of forward protection analyzed here in a setup with persistent leadership differ from those in models with leapfrogging as analyzed by O ´Donoghue and Zweimüller (2004) and O ´Donoghue, Scotchmer and Thisse (1998). In these models firms only get profits π_1 without collusion and forward protection (which is assumed to be the only way to make collusion possible) allows to increase joint market power and profits to π_2. If the bargaining power of incumbents is not too large so that entrants get a large enough share of the increased profit pie right upon entry (that means if profit flows are not too backloaded), forward protection therefore increases the value of an innovation and equilibrium growth. The mechanism is therefore different than in the case of persistent leadership where forward protection cannot increase joint market power and mainly acts as a barrier to entry (unless maybe if it allows to reduce joint R&D costs...).

It should be noted that in both the case of leapfrogging and the case of persistent leadership simply permitting collusion without granting forward protection unambiguously increases the value of an innovation (for an entrant) and equilibrium growth. The reason for this is that forward protection greatly increases the bargaining power of the incumbent which has already been shown to decrease the value of an innovation (for an entrant) and equilibrium growth. And again, introducing a patent transfer scheme which makes private negotiations unnecessary and gives all the market power to entrants right upon entry leads to even bigger innovation incentives in both cases. (For a more detailed discussion of the case of leapfrogging see the *Appendix 17.1*).

6.1 Increasing profit flows

It would clearly be more realistic to assume that profits flows (π_1 and π_2) increase in the quality of the good (like it is the case in Denicolo (2001)[11]) so that even firms with a two -step lead have an incentive to cooperate with entrants who have invented an improved version of the good. In such a setup, there are incentives to innovate even without entry pressure. However, as long as a monopolistic firm cannot appropriate the entire increase in social welfare resulting from its innovation, it will conduct less than the socially desirable amount of R&D if there is no entry pressure. Contrary to incumbents, entrants don ´t take into account that their innovation makes the previous generation of the good obsolete[12] so that their R&D incentives are larger than those of a monopolist who doesn ´t face entry pressure and this is why even in such a more sophisticated model the equilibrium amount of R&D still depends on the R&D incentives of entrants and not on the stand - alone innovation incentives of leaders. Even in this setup, the results that a patent transfer scheme maximizes growth and that ex post collusion increases entry pressure and growth therefore hold due to the same reasons as in the simple model analyzed above. Also the analysis of forward protection in this setup is very

[11] In order to obtain a balanced growth path he assumes that R&D costs also rise at the same rate as profits over time.

[12] In the case where $\epsilon > 0$ entrants moreover do more R&D as entry occurs until the value of an innovation is equal to the average and not the marginal costs of innovating. This "tragedy of the commons" effect might lead to excessive R&D even if there are problems of appropriability. However, in the case considered here ($\epsilon \to 0$) equilibrium R&D is always insufficient given that $\frac{\mu}{\rho c} > 1$.

similar but now collusion allows to increase joint profits by increasing π_2 even in the case of persistent leadership. If collusion however doesn ´t allow to decrease joint R&D expenditures at the same time, forward protection still reduces growth compared to the case without forward protection and without collusion. The reason for this is that even if the entrant has all the bargaining power, he only gets the increase in π_2 resulting from his innovation until he advances to a 2 - step lead, and that this is - due to the Arrow replacement effect - still lower than π_1 which he would get without forward protection. Therefore, forward protection decreases the value of an innovation for an entrant and equilibrium growth (at least in the case where it doesn ´t allow for a considerable reduction in joint R&D costs).

7 Patentability requirement

So far it has been assumed that each innovation increases the quality of the good by the fixed amount μ. However, it might be possible for R&D firms to target different innovation sizes at different costs. In this case there is another instrument that patent policy can use: a patentability requirement that sets a lower bound on the innovative step below which an innovator cannot obtain a patent. Let us assume that the R&D production function is given by $\phi(t) = \left(\frac{\lambda(\mu)n(t)}{c}\right)^{\frac{1}{1+\epsilon}}$ with $\frac{\partial\lambda(\mu)}{\partial\mu} < 0$ so that the arrival rate ϕ decreases if larger innovation sizes μ are targeted and the same amount of R&D labour $n(t)$ is used. An incumbent who ´s lead is large enough to allow her to charge the maximal price π_2 does not care about the size of her innovations but only about the total number of R&D workers she needs to hire in order to make entry unprofitable. The incentives to innovate of an entrant however depend on the sequence of innovative steps chosen to reach a lead that is large enough to charge price π_2. Without any patentability requirement entrants chose the sequence and sizes of inventive steps that maximize the present discounted value of their R&D activity. Therefore, any restrictions imposed by patent policy on entrants R&D decisions necessarily decrease the value entrants derive from undertaking R&D and therefore reduce entry pressure. And if entry pressure is reduced, incumbents need to do less R&D in order to discourage entry and the number of researchers $n(t)$ that is hired in equilibrium decreases.

Taking the simple example where entrants find it optimal to do two steps in order to get the maximal profits, the expected profits of a potential entrant are given by $E\Pi_E = \phi_0 \left(\frac{\pi_1 - \frac{\phi_1}{\lambda(\mu_2)}}{r+\phi_1} + \frac{\phi_1}{r+\phi_1} \frac{\pi_2 - n}{r} \right) - \frac{\phi_0}{\lambda(\mu_1)}$ where ϕ_0 and ϕ_1 indicate the arrival rates chosen if the entrant has not made any innovation and if he is one step ahead and where again that assumption is made that $\epsilon \rightarrow 0$. Note that $\pi_1 = \mu_1$ and $\pi_2 = \mu_1 + \mu_2$ if the entrant can freely chose the inventive steps μ_i but that in the case of a patentability requirement that imposes $\mu_i > \overline{\mu}$ it might well be that $\pi_2 < 2\overline{\mu}$, especially if there is a price cap implying that $\pi_2 < 2\pi_1$. For a given value n of the R&D costs of a firm with a two - step lead, $E\Pi_E$ decreases if patent policy forces entrants to select different innovation sizes μ_i than their preferred ones. As in equilibrium $E\Pi_E = 0$ must hold n decreases if a binding patentability requirement for entrants is imposed. The same reasoning holds if entrants find it optimal to make more than two innovative steps in order to obtain the maximal lead.

