Cheap talk, Information acquisition and conditional Delegation

Mark Le Quement

March 30, 2009

Abstract

We examine a Principal-Agent problem in which the Agent holds private information about a non-contractible action to be chosen either by the Principal (Authority) or by the Agent (Delegation). We analyze the Agent's incentives for information acquisition under convex, linear and concave information costs. The Agent always acquires more information under Delegation and always prefers the latter decision scheme to Authority. In contrast, the Principal only prefers Delegation when the Agent's information level is sufficiently high. We study conditional Delegation games where the Principal conditions Delegation on the observed amount of information acquired by the Agent. In such games, when the Principal can commit ex ante to a potentially ex post inefficient conditional delegation strategy, if costs are intermediate, his prefered Agent always has a positive bias for concave, linear or convex cost functions. Similarly, if the Principal can only choose Delegation when this is ex post optimal, when costs are intermediate, his favorite Agent still has a positive bias under concave and linear costs.

JEL classification: C72, D82, D83

Keywords: communication, cheap talk, information acquisition

1 Introduction

In a variety of organizational environments, information is local, in the sense that expertise on particular issues is only held by a limited number of specialized individuals. An organization may thus be conceived of as a particular form of community of local information holders. A main problem faced by an optimizing "organizational designer " is to create an organizational form that makes existing information available to decision makers. The major impediment to the realization of such a goal is individual rationality and heterogeneous preferences among members of organizations. Individuals holding decision relevant information may not fully share the incentives of decision makers and may therefore not have any incentives to reveal their information truthfully. In the face of such communicational imperfections within organizations, as demonstrated by Dessein (2002), delegation may offer an efficient alternative to communication. Principals, rather than extracting very imperfect information from Agents, may prefer to let Agents decide by themselves. Indeed, the improvement in the quality of the information underlying the decision may more than compensate for the Agent's bias in decision making.

A problem implictly underlying issues of information transmission is that of information acquisition. Before even worrying about how information is transmitted, we need to understand how and why information is available in the first place. In the existing litterature, the issue of information transmission among agents with diverging preferences has been examined mostly under the assumption of exogenous information of the sender. Thus, in most treatments of the question, the sender's informational status (the degree of imperfection of his information) is predetermined and does not depend on the equilibrium communication taking place between sender and receiver. This simplifying assumption may however be very misleading when there exists a strong feedback effect of equilibrium communication on the incentives for information acquisition of the sender. Indeed, taking as a point of departure the classical Crawford and Sobel (1982) model of cheap talk communication, think of the case of a sender who's commonly known bias is so great that even when perfectly informed of the state of the world, he can only transmit his information very imperfectly. Such a sender may in turn be tempted to acquire very little or no information in the first place. Diverging interests thus not only have a negative first order effect on communication but also have an adverse second order effect in possibly leading to less information being held by the sender. On the contrary, one could also suppose that a very biased sender may want to compensate for his bias by acquiring more information than a less biased sender.

The Principal's choice between communication and delegation may also be reexamined in the light of the information acquisition problem. Indeed, when a Principal can observe the information acquisition of the Agent (as opposed to the contents of the information acquired), he naturally conditions his choice of decision scheme on the Agent's level of expertise. For example, if the Agent is perfectly uninformed of the ex post realization of the state of the world, Delegation is obviously a dominated option. In such a case, whomever makes the decision will do so on the basis of the sole prior distribution but the Agent will however choose according to his bias. On the other hand, a Principal who is able to observe the Agent's information choice may be able to use this knowledge strategically by proposing a contract in which his delegation choice is explicitely conditioned on the amount of information acquired by the Agent.

Results:

We conduct our analysis on the uniform-quadratic version of the classical Crawford and Sobel (1982) model and allow for a variety of cost functions (concave, linear or convex). We first determine the amount of information acquisition chosen by an Agent when the decision scheme (Authority or Delegation) is exogenously determined ex ante. We go on to analyze the case of endogenous decision scheme choice by analyzing games in which the Principal may stimulate information acquisition by the Agent and thereby increase his revenue.