To sum up, imposing a patentability requirement on entrants decreases entry pressure and the number of researchers hired by incumbents to prevent entry. However, it might be optimal to impose a patentability requirement on incumbents in order to induce them to pursue the socially optimal size of innovations[13].

This result differs from the ones of O ´Donoghue (1998), O ´Donoghue, Scotchmer and Thisse (1998) and O ´Donoghue and Zweimüller (2004) who show that in the case of leapfrogging a minimal patentability requirement is not only welfare -, but also growth - increasing. The reason for their finding is that in the case of leapfrogging turnover becomes higher and profit flows are reduced if firms choose smaller innovative steps which decreases the value of an innovation to an entrant. Contrary to that, not imposing a patentability requirement in my model of preemptive patenting only increases entry pressure while the small innovations are never realized in equilibrium and therefore don ´t lead to lower markups.

[13] In a model where incumbents can increase their profit flows through R&D Denicolo (2001) shows that incumbents tend to pursue too little innovative steps.

7.1 Collusion

If there is no patentability requirement for entrants but incumbents cannot commit to compete once entry occurs, is collusion still growth enhancing? The smaller the inventive step of the entrant the smaller is the profit he can earn in competition with the previous incumbent but the easier it is to replace the incumbent. Whatever the size of the innovation, the incumbent loses all profits upon entry if there is competition. In the case of collusion, entrant and incumbent again share the value V_I of incumbency (with maximal profit flows π_2) according to their bargaining power. As long as the incumbent has some bargaining power, an entrant cannot get the whole value of incumbency right upon entry and this makes his ex ante incentives to innovate and to replace the incumbent lower than the incumbent ´s incentives to prevent entry so that there is again the preemption equilibrium. Clearly, entry pressure increases if entrants are allowed to patent very small innovations and if their bargaining power is large. In this case the incumbent has to do a lot of R&D in order to prevent entry and this drives the equilibrium value of incumbency down. But again, inefficiently small innovations need not be realized in equilibrium if there is a patentability requirement for the incumbent.

It should also be noted that making a patentability requirement conditional on whether an innovation is realized by an entrant or an incumbent would be easy to implement as the patent office can observe whether the firm who is filing for the patent already holds a patent for the previous generation of the good.

7.2 Imperfect preemption

A strong assumption of the analysis so far has been that incumbents are perfectly informed about the R&D costs of entrants and can therefore exactly compute the level of R&D they need to undertake to make entry unattractive. If incumbents however only know the distribution of possible R&D costs c of entrants they might find it profitable to allow entry with a positive probability if this saves a lot of R&D costs. In such a case there would therefore not be perfect preemption and from time to time entrants would do R&D and replace leaders[14]. In order to avoid that very small innovations are actually patented by entrants, one might consider to also introduce a patentability requirement for entrants in this case. However, as long as entrants don ´t do much R&D in equilibrium, the patentability requirement for them should be weaker than that imposed on incumbents in order to keep entry pressure high.

8 Patent buyout schemes (preliminary)

Hopenhayn, Llobet and Mitchell (2006) study the optimal patent system in a quality ladder model where the patent office cannot observe the size of the inventive step and therefore cannot impose a patentability requirement[15]. Under the assumption that innovations are always realized by entrants they find that the optimal R&D incentives can be obtained through a patent buyout scheme which requires that an innovator pays a certain fee to the previous patent holder in order to be allowed to replace her, and that in addition gives a menu of contracts indicating the level of the fee an innovator has to pay to the patent office now in order to fix the level of the fee that future innovators are required to pay to him in order to replace him in the future. As buying stronger protection against future innovations is more costly, only firms with a large innovative step that allows to charge a larger price and to earn larger profits find it profitable to buy strong protection and this encourages large innovative steps. Now the question is whether such a buyout scheme would work in the setup of preemptive patenting by leading firms. Given an incumbent can buy protection against future entry by paying a fee today, this just gives an additional instrument to prevent entry which is likely to reduce innovation incentives as it can be used as a substitute for incumbent R&D.

If the incumbent does all the R&D and charges the unconstrained monopoly

[14] Incumbents face the trade - off that if they want to reduce the probability of being replaced they have to do more R&D which is costly and doesn ´t increase the own profits. If the distributio of c is such that low values of c are realized with a low probability, incumbents therefore select a threshold $\widetilde{c}$ and only undertake as much R&D as needed to deter entry if $c > \widetilde{c}$ so that there is entry in the case where $c < \widetilde{c}$.

[15] They use a specification where R&D productivity $\lambda(\mu)$ is random so that large innovations should only be targeted if this productivity is large.

price π_2 and if the patent office cannot observe the size of the inventive steps undertaken by the incumbent it is in fact not possible to give incentives to pursue larger inventive steps with such a buyout scheme (where the incumbent would pay a fee to herself if she innovates).

9 Fixed costs of entering the R&D sector

So far it has been assumed that there is free entry into the R&D sector and that there are only variable costs of undertaking R&D. Let us now look at the case where a firm which has not itself invented the currently newest quality good has to pay a fixed cost before being able to do R&D targeted at improving the quality of the good by one more step. Given entry into the R&D sector has occurred, the analysis is similar as above and an incumbent firm with a lead of two steps does exactly as much R&D as needed in order to make entrants get (slightly less than) zero expected profits if they do R&D. Entrants therefore do not do any R&D in equilibrium even if they paid the fixed costs of entering the R&D sector. Expecting this outcome, there are therefore no incentives for entrants to pay the fixed costs of entering as they know that given they enter, the incumbent will undertake so much R&D that they cannot make any profits. Therefore, there is no entry in equilibrium and an incumbent with a two - step lead does no R&D at all (as this would only increase costs without rising profits above the current level π_2). Equilibrium growth ϕ_2 is therefore zero[16] if there are (arbitrarily small) positive fixed costs of entering the R&D sector.