First, we show that under Authority, the Agent's information acquisition decision is negatively affected by his bias. The larger his bias, the less information does he acquire, and if information costs are sufficiently high, there is a positive level of bias for which he acquires no information. Under Delegation, in contrast, the information acquisition decision of the Agent exclusively depends on the cost of information. Our analysis thus highlights a new adverse effect of Agent bias on communication, residing in decreased incentives for information acquisition. Once accounting for the (possibly very limited) amount of information information acquired by the Agent, Delegation is not unilaterally the best option for the Principal in contrast to Dessein (2002). Indeed, if the Agent chooses to remain uninformed under Delegation because information costs are too high, the Principal prefers Authority which allows him to decide on the basis of his prior. On the contrary, if the Agent's bias is intermediate, the Agent may choose to acquire information under Delegation, while choosing to remain uninformed under Authority. In such a case, Delegation may dominate Authority due to its superior capacity to stimulate information acquisition.

The main result of this paper is arises from our study of conditional delegation games in which the Principal's decision scheme is chosen upon observation of the Agent's level of information acquisition. In such games, whether or not the Principal is allowed to commit ex ante to a conditional delegation rule, we show that the Principal's prefered agent has a strictly positive bias, in the case of intermediate information costs and for a wide variety of cost functions. The feature underlying this result is that that Delegation is more attractive to an Agent, the more biased he is. More biased Agents are thus ready to acquire more information to ensure that the decision is delegated, which more than compensates for their high bias in making a delegated decision, under a variety of cost functions.

Related litterature:

The model upon which we build our analysis is the seminal contribution of Crawford and Sobel (1982), (CS in what follows). The authors propose a model where an uninformed Principal faces a perfectly informed Agent with diverging preferences. They show that the level of noise characterizing equilibrium communication is increasing in the interest divergence between Agent and Principal. Building on CS, Dessein (2002) studies delegation as an alternative to communication. He shows that a principal prefers to delegate the decision to an Agent rather than communicate if the interest divergence between the two is not too large relative to the uncertainty about the state of the world. Krähmer (2006), using a version of the CS model, studies a problem of incomplete contracting, where Delegation is made Message contingent. The author shows that partial delegation stimulates information revelation and can therefore outperform unconditional Authority and unconditional Delegation. In these works, the Agent's information is both perfect and exogenous, in the sense that it does not follow from a chosen and costly effort produced by the Agent.

A series of works has examined the question of information acquisition and related this issue to the question of the optimal decision scheme of the Principal. Aghion and Tirole (1997) examine a delegation problem between a principal and an agent. They show that the principal may profitably delegate formal authority to an agent with the aim of increasing the agent's incentives to acquire information. We reach the same conclusion in the first part of our analysis, where we study unconditional decision schemes. In Szalay (2005), a principal delegates a decision right to an agent who has misaligned interests. The author shows that constraining the choice set of the agent to a subset of extreme options may improve the expected payoff of the Principal, by increasing the Agent's incentives to acquire information. Persico (2006) and Gerardi (2007) study the problem of information acquisition within committees. The underlying insight in the two latter contributions is that the Principal may want to commit to a decision rule that is some times ex post inefficient (given the available information) in order to attain ex ante efficiency, by adopting rules that stimulate information acquisition by the Agent(s). A "good" decision rule must balance between the incentives to invest in information and the aim of aggregating information in an efficient way.

Contribution

This paper is to my knowledge the first to consider the possibility of conditioning the delegation decision of the Principal on the amount of information acquired by the Agent. Typically, the decision mechanism (Authority or Delegation) is set ex ante, anticipating that this will subsequently affect the Agent's incentives for information acquisition, as in for example Aghion and Tirole (1997). Here in contrast, it is the anticipation of the fact that the Principal will condition his choice of decision scheme on the Agent's information level that incentivizes the Agent to acquire more information. The conditional delegation games that we study thus offer an innovative approach to the Delegation vs Authority problem by making a new tool available to the Principal. The assumption of observability of the Agent's information level yields in particular the robust finding that for intermediate cost levels, the Principal generally prefers Agents with a strictly positive bias to perfectly unbiased Agents.

Structure of the paper:

In section 3, we first conduct an analysis of the the information acquisition problem under unconditional (exogenously determined) decision schemes. In section 4, we consider a conditional Delegation game in which the Principal may ex ante commit to an optimal conditional Delegation rule. In section 5, we consider a conditional Delegation game in which the Principal unable to commit ex ante to a conditional Delegation rule but only can choose the ex post optimal decision scheme. In all sections, we allow for a large variety of increasing cost functions, both concave, linear and convex.

2 The model

We adopt the standard framework originally proposed by Crawford and Sobel and later extended to the case of Delegation by Dessein (2002) and Krähmer (2006). The specific modelling of the extension to an imperfectly informed Agent is taken from Ottaviani (2000). The state of the world is represented by a variable x that is uniformly distributed on the interval $[0, 1]$ and an action a must be chosen. There is a Principal (P) and an agent (A), whose utility functions are given by respectively:

$$U^P = -(a - x)^2 \text{ and } U^A = -(a - (x + b))^2 \tag{1}$$

The Agent has access to information about the state x while the Principal is uninformed. The expertise of A is represented by a variable $t \in [0, 1]$. The variable t describes the probability that A receives a signal $s = x$ while he receives a signal s randomly drawn from the prior distribution with probability $1 - t$. We suppose that when A receives a signal s, he does not know whether the signal is perfectly informative or randomly drawn from the prior distribution. A decision scheme allows for an action a to be chosen, which subsequently determines the payoffs of both Principal and Agent according to the payoff functions given in (1). We study two possible decision schemes. In the first decision scheme, which we call Authority (ND as in no Delegation), the Principal chooses and action, possibly using some information transmitted to him by the Agent. In the second scheme, called Delegation (D), the Principal delegates the decision to the Agent who freely chooses an action. The decision stage is preceded by an information acquisition stage in which the Agent may acquire $t \in [0,1]$ at a cost $C(t)$. The level of t chosen by A is observable to the Principal. We suppose that $C(t)$ is such that $C'(t) > 0$ and belongs to one of the following sets:

$$\text{set } X \quad : \quad C^{''}(t) > 0 \text{ s.t } C'(0) = 0 \text{ and there is a unique } \bar{t} \text{ s.t for } t > (\leq)\bar{t},\ C'(t) < (\geq)\frac{1}{6}t \tag{2}$$

$$\text{set } L \quad : \quad C'(t) = k, \text{ for some } k < \frac{1}{6} \tag{3}$$

$$\text{set } W \quad : \quad C^{''}(t) < 0 \text{ s.t } C'(1) < \frac{1}{6} \tag{4}$$

Note that each of these categories may be subdivided into various subcategories that we present here. We assume that the third derivative $C'''(t)$ has the same sign for any $t \in [0,1]$. For a function $C \in X$, either $C'''(t) = 0$, or $C'''(t) > 0$ or $C'''(t) < 0$. We call the corresponding subsets $X.1$, $X.2$ and $X.3$. Furthermore, if $C \in W$, we may again distinguish between three cases according to the sign of $C'''(t)$. We call these respectively $W.1$, $W.2$ and $W.3$.

All elements of the game are common knowledge. Following Dessein (2002), we interpret the model as a representation of Principal-Agent relations within a firm. The Agent gains local information about some specialized activity. This information is not directly available to the Principal and is neither verifiable. In such a context, the Principal may rely on two different decision schemes in order to make use of the information gathered by the Agent: he may either communicate with the Agent and attempt to acquire some of the latter's information, or he may let the Agent decide directly. The relative benefits of the two options will of course depend on the extent of interest misalignment between Principal and Agent.

A comment on the main assumption of our model is here warranted: we crucially assume that the information level of the Agent is observable to the Principal, despite the non-verifiability of the information held by the Agent. These two dimensions are not incompatible, within the organizational interpretation that we adopt. While the Principal may not be able to evaluate the truth value of statements made by Agents in contact with highly specialized information, a Principal can typically monitor the effort level of Agents. Good proxies for the Agent's effort in acquiring information are for example the number of hours that the Agent spends in his office, the number of reports he produces, or the number of "improvement courses" that he follows.

In what follows, we first establish two lemmas that allow us to subsequently engage in the study of the information acquisition problem. We first briefly characterize equilibrium communication under Authority, when the Agent's informational level is exogenously given by a level t (known to both Principal and Agent). Equilibrium communication, as shown by Ottaviani (2002) is partitional and strongly echoes that of the original Crawford-Sobel model with a perfectly informed Agent. Below, we clarify the concept of partitional communication equilibrium:

Definition 1 ***Partitional communication equilibrium***

A partitional equilibrium with N partitions is defined by a set of partition thresholds $S = \{s_0, s_1, .., s_N\}$ *and a set of messages* $M = \{m_1, m_2, .., m_N\}$ *such that if* $x \in [s_i, s_{i+1}]$ *such that A sends message* m_i *if* $x \in [s_{i-1}, s_i]$.