9.1 Expiring patents

The result that equilibrium growth is zero if there are fixed costs of entering the R&D sector was derived under the assumption that patents never expire. However, if patents expire and if in the case of expiration the newest available quality of the good falls in the public domain, firms might again have incentives to undertake R&D in order to become the next leader by inventing a new patentable version of the quality good. Once a firm has obtained a sufficient lead there is again no

[16] In a model where R&D increases profit flows equilibrium growth would depend on the incentives of a monopolist to undertake R&D, and these are generally insufficiently small compared to the social optimum if there are problems of appropriability.

entry pressure and no R&D is undertaken until the patent expires. If patents don ´t expire too quickly and if the fixed costs of entering the R&D sector are not too high, average growth can therefore be positive as incumbents regularly lose their lead which gives incentives for new entrants to innovate in order to become the next incumbents. If patents however expire too quickly no firm finds it profitable to do R&D so that equilibrium (average) growth is again zero. Average growth is therefore maximal for an intermediate strength of patent protection.

For a more detailed analysis see *Appendix A*

10 Ex ante agreements

Assume now that there is just a limited number of firms which are able to do R&D and that they can make ex ante agreements about their joint R&D effort and about how to split profits. Given these firms already have a lead of two steps and can charge price π_2 they clearly don ´t have any incentives to do further R&D (as they cannot get larger profits than π_2) and growth is zero. If it is difficult to prevent such ex ante agreements and to make firms compete, another possibility to increase growth above zero is to make patents expire with a positive probability (like in the case of fixed costs of entering the R&D sector): If no firm owns a patent and if the probability of patent expiration is not too large, firms capable of doing R&D find it profitable to innovate (even if there are ex ante agreements) as they can only make profits if they possess at least one patent on a version of the quality good that is newer than these that are in the public domain.

Here again the main assumption of the model that - once a two - step lead is obtained - nothing of the increase in consumer welfare arising from an innovation can be appropriated by innovators is clearly unrealistic. Moreover, one can argue that ex ante agreements might improve R&D efficiency by exploiting synergies and avoiding costly duplication. But even in a setup where monopolists do some R&D in order to increase their profit flows these R&D incentives tend to be low as a monopolist takes into account that she replaces her old innovation by introducing a new one. If the monopolist doesn ´t hold a patent on the old innovation anymore, this Arrow replacement effect disappears so that average growth might be increased if patents expire with a positive probability. And given that a monopolistic firm can only appropriate part of the social surplus created by an innovation it does less R&D than socially optimal (if patents never expire), so that in the case where R&D and growth can be increased by making patents expire more quickly this also tends to increase social welfare.

11 Heterogenous R&D labour

If labour is heterogenous in the sense that some workers are better researchers than others but all are equally good in producing the homogenous good the costs of hiring a researcher increase in the total number of researchers hired as less and less able people have to be recruited. In this case preemption is not only possible if a certain amount of R&D is actually undertaken by the incumbent (like in the case with a perfectly elastic (R&D -)labour supply but decreasing R&D productivity) but also if the incumbent hires the most able researchers and makes them produce the homogenous good instead. Given that the pool of workers from which entrants can hire only contains sufficiently untalented researchers they don ´t find it profitable to engage in R&D and there is again no growth in equilibrium. Again, growth will be maximized for an intermediate strength of patent enforcement like in the previous two sections. An interesting point here is that although there are no deadweight losses due to intellectual property rights the government still has incentives to make patents expire more quickly ex post then promised ex ante as this encourages follow - on R&D and growth (as long as future innovators don ´t fear that the government won ´t enforce their patents either).

12 Intermediate R&D inputs

Let us now look at the case where in order to improve the quality of the good by one step two R&D stages have to be completed. In the first stage an intermediate R&D input (which might be thought of as an idea) has to be invented and this input is used in the second stage to invent an improved version of the quality good. The R&D technology at each stage is again stochastic and assumed to be of the same form as in the main model, allowing for the possibility of preemption due to decreasing R&D productivity at the industry level. In the following I analyze the implications of granting patents on intermediate R&D inputs on the incentives to innovate.

If an incumbent with a two - step lead has patented the intermediate good required to invent the next generation of the quality good she can block further entry as in order to replace her as the leader entrants need to have access to (a license for) the intermediate R&D input which can be denied if is protected by a patent. As the incumbent cannot increase her profits through further innovation and as the value of an innovation for an entrant is lower than the value the incumbent attaches to keeping her leadership position, she never agrees to license the intermediate R&D input so that there is no entry. If an entrant has a patent on the intermediate good he can either license it to the incumbent or use if to proceed with the second R&D stage. Both firms clearly have incentives to license the input as the incumbent values the possibility to block entry more than the entrant values the possibility to undertake follow - on R&D. As a result, there is again no follow - on research in equilibrium. If it is however not possible or not allowed to license to the incumbent, the entrant uses the input in order to start the second stage of R&D aimed at inventing a new generation of the quality good which allows him to earn profit flows π_1 (until he advances to a two - step lead). Again, the incumbent values a patent on the intermediate good more than an entrant and if no firm has yet invented the intermediate good she will preempt entry (into the intermediate R&D goods sector) by undertaking the level of R&D needed to discourage R&D of entrants. Once the incumbent has invented the intermediate good, she can block further entry and equilibrium growth is zero.

In the case where no patents are granted to intermediate R&D inputs no firm has an incentive to invent such a good: an entrant who invents it cannot make any profits out of it as it would fall in the public domain and would be supplied at marginal cost which also implies that there is free entry into the second stage R&D race which pushes expected profits in this stage down to zero. Also an incumbent with a two - step lead has no incentive to invent the intermediate good if no entrant does as this doesn ´t allow to increase her profits. Therefore equilibrium growth in this case without patents is zero as well.

However, it is possible to have sustained growth if the leader is not allowed to patent intermediate R&D inputs but if the entrants are with the restriction that they are not allowed to license to the leader. In this case, the incumbent has incentives to preempt entrant R&D at each stage without ever being able to block future entry completely: the main point is that while the invention of the intermediate good by the incumbent doesn ´t prevent entrants from participating in the race for the second R&D stage, it is still worthwhile as it prevents that entrants invent the intermediate good which would allow them to replace the incumbent in the future. And once the intermediate good is invented by the incumbent, the incumbent has again incentives to preempt entry in the race for the second R&D step (which leads to an improvement in the quality of the good) due to the standard reasoning in the main model. Once the next version of the quality good is invented the whole process starts again. To sum up, granting patents on intermediate goods only to entrants doesn ´t reduce the incentives for leaders to invent these goods and furthermore guarantees entry pressure and sustained R&D by leaders also in the second R&D stage. While collusion in the intermediate good stage is bad for R&D incentives as it allows incumbents to block entry, allowing collusion (or licensing) if entrants have invented a new version of the quality good again increases entry pressure and innovation incentives due to the same mechanism as discussed above.

13 State dependent intellectual property protection

In the following I analyze the effects of patent expiration in the basic model. Like Acemoglu and Akcigit (2008) I allow for "state dependent intellectual property protection" meaning that the probability of patent expiration can depend on whether the firm has a one - step or a two - (or more -) step lead over its rivals. The patents of an incumbent with a one - step lead expire with the instantaneous Poisson arrival rate γ_1 and and those of an incumbent with a two - step lead with the instantaneous arrival rate γ_2. In the case of patent expiration the newest innovation of the incumbent falls in the public domain allowing entrants to fully catch up. This specification implicitly assumes that patents on second newest goods never expire and the case in which firms with a two - step lead can lose this lead for a one - step lead (because their patent on the second newest good expires) is not considered.

The value of an innovation for an entrant V_E (that means the value of being one step ahead) and the value V_2 of being two (or more) steps ahead and can now be derived from the following arbitrage conditions:

$rV_2 = \pi_2 - c\phi_2 - \gamma_2 V_2$

$rV_E = \pi_1 - c\phi_1 - \gamma_1 V_E - \phi_1 V_E + \phi_1 V_2$

from which we get:

$V_E = \frac{\pi_1 - c\phi_1}{r+\gamma_1+\phi_1} + \frac{\phi_1}{r+\gamma_1+\phi_1}\frac{\pi_2 - c\phi_2}{r+\gamma_2}$

This value decreases in both expiration rates, also if ϕ_1 is chosen as an optimal response to γ_1 and γ_2. The equilibrium arrival rate of an innovation might now depend on the state in which the economy is. If there is an incumbent with a two - step lead the arrival rate is ϕ_2 and it is determined by the zero profit condition $V_E = c$. Again looking at the limit case where $\epsilon \rightarrow 0$ in the more general model with decreasing R&D productivity, the arrival rate in the case where the newest generation of the good is in the public domain is also given by $\phi_0 = \phi_2$. While R&D is now undertaken by entrants and not by the incumbent (as there is none) the arrival rate is the same because the value of an innovation for an entrant is independent of whether he replaces an incumbent or not (if there is no collusion) and because the same zero profit condition holds in both cases. In the case where there is an incumbent with a one - step lead, the arrival rate is either given by the minimal amount necessary to preempt entry ($\phi_1 = \phi_2$) or by the maximally feasible rate $\phi_1 = \phi_m$. In the first case the arrival rate is therefore the same in all possible states. As V_E decreases in the arrival rate if $\phi_1 = \phi_2$, the zero profit condition indicates that in this case an increase in any of the probabilities of patent expiration reduces the equilibrium arrival rate as long as the change in patent policy doesn ´t induce firms to chose $\phi_1 = \phi_m$ instead of $\phi_1 = \phi_2$.

In the case where $\phi_1 = \phi_m$ the arrival rate is dependent on the state of the economy and is higher in the case where the leader has a one - step lead than in the cases where no firm has a lead or where the leader has a two - step lead (in which it is given by $\phi_0 = \phi_2$). Now the question is whether patent policy can affect the choice of ϕ_1 and whether, given $\phi_1 = \phi_m$, increasing any of the rates of patent expiration can increase the probability of being in the state of high growth and even increase average growth through such a composition effect. In order to analyze this question I first derive conditions under which firms with a one - step lead actually choose the high R&D effort $\phi_1 = \phi_m$ in equilibrium:

Lemma 2 *A firm with a one - step lead selects R&D effort* $\phi_1 = \phi_m$ *if* ϕ_m *is large enough and if* $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$ *(Condition 1)*

Lemma 3 Proof. *see Appendix B* ■

As *Condition 1* implies that $\gamma_1 > \gamma_2$ must hold firms that are one step ahead therefore only have incentives to do more R&D than necessary to prevent entry if patents expire sufficiently more quickly in the case of a one - step lead then in the case of a two - step lead (and if collusion is not possible). Like in Acemoglu and Akcigit (2008) protecting patents of firms with a bigger lead more induces firms with a smaller lead to race faster (increase ϕ_1) in order to obtain a bigger lead and stronger protection more quickly.

13.1 Average arrival rate

In the case where $\phi_1 = \phi_m > \phi_2 = \phi_1$ the arrival rate depends on the size of the lead which itself changes stochastically over time. In order to calculate the average rate of growth (arrival rate) we need to compute for which fraction of the time the economy is in which state (on average). To simplify the intuition one can also think about a slightly modified model in which there is a continuum of symmetric quality good sectors of mass one[17] and compute in which fraction of the sectors the lead is equal to 0, 1 or 2 steps. Denoting the proportion of time or the fraction of sectors in which the lead is equal to k steps by σ_k the following conditions need to be satisfied in order to guarantee that the average entry into the state k equals the average exit to other states:

$$\sigma_o\phi_2 = \gamma_1\sigma_1 + \gamma_2\sigma_2 \quad (k = 0)$$
$$\sigma_1(\phi_m + \gamma_1) = \sigma_0\phi_2 \quad (k = 1)$$
$$\sigma_2\gamma_2 = \sigma_1\phi_m \quad (k = 2)$$

---

[17] Utility in this case is given by: $U(\tau) = \int\limits_{t=\tau}^{\infty} \left(x(t) + \int\limits_{j=0}^{1} q_j(t)dj \right) e^{-\rho(t-\tau)}dt$ and the production technologies are the same for each sector.