We now characterize equilibrium communication with an imperfectly informed Agent.

Lemma 1 ***Communication under Authority with an imperfectly informed Agent (Ottaviani (2000))***

Suppose A holds information of quality t. Equilibrium communication is partitional and partition thresholds are defined by:

$$s_i = \frac{i}{N} + \frac{2bi(i-N)}{t}, \text{ for } i = \{0, 1, ., N(b,t)\} \tag{5}$$

, where $N(b,t)$ *is the lowest integer number larger or equal to* $-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2t}{b}}$

Proof: see in the appendix.

Equilibrium communication with an imperfectly informed Agent is thus partitional as in CS and takes a form that is very similar to the original model. Indeed, the only change in the formula for s_i determining equilibrium partitions is the addition of the term t beneath the element b. The Agent truthfully reveals that he has received a message belonging to the interval $[s_i, s_{i+1}]$, for some $i \in \{0, 1, ., N(b,t) - 1\}$. The Principal, on that basis, infers that the expected value of the state is $\frac{t(s_i+s_{i+1})+(1-t)}{2}$, as he takes into account the possibility that the Agent received a randomly drawn signal with a probability $1 - t$.

We may now establish the expected payoffs of P and A under Authority and Delegation, when the Principal commits ex ante to Authority or Delegation, and when a level t of information is acquired by the Agent.

Lemma 2 ***Expected payoffs under unconditional decision schemes.***

- *The expected payoff of P and A under unconditional Authority, given an acquired information level t and a number of partitions N, is given by:*

$$V^P_{ND}(t,N) = -\frac{t^2}{12N^2} - \frac{1-t^2}{12} - \frac{b^2(N^2-1)}{3} \text{ and } V^A_{ND}(t,N) = -\frac{t^2}{12N^2} - \frac{1-t^2}{12} - \frac{b^2(N^2+2)}{3} - C(t) \tag{6}$$

- *The expected payoff of P and A under unconditional Delegation is given by:*

$$V^P_D(t,b) = -\frac{1}{12}(1-t^2) - b^2 \text{ and } V^A_D(t) = -\frac{1}{12}(1-t^2) - C(t) \tag{7}$$

Proof: see in the appendix.

Note that as in the original CS (1982), the expected payoff of A and P is increasing in the number of partitions N for $N \leq N(b,t)$ (where $N(b,t)$ is the highest number of partitions that can be sustained in equilibrium for a given combination of t and b). We may thus expect that for every value of t, P and A will coordinate on the communication equilibrium displaying the highest possible amount of partitions. The payoff functions of P and A are thus concatenations of a set of payoff functions, corresponding to increasing values of N, as N increases. Thus, for small values of t, the payoff function of P may be $V_{ND}^{P}(t,2)$, while for larger values of t, it may become $V_{ND}^{P}(t,2)$. With a sight abuse of notation, we thus define the following payoff functions of P and A under Authority:

$$V_{ND}^{i}(t) = V_{ND}^{i}(t, N(b,t)) \text{ for } i \in \{A, P\} \tag{8}$$

, where $N(b,t)$ is the highest number of partitions that can be supported for a given combination of b and t. (see in the appendix for further details). In the figure below, we depict the payoff function of the Agent for different values of t, for a cost function $C(t) = k(ln(1-t)+t)$, $k = 0.03$ and $b = 0.05$. For different values of t, different maximum numbers of partitions may be sustained (one for $t < 0.2$, two for $0.2 \leq t \leq 0.6$ and three for $t \geq 0.6$), and $V_{ND}^{A}(t, N(b,t))$ corresponds to the highest of the three functions, for each interval.

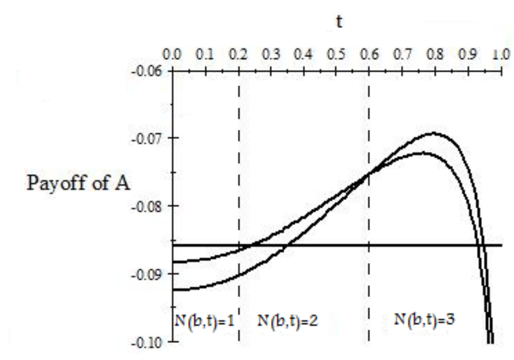


Figure 1

Below, we clarify further P and A's preference for Authority and Delegation, given an exogenously fixed level of information acquisition t.