The left hand sides stand for the exit from the corresponding states k and the right hand sides for the entry into these states. Taking as an example the case where $k = 0$ the average number of sectors leaving this state is given by the probability of an innovation in this state $\phi_2 = \phi_0$ times the fraction of sectors where the state is given by a lead of zero (σ_0). Entry into this state occurs due to the expiration of patents in sectors with a lead of one or two steps and is given by the arrival rate of patent expiration in the case of a one - step lead times the fraction of sectors with a one - step lead ($\gamma_1\sigma_1$) plus the corresponding expression for $k = 2$ ($\gamma_2\sigma_2$). Using the condition $\sigma_0 + \sigma_1 + \sigma_2 = 1$ and these three equations we can compute:

$$\sigma_0 = \frac{(\phi_m+\gamma_1)\gamma_2}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

$$\sigma_1 = \frac{\phi_2\gamma_2}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

$$\sigma_2 = \frac{\phi_2\phi_m(\phi_m+\gamma_1)\gamma_2}{(\phi_m+\gamma_1)\gamma_2((\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m))}$$

The average arrival rate $\widehat{\phi}$ is now given by the weighted sum of arrival rates in the different stated with the weights given by σ_k:

$$\widehat{\phi} = \phi_m\sigma_1 + \phi_2(\sigma_0 + \sigma_2) = \frac{\phi_2(\gamma_2(\phi_m+\gamma_1)+\phi_m(\phi_2+\gamma_2))}{(\phi_m+\gamma_1)\gamma_2+\phi_2(\gamma_2+\phi_m)}$$

13.2 The effect of expiring patents on average growth

Proposition 4 *Given the conditions from Lemma 1 hold so that $\phi_1 = \phi_m$, increasing the rate of patent expiration γ_1 or γ_2 decreases the average arrival rate $\widehat{\phi}$ (that means $\frac{\partial\widehat{\phi}}{\partial\gamma_1} < 0$ and $\frac{\partial\widehat{\phi}}{\partial\gamma_2} < 0$).*

Proof. *See Appendix C* ■

Increasing any of the instantaneous expiration rates decreases the value of an innovation for an entrant and therefore the arrival rate $\phi_2 = \phi_0$ in the states where no firm has a lead and where the leading firm has a two - step lead and preemptively innovates. Increasing γ_1 decreases the probability σ_1 that the economy is in the state with a one - step lead where the arrival rate is maximal $\phi_1 = \phi_m$ (see *Appendix C*). Therefore, increasing γ_1 unambiguously reduces the average arrival rate. Increasing γ_2 can however increase σ_1 so that there is a composition effect: reducing patent protection for firms with a two - step lead can increase the probability that the economy is in the state where there is a one - step lead and where the arrival rate is maximal. However, this composition effect is not strong enough to compensate the negative effect that an increase in γ_2 has on the arrival rate ϕ_2 in the other states so that an increase in γ_2 still leads to a reduction in the average arrival rate $\widehat{\phi}$.

For γ_1 given and large enough so that $\phi_1 = \phi_m$ initially holds, increasing γ_2 first continuously decreases the average arrival rate and at a certain threshold induces firm to choose $\phi_1 = \phi_2$ so that $\widehat{\phi}$ falls discontinuously[18]. After this threshold is passed, increasing γ_2 again continuously decreases $\widehat{\phi}$ and if γ_2 is sufficiently larger than γ_1 there is a switch to a leapfrogging equilibrium as incumbents with a one - step lead don ´t find it profitable anymore to obtain a two - step lead and prefer not to do any R&D and to be replaced by entrants.

For γ_2 given and starting from γ_1 low enough so that $\phi_1 = \phi_2$, increasing γ_1 first reduces the average arrival rate but when a certain threshold of γ_1 is reached an upward jump of $\widehat{\phi}$ occurs while $\widehat{\phi}$ again decreases if γ_1 is increased beyond the threshold. There is therefore a non-monotonic relation between the average arrival rate and the probability of patent expiration in the state of a one - step lead.

14 Entry costs and expiring patents

In the previous section it was shown that state - dependent intellectual property protection can induce firms with a one - step lead to exert the maximal R&D effort. However, this is not the only possibility to get such a situation. In fact, we can also have $\phi_1 = \phi_m$ in the case where there are sufficiently high fixed costs of entering the production sector or compulsory licensing fees. In the following I analyze the effect of such entry costs on the average arrival rate if patents of firms with a two - step lead (but not those of firms with a one - step lead) regularly expire:

The value of an innovation for an entrant is given by: $V_E = \frac{\pi_1 - c\phi_1}{r+\phi_1} + \frac{\phi_1}{r+\phi_1}\frac{\pi_2 - c\phi_2}{r+\gamma} - F$ where γ stands for the instantaneous expiration rate of patents of firms with

[18] While the arrival rate ϕ_1 jumps discontinuously at this threshold, this is not the case for the value of an innovation and ϕ_2 as firms are indifferent between choosing $\phi_1 = \phi_m$ and $\phi_1 = \phi_2$ at the threshold level of γ_2.

a two (or more) - step lead[19] and F for the fixed entry cost or licensing fee that even has to be paid if no firm has a lead. The zero profit condition is given by: $V_E = F + c$ and for simplicity it is assumed that $\pi_2 = 2\pi_1$.

Proposition 5 *$\phi_1 = \phi_m$ holds in equilibrium if $F > \frac{\pi_1}{r} - c$ and $\phi_m > \frac{F}{c}(r - \gamma) - 2\gamma > 0$.*
Proof. *See Appendix D* ■

The fixed fee F therefore needs to exceed a certain threshold in order to induce firms with a one - step lead to exert the maximal R&D effort. The intuition for this is that increasing F reduces the value of an innovation for an entrant and therefore the amount of R&D that an incumbent with a two - step lead needs to undertake in order to preempt entry. This again increases the value of being two steps ahead and induces firms with a one - step lead to try harder to reach a two - step lead.

As derived in the section on state dependent intellectual property rights (but taking the case where $\gamma_1 = 0$ and $\gamma_2 = \gamma$) the average arrival rate is given by $\widehat{\phi} = \frac{\phi_2(\gamma\phi_m + \phi_m(\phi_2+\gamma))}{\phi_m\gamma + \phi_2(\gamma+\phi_m)}$. Given that $\phi_1 = \phi_m$ this rate decreases in F as ϕ_2 decreases in F (and $\widehat{\phi}$ increases in ϕ_2). Therefore, starting from $F = 0$ so that initially $\phi_1 = \phi_2$, increasing F first reduces the average arrival rate, but at a critical level, induces firms with a one - step lead to choose $\phi_1 = \phi_m$ which makes $\widehat{\phi}$ jump upward. Once this threshold level of F is surpassed, $\widehat{\phi}$ again decreases in F while firms keep their choice $\phi_1 = \phi_m$. In the case where $\phi_1 = \phi_m$ increasing the rate of patent expiration γ unambiguously reduces $\widehat{\phi}$ due to the same reasons due to which in the case of state dependent intellectual property protection an increase in γ_2 led to a decrease in $\widehat{\phi}$ (Proof: see *Appendix E*).

15 Concluding remarks

While most of the literature analyzing the role of patents in models of cumulative innovation focuses on the case of leapfrogging, this paper looks at the other

[19] This rate cannot be too large, as there would then be a leapfrogging equilibrium where no firm has incentives to ever reach a two- step lead where patents expire very quickly.

extreme of persistent leadership and preemptive patenting and comes to different conclusions in some points. While there are certainly cases where the assumption of leapfrogging makes sense and where preemption is not possible due to technological or informational reasons (or where full preemption is not desirable, like in section 8.2), it still seems to be a relevant case and is furthermore consistent with the evidence that many incumbent firms do innovate in reality. And in the case where patents expire or innovations are drastic the model can also explain some turnover.

It is also important to note that the result that innovation incentives are increased if a larger share of total profits is allocated to entrants at the expense of previous innovators depends on the assumption made in quality ladder models that each innovation builds on a previous innovation. If there is however an initial product innovation on which all following innovations build the incentives to make this initial innovation clearly decrease if follow - on innovators can easily replace the initial innovator without any compensation. In such a case there is therefore a trade - off for patent policy and encouraging initial R&D (through compulsory licensing fees or forward protection for example) comes at the cost of reducing follow - on R&D. Also in the case without an initial innovation but where innovating requires access to the knowledge base of previous innovators and in which this knowledge depreciates without costly maintenance effort, some compensation for earlier innovators implying a more backloaded profit stream for an entrant might be required in order to give incentives to invest in the conservation of previous knowledge on which future innovations build.

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17 Appendix

17.1 The case of Leapfrogging

If R&D can only be undertaken by firms that have ideas and if incumbents happen to never have ideas for follow - on R&D but there are enough entrants who do, there is leapfrogging and R&D is only carried out by entrants. Therefore, no firm ever gets a lead of more than one step and profit flows are given by π_1. If the (correctly anticipated) rate of arrival of an innovation is given by ϕ the following arbitrage condition determines the value of an innovation in this setup: $rV = \pi_1 - \phi V$ so that $V = \frac{\pi_1}{r+\phi}$. If the costs of R&D are again given by $n(t) = c\phi(t)^{1+\epsilon}$ entry occurs until average costs are equal to V (which only depends on the future realization of ϕ) so that the zero profit condition is given by: $V = c\phi(t)^{\epsilon}$. Taking again the limit where $\epsilon \to 0$ this condition can be solved for the equilibrium rate of R&D: $\phi = \frac{\pi_1}{c} - r$.

17.1.1 Compulsory licensing

If a licensing fee F has to be paid to the previous innovator, the value of an innovation becomes $V = \frac{\pi_1}{r+\phi} - F(1 - \frac{\phi}{r+\phi})$. It decreases in F as the present discounted value of paying the licensing fee upon entry and receiving it back in the future if another firm enters is negative if the interest rate r is positive[20]. V also decreases in F (given that $F < \frac{\pi_1}{r}$ which needs to hold for V to be positive) so that the zero profit condition $V = c$ implies that the equilibrium rate of innovation ϕ decreases in F. Introducing compulsory licensing therefore reduces growth like in the case with persistent leadership.

17.1.2 Collusion and patent transfer scheme

Permitting collusion between an entrant and the previous incumbent allows them to charge price π_2 so that they can share the value $V_c = \frac{\pi_2}{r+\phi}$. As the entrant can get $V_E = \frac{\pi_1}{r+\phi}$ without colluding he must at least receive the same amount

[20]Here it is assumed that the licensing fees don ´t depend on the level of quality realized through the innovation. If they would rise sufficiently with each innovation there could be a positive present discounted value due to these licensing fees. However, in our model where profits don ´t increase over time such a scheme has to be ruled out as it would resemble a Ponzi game.

with collusion while the previous incumbents´ outside option is zero (the profits derived if there is competition). Let us assume that the relative bargaining power between incumbents and entrants is always the same so that entrants get the share α of total profits and incumbents the share $1-\alpha$. In order to guarantee that the entrant gets more than his outside option the condition $\alpha > \frac{1}{2}$ needs to hold (as under this condition $\frac{\alpha\pi_2}{r+\phi} > \frac{\pi_1}{r+\phi}$ is satisfied because $\pi_2 \leq 2\pi_1$). The value of an innovation is now given by $V_E = \frac{\alpha\pi_2}{r+\phi} + \frac{\phi(1-\alpha)\pi_2}{(r+\phi)^2}$ and it increases in α and decreases in ϕ if $\alpha \geq \frac{1}{2}$. The zero profit condition $V_E = c$ therefore implies that the equilibrium rate of innovation ϕ increases in the bargaining power α of the entrants. The intuition for this result is that profit flows for an entering firm become more frontloaded and have a higher present discounted value (for a given replacement rate ϕ) if the bargaining power of entrants is increased (this is basically the result of O ´Donoughue and Zweimüller (2004)). Again, growth is maximal if entrants have all the bargaining power and get the full profit flow π_2 right upon entry. This maximal rate of growth can again be implemented through a patent transfer scheme which forces previous inventors to hand their patents over to an entrant who successfully improves upon their innovation.