Remark 1 ***Relative preferences for Authority and Delegation***

For a given information level t:

- P prefers Delegation to Authority if $N = 1$ and $t \geq \sqrt{12}b$ or if $N(b,t) \geq 2$, i.e. if t is such that Authority implies the possibility of communication.

- A always prefers Delegation to Authority

Proof: Note that:

$$V^P_{ND}(t,N) - V^P_D(t) = -\frac{t^2}{12N^2} + \frac{b^2\left(4-N^2\right)}{3} \tag{9}$$

, which is clearly always negative whenever $N \geq 2$. Note also that if $N = 1$, this expression is negative if $t \geq \sqrt{12}b$. Note that $N(b,t) \geq 2$ for $t \geq 4b$. Given that $\sqrt{12}b \leq 4b$. Thus for $\sqrt{12}b \leq t < 4b$, P prefers Delegation while no communication is possible given t. On the other hand, for $t \geq 4b$, P prefers Delegation although communication is possible given t. For $t \geq \sqrt{12}b$, P thus always prefers Delegation to Authority.

$$V^A_{ND}(t,N) - V^A_D(t) = -\frac{t^2}{12N^2} - \frac{b^2\left(N^2+2\right)}{3} < 0 \tag{10}$$

■

Note that A always prefers Delegation to Authority, whatever the amount of information that he acquires, while the Principal only prefers Delegation when the information acquired is superior to some minimum positive threshold which is increasing in the bias of the Agent. The Agent's unconditional preference for Delegation, as we will see in sections 4 and 5, may be exploited by the Principal through the threat of not delegating the decision. In the following section, we characterize the information acquisition choice of the Agent under Authority and Delegation, for given levels of information costs and bias b, when the decision scheme is chosen exogenously. In the subsequent sections, we examine games in which the Principal chooses the decision scheme after having observed the information acquired by the Agent.

3 Unconditional Decision schemes

The decision scheme by which the action may be chosen, given the available information, is here fixed exogenously. The timing of the game is here given as follows:

1. The principal sets a decision scheme $\Omega \in \{ND, D\}$
2. The Agent chooses t after observing Ω and receiving a signal s
3. The decision is made according to the chosen scheme Ω

It turns out that the the information acquisition behavior is qualitatively identical for all types of cost functions that we study. The only minor distinction is that information acquisition is binary (i.e. $t \in \{0,1\}$ in the case of all cost functions but type X.1. For the the latter type of cost function, the acquired information can take an interior value $0 < t < 1$.

Proposition 1 ***Information acquisition under unconditional Decision schemes.***

Suppose that $C \in \{X.2, X.3, L, W\}$. Let t_1 and t_2 denote the amount of information acquired by the Agent under respectively Authority and Delegation.

- ***Authority***

  a)If $C \in \{X.2, X.3, L, W\}$*,* $C(1) > \frac{1}{12}$*,* $t_1 = 0$*. If* $C(1) \leq \frac{1}{12}$*,* $t_1 = 0$ *or* $t_1 = 1$*. If* $C \in X.1$*, either* $t_1 = 0$ *or* t_1 *s.t* $\frac{t_1}{6}\left(\frac{N(b,t_1)^2-1}{N(b,t_1)^2}\right) = C'(t_1)$ *for some integer* $N(b, t_1) \leq N(b, 1)$ *or* $t_1 = 1$*.*

  b) For $b' < b$*,* $t_1(b') \geq t_1(b)$*.*

  c) For $C(t)$ *sufficiently large, there exists a* $\overline{b}$*, such that for* $b < \overline{b}$*, the Agent acquires full information and for* $b > \overline{b}$*, the Agent acquires no information.*

  d) The Agent's payoff under Authority $V_{ND}^{A}(t_1)$ *is decreasing in* b*.*

- ***Delegation***

  $t_2 = 1$ *if* $C(1) \leq \frac{1}{12}$*, otherwise* $t_2 = 0$

- ***Authority vs Delegation***

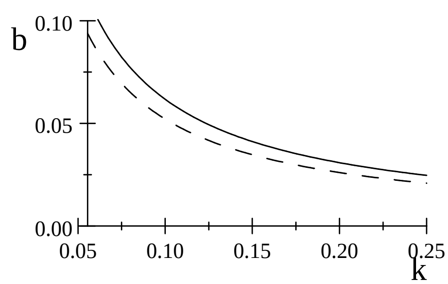
  The Agent always acquires strictly less information under Authority than under Delegation, i.e. $t_1 < t_2$

  The Agent's optimal payoff under Authority is strictly lower than his optimal payoff under Delegation, i.e.$V_{D}^{A}(t_2) > V_{ND}^{A}(t_1)$

Proof: see in the appendix.

We here give an example of the information acquisition behavior of the Agent under both decision regimes.

Example 1: Suppose that $C(t) = kt^3$ such that $k > \frac{1}{18}$ (where the latter condition ensures that $C\prime(t) = \frac{1}{6}t$ for $0 < t < 1$). The derived bounds on b governing the Agent's acquisition of information are given by $\underline{b} = \frac{1}{192k}$and $\overline{b} = \frac{1}{162k}$. Below, we graph these bounds as a function of k:



Bounds on b as a function of k

Figure 2

We see that the bounds $\underline{b}$ and $\overline{b}$ are expectedly decreasing in k, so that the higher the cost of information, the lower the maximum Agent bias for which the Agent acquires a positive level of information. Note furthermore that the bounds obtained are close to each other.

Our analysis, by taking into account the Agent's incentives for information acquisition, sheds light on a new adverse consequence arising from the preference misalignment between P and A, which adds to the classical "noisy information transmission" result of CS. Indeed, not only does the interest divergence between A and P cause imperfect communication given the information available to A, but the Agent's bias also decreases the amount of information acquired by the Agent. Acknowledging that his acquired information will yield him little benefit due to the inefficient transmission of information, the Agent's incentive to acquire information in the first place is reduced.

Another important feature emerging from our analysis is that the Agent's utility under Authority is decreasing in his bias. This, taking into account that A's utility under Delegation is unaffected by his bias, implies that the relative attractiveness of the two decision schemes changes as the Agent's bias increases. In the next sections, we explore the implications of this feature in settings where the Principal's delegation choice depends on the amount of information obtained by the Agent.

4 Conditional Delegation with commitment

The Principal may be dissatisfied with the insufficient amount of information acquired by the Agent under either of the available unconditional decision shemes. We now propose a simple conditional delegation strategy of the Principal that can succeed in incentivizing the Agent to acquire more information under Delegation than he would be acquiring under unconditional Delegation. The conditional strategy consists in committing to delegate the decision to the Agent only if he acquires more than an given minimum level of information. Under such a conditional decision scheme, given that the Agent intrinsically prefers delegation to Authority, he will be willing to exert some supplementary effort in order to ensure delegation of decision making. A crucial feature of our model is that an Agent's relative preference for Delegation over Authority becomes larger, the greater his bias. The threat of not delegating the decision should thus becomes more powerful, the greater the interest divergence between Agent and Principal. A Principal is thus able to extract more information acquisition out of Agents with a larger bias. More information acquisition by more biased Agents however comes at a cost: given the acquired information, the quality of an Agent's decision, from the point of view of the Principal, is decreasing in the Agent's bias. It thus remains unclear a priori whether a Principal may prefer to collaborate with Agents with a positive bias than with perfectly unbiased Agents. This will be the case if the increase in the information acquisition of a biased agent more than compensates for his increased bias in decision making. In what follows, we show that this is true for all cost functions that we study, whether concave, linear or convex, if costs are intermediate (i.e. neither too high nor too low).

We first define the conditional delegation mechanism that we study in this section.

Definition 2 ***Conditional Delegation with commitment:***

A Conditional Delegation Mechanism (CDM) is characterized by a threshold T such that the Principal delegates the decision to the Agent if and only if the Agent acquires information $t \geq T$. The optimal CDM, given b and $C(t)$, is the CDM that maximizes the ex ante utility of the Principal.

The timing of the Conditional Delegation with commitment game is given as follows:

1. The Principal announces with commitment a Delegation threshold T
2. The Agent chooses t
3. The decision is delegated if $t \geq T$
4. An action is chosen according to the selected decision scheme

In what follows, we analyze the outcomes of the game defined by the Conditional Delegation Mechanism defined above.