We can therefore conclude that also in the case of leapfrogging licensing fees are bad for growth but collusion is growth enhancing and that the maximal rate of growth can be obtained with a patent transfer scheme.

17.2 A

In the following, I look at the case where patents expire with a constant (flow) probability γ and where in the case of expiration the newest available quality of the good falls in the public domain and is supplied at marginal cost of zero[21]. In this case the incumbent loses all profits and becomes equal to all other potential entrants.

In the case where the currently newest good is in the public domain the R&D incentives depend on the value of getting a one - step lead which is given by

[21] The specification is chosen for reasons of simplicity and neglects the case where firms with a two - step lead lose this lead for a one - step lead because their patent on the second newest good expired. It is therefore implicitely assumed that patents on second newest goods never expire.

$V_1 = \frac{\pi_1 - c\phi_1}{r+\gamma+\phi_1} + \frac{\phi_1 \frac{\pi_2}{r+\gamma}}{r+\gamma+\phi_1}$. A firm with a one - step lead that faces no entry pressure sets it ´s R&D level equal to $\phi_1 = \phi_m$ if $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ (as then $\frac{\partial V_1}{\partial \phi_1} \geq 0$) and doesn ´t do any R&D ($\phi_1 = 0$) if $\gamma > \frac{\pi_2 - \pi_1}{c} - r$. If there is no incumbent monopolist and if at least one firm has sunk the fixed cost of entering the R&D sector it chooses the R&D level $\phi_0 = \phi_m$ if $V_1 > c$ (as then the marginal benefit of R&D is larger than the marginal cost c). If $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ this condition is satisfied and we have $\phi_0 = \phi_1 = \phi_m$. If $\frac{\pi_1}{c} - r > \gamma > \frac{\pi_2 - \pi_1}{c} - r$ we get $\phi_1 = 0$ and $\phi_0 = \phi_m$ and if $\gamma > \frac{\pi_1}{c} - r$ we get $\phi_0 = \phi_1 = 0$. While a firm with a two -step lead does not do any R&D ($\phi_2 = 0$), a firm with a one - step lead therefore does the maximal amount of R&D ($\phi_1 = \phi_m$) if $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ and in the case where no firm has a lead, we get the maximal R&D effort ($\phi_0 = \phi_m$) if $\gamma < \frac{\pi_1}{c} - r$. However, in the latter case there is only entry into the R&D sector if the expected value of entering is larger than the fixed costs R of entering, that means if $V_0 = \frac{\phi_m(V_1 - c)}{r} > R$ which is more likely to be satisfied if the probability of patent expiration γ is low and if the fixed costs R are low[22]. From the results above we can calculate the long run average rate of growth g. If patents never expire ($\gamma = 0$), the incumbent firm does not do any R&D once it has reached a lead of two steps as there is no entry pressure and average growth is zero. If patents expire too quickly ($\gamma > \frac{\pi_1}{c} - r$), firms don ´t find it profitable to race for the next innovation once old patents have expired so that average growth is again zero. For an intermediate probability of patent expiration (and low enough fixed costs of entering the R&D sector) average growth is however positive as incumbents regularly lose their lead which gives incentives for new entrants to innovate in order to become the next incumbents. Average growth is given by[23] $g = \frac{\gamma^3 + 2\gamma^2\phi_m}{(\phi_m+\gamma)^2}$ if $\gamma \leq \frac{\pi_2 - \pi_1}{c} - r$ and by $g = \frac{\phi_m \gamma}{\phi_m + \gamma}$ in the case

[22] If entry into the R&D sector is unprofitable if already one firm has entered and undertakes the maximal amount of R&D, there is clearly the issue of which firm can enter first and get some rents. For simplicity it could be assumed that one of the potential entrants (not including the previous incumbent) is drawn randomly to get the right to move first. Another way of modeling would be that there are no fixed costs of entering the R&D sector if the currently newest good is in the public domain. Even though there is the upper bound on the total arrival rate ϕ_m the zero profit condition can be satisfied with equality if firms in the aggregate undertake more R&D than necessary to obtain ϕ_m so that average costs increase above c. This can happen if the individual probability of obtaining a patent depends on the ratio between individual and total R&D spending.

[23] Average growth is calculated in the following way (see also section 13.1): the proportions of

where $\frac{\pi_1}{c} - r > \gamma > \frac{\pi_2 - \pi_1}{c} - r$. In both cases g can be shown to increase in γ.

17.3 B

A firm with a one - step lead selects R&D effort $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial \phi_1} > 0$ which is satisfied if $\phi_2 < \frac{\pi_2}{c} - r - \gamma_2 - \frac{\pi_1(r+\gamma_2)}{c(r+\gamma_1)}$ (*Condition A*). Given $\phi_1 = \phi_m$ we can solve the zero profit condition $V_E = c$ for ϕ_2 to get: $\phi_2 = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. Plugging this expression into *Condition A* gives the condition $\phi_m(\pi_1 - cr - c\gamma_1) < -(r+\gamma_1)(\pi_1 - cr - c\gamma_1)$. In the case where $\pi_1 - cr - c\gamma_1 < 0$ this implies that $\phi_m > r + \gamma_1 > \frac{\pi_1}{c}$ (*Condition B*) must hold in order to get $\phi_1 = \phi_m$. So the question is whether *Condition B* is compatible with the condition $\phi_m > \phi_2 > 0$ that must be be satisfied in order to have an equilibrium. $\phi_m > \phi_2$ is satisfied if ϕ_m is large enough and for ϕ_m large the condition $\phi_2 > 0$ holds if $\frac{\pi_2}{c} - 2(r+\gamma_2) > 0$ (*Condition C*). Taken together, *Conditions B* and *3* therefore require that $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$. As $\pi_2 \leq 2\pi_1$ this condition implies that $\gamma_1 > \gamma_2$ must hold.

Summing up, a firm with a one - step lead selects R&D effort $\phi_1 = \phi_m$ if ϕ_m is large enough and if $\frac{\pi_2}{2c} - \gamma_2 > r > \frac{\pi_1}{c} - \gamma_1$ which implies that $\gamma_1 > \gamma_2$ must hold.

17.4 C

Given $\phi_1 = \phi_m$ we can solve the zero profit condition $V_E = c$ for ϕ_2 to get: $\phi_2 = \frac{\pi_2}{c} - \frac{r+\gamma_2}{\phi_m c}(2c\phi_m + cr + c\gamma_1 - \pi_1)$. From this we obtain $\frac{\partial \phi_2}{\partial \gamma_1} < 0$ and $\frac{\partial \phi_2}{\partial \gamma_2} < 0$ given the condition $r > \frac{\pi_1}{c} - \gamma_1$ from *Lemma 1* holds. The average arrival rate is given by: $\widehat{\phi} = \phi_m \sigma_1 + \phi_2(\sigma_0 + \sigma_2) = \phi_m \sigma_1 + \phi_2(1 - \sigma_1)$. Using $\sigma_1 = \frac{\phi_2 \gamma_2}{(\phi_m + \gamma_1)\gamma_2 + \phi_2(\gamma_2 + \phi_m)}$ we obtain $sign \frac{\partial \sigma_1}{\partial \gamma_1} = sign \left\{ \frac{\partial \phi_2}{\partial \gamma_1} (\phi_m + \gamma_1)\gamma_2 - \phi_2 \gamma_2 \right\} < 0$ so that we get $\frac{\partial \widehat{\phi}}{\partial \gamma_1} = \frac{\partial \sigma_1}{\partial \gamma_1}(\phi_m - \phi_2) + \frac{\partial \phi_2}{\partial \gamma_1}(1 - \sigma_1) < 0$.

Deriving the average arrival rate $\widehat{\phi} = \frac{\phi_2(\gamma_2(\phi_m + \gamma_1) + \phi_m(\phi_2 + \gamma_2))}{(\phi_m + \gamma_1)\gamma_2 + \phi_2(\gamma_2 + \phi_m)}$ with respect to γ_2 gives $sign \frac{\partial \widehat{\phi}}{\partial \gamma_2} =$ $sign \frac{\partial \phi_2}{\partial \gamma_2} \left[(\gamma_2(\phi_m + \gamma_1) + \phi_m(\phi_2 + \gamma_2)) \gamma_2 (\phi_m + \gamma_1) \;\; + \phi_2 \phi_m (\gamma_2(\phi_m + \gamma_1) + \phi_2(\gamma_2 + \phi_m)) \right] +$

time σ_k in which the lead is equal to k steps can be derived by setting the probability of losing a k - step lead due to patent expiration or R&D equal to the probability of obtaining such a lead coming from states with different sizes of the lead. Average growth is then simply given as $g = \sum_{i=1}^{k} \sigma_i \phi_i$.

$\phi_m\phi_2^2(\phi_m - \phi_2) < 0$. The derivative is negative as $\frac{\partial\phi_2}{\partial\gamma_2} = -2 - \frac{r+\gamma_1-\frac{\pi_1}{c}}{\phi_m} < -2$ under *Condition 1* (from *Lemma 1*)■

17.5 D

We have $\phi_1 = \phi_m$ if $\frac{\partial V_E}{\partial\phi_1} > 0$ which holds if $\phi_2 \leq \frac{\pi_2}{c} - \frac{(\pi_1+cr)(r+\gamma)}{rc}$ (*Condition 0*). Inserting the equilibrium value $\phi_2 = \frac{\pi_2}{c} - 2r - 2\gamma - \frac{r+\gamma}{c\phi_m}(cr + Fr + F\phi_m - \pi_1)$ obtained from the zero profit condition into *Condition 0* implies that $r(\pi_1 - cr - rF) \leq -\phi_m(\pi_1 - cr - rF)$ which can only hold if $\pi_1 - cr - rF < 0$ (*Condition 1*). Taking the case where $\pi_2 = 2\pi_1$ we get $\phi_2 > 0$ if $\frac{F}{c}(r-\gamma) - 2\gamma > 0$ (*Condition 2*). $\phi_m > \phi_2$ holds if $\phi_m^2 > \phi_m\left(\frac{\pi_2}{c} - 2r - 2\gamma - \frac{F(r+\gamma)}{c}\right) - \frac{r+\gamma}{c}(cr + Fr - \pi_1)$ which is satisfied even for the maximum values of π_1 and π_2 (given by *Condition 1*) if $\phi_m > \frac{F}{c}(r-\gamma) - 2\gamma$ (*Condition 3*).

Summing up, $\phi_1 = \phi_m$ holds in equilibrium if $\pi_1 - cr - rF < 0$ and $\phi_m > \frac{F}{c}(r-\gamma) - 2\gamma > 0$■

17.6 E

Using the same analysis as in *Appendix C* but with $\gamma_1 = 0$ and $\gamma_2 = \gamma$ we can derive: $sign\frac{\partial\widehat{\phi}}{\partial\gamma} = sign\left\{\frac{\partial\phi_2}{\partial\gamma}\left[\gamma\phi_m + \phi_m^2(\phi_2+\gamma)\gamma + \phi_2\phi_m(\gamma\phi_m + \phi_2(\gamma+\phi_m))\right] + \phi_m\phi_2^2(\phi_m - \phi_2)\right\} < 0$ as $\frac{\partial\phi_2}{\partial\gamma} = -2 - \frac{cr+Fr+F\phi_m-\pi_1}{c\phi_m} < -2$ due to *Condition 1* from *Appendix 3*■