4.1 Concave cost function, linear and subcases X.2 and X.3 of convex cost function

Proposition 2 ***Optimal CDM for intermediate costs***

Suppose that $C \in \{X.2, X.3, L, W\}$ and $\frac{1}{6} > C(1) > \frac{1}{12}$ so that A acquires no information under unconditional Delegation. Define $\widetilde{b} = \sqrt{C(1) - \frac{1}{12}} > 0$.

- *If $b < \widetilde{b}$, either P sets $T^*(b) < 1$ such that $V^A_{ND}(T(b), N(b, T(b)) = V^A_{ND}(0, 1)$ if $T^*(b) \geq \sqrt{12}b$, or sets $T^*(b) = 1$. The decision is delegated in the first case and not delegated in the second case.*

  If $\frac{1}{\sqrt{12}} \geq b \geq \widetilde{b}$, $T^*(b) = 1$ and the decision is always delegated.

  If $b > \frac{1}{\sqrt{12}}$, $T^*(b) > 1$ and the decision is never delegated.

- *The Principal's expected payoff is maximized for an Agent with bias $\widetilde{b} > 0$*

Proof:

Part 1: $C \in \{X.3, L, W\}$

The condition $C(1) > \frac{1}{12}$ implies that $V^A_D(0) > V^A_D(1)$, which means that the Agent acquires no information under Delegation. We furthermore know that $V^A_D(t)$ is decreasing in t until $\bar{t}$ such that $\frac{1}{6}\bar{t} = C'(\bar{t})$. Now, this implies that there is a unique τ such that for $t < \tau$, $V^A_D(t) > V^A_D(1)$ while for $t \geq \tau$, $V^A_D(t) \leq V^A_D(1)$. Note that if Delegation does not take place, we know that the Agent chooses $t = 0$ under Authority, obtaining $V^A_{ND}(0) = -\frac{1}{12} - b^2$. Suppose that a given Delegation threshold T is chosen by the Principal. The Agent will thus choose to exert effort T and obtain Delegation if $V^A_D(T) \geq V^A_{ND}(0)$.

There are now two possibilities Either $T > \tau$, in which case A's best choice of t, under the restriction that $t \geq T$, is $t = 1$. Or $T \leq \tau$, in which case A's best choice of t, under the restriction that $t \geq T$, is $t = T$. On the other hand, whatever the value of T, under the restriction that $t < T$, the best value of t for the Agent is given by $t = 0$. There are now two scenarios that need to be considered, which we call S.1 and S.2. In S.1, the bias b of the Agent is such that $V^A_D(1) > V^A_{ND}(0)$, i.e. $C(1) < \frac{1}{12} + b^2$, in which case the Agent will choose $t = 1$ if the Delegation threshold is $T = 1$. In scenario S.2, the bias b of the Agent is such that $V_D^A(1) < V_{ND}^A(0)$, i.e. $C(1) > \frac{1}{12} + b^2$, in which case the Agent will choose $t = 0$ if the Delegation threshold is $T = 1$. He will however choose $t = T$ if T is set in such a manner that $V_D^A(T) = V_{ND}^A(0)$. This implies that in Scenario S.2, given the behavior of $V_D^A(t)$ over the interval $[0, 1]$, T is set s.t $T \leq \tau$. We have thus characterized the highest information acquisition that the Principal may obtain from the Agent under Conditional Delegation with commitment, for different values of b. There is thus a threshold value $\widetilde{b} = \sqrt{C(1) - \frac{1}{12}}$ such that for $b \geq \widetilde{b}$, the Principal can incentivize the Agent to acquire $t = 1$ while for $b < \widetilde{b}$, the Principal only succeeds in incentivizing a partial information acquisition $t < \tau$. Now, we examine the evolution of the payoff of the Principal over different values of b, analyzing in turn S.1 and S.2. In scneario S.2, note that, applying the Implicit Function theorem to the equality relation $V_D^A(T(b)) = V_{ND}^A(0)$, we obtain the following :

$$\frac{\partial T(b)}{\partial b} = \frac{-2b}{\frac{1}{6}T(b) - C'(T(b))} > 0 \tag{11}$$

Now, given that $V_D^P(t, b) = -\frac{1}{12}(1 - t^2) - b^2$, it follows from the Envelope theorem that:

$$\begin{aligned} \frac{\partial V_D^P(T(b), b)}{\partial b} &= \underbrace{\frac{1}{6}T(b)\frac{\partial T(b)}{\partial b}}_{\text{Benefit of increased information acquisition}} - \underbrace{2b}_{\text{Cost of increased bias in delegated decision making}} && (12) \\ &= -2b\left(\frac{\frac{1}{3}T(b) - C'(T(b))}{\frac{1}{6}T(b) - C'(T(b))}\right) && (13) \end{aligned}$$

Now, note that $\frac{1}{6}T(b) - C\prime(T(b))$ is always negative given that $T(b) < \tau$. Furthermore, note that given the assumptions on $C(t)$, $\frac{1}{3}T(b) - C\prime(T(b))$ may either be positive up to some value $\widetilde{t} < \tau$ and negative thereafter or be negative for all $t \geq 0$. In the first case, $\frac{\partial V_D^P(T(b),b)}{\partial b}$ is negative up to $T(b) = \widetilde{t}$ and then positive for $T(b) > \widetilde{t}$. In the second case, $\frac{\partial V_D^P(T(b),b)}{\partial b}$ is negative for all $T(b) < \tau$. The prefered values of b for the Principal in Scenario S.2 are thus given by either $b = 0$ or b arbitrarily close to $\widetilde{b}$. Now, note that $V_D^P(0, 0) = -\frac{1}{12}$, while $\lim_{t \to \widetilde{b}^-} V_D^P(T(b), b) = V_D^P(\tau, \widetilde{b})$ given that $\lim_{t \to \widetilde{b}^-} T(b) = \tau$, in scenario S.1. In scenario S.1., on the other hand, note that $T(b) = 1$ for $b \geq \widetilde{b}$. Given that $\frac{\partial V_D^P(1,b)}{\partial b} < 0$, it thus follows that in scenario S.1., $\frac{\partial V_D^P(T(b),b)}{\partial b} < 0$. We may conclude that the Principal's prefered agent has a bias $\widetilde{b}$, given that obviously $V_D^P(1, \widetilde{b}) > V_D^P(\tau, \widetilde{b})$.

Part 2: $C \in \{X.2\}$

Note that the assumption that $\frac{1}{6} > C(1) > \frac{1}{12}$ implies $\frac{1}{3}t > C'(t) > \frac{1}{6}t$, $\forall t \in [0, 1]$, which implies that the Agent acquires information $t = 0$ under unconditional Authority and Delegation. It follows that the Conditional Delegation Threshold $T(b)$ must respect tthe condition that $V_D^A(T(b), b) = V_{ND}^A(0)$. The solution to this equality condition is given by:

$$T(b) = \frac{b}{\sqrt{C(1) - \frac{1}{12}}} > 0 \tag{14}$$

Now, it is easily shown that $T(b) > \sqrt{12}b$ if $k \leq \frac{1}{6}$ which we have assumed from which it follows that the Principal will always chooses to set $T(b)$ and delegate the decision to A. Furthermore, note that:

$$\frac{\partial V_D^P(T(b),b)}{\partial b} = \frac{b^2(2-12k)+\frac{1}{12}-k}{(12k-1)} > 0 \tag{15}$$

Which is easily shown to be positive for $\frac{1}{6} < C^{'}(1) \leq \frac{1}{3}$, i.e. $\frac{1}{12} < k \leq \frac{1}{6}$. Note however that for $b > \widetilde{b}$ such that $\frac{\widetilde{b}}{\sqrt{k-\frac{1}{12}}} = 1$, an increase in the Agent's bias is not beneficial to the Principal anymore, as the Agent cannot acquire more than information $t = 1$. From that point onwards, an increase in Agent bias only decreases the quality of the Agent's decision, given his perfect information, and it thus only affects negatively the Principal's payoff. ■

Expressions (11) and (12) are key to the understanding of the conditions under which the Principal's payoff increases in the Agent's bias. When costs respect the conditions of Proposition 2, expression (11) tells us that the rate of increase of the optimal conditional threshold $T(b)$ depends on the following ratio:

$$\text{Rate of growth of } T(b) \text{ as } b \text{ increases} = \frac{\text{Rate of decrease of } V_{ND}^A(0,1) \text{ as } b \text{ increases}}{\text{Rate of decrease of } V_D^A(T(b)) \text{ as } b \text{ increases}}$$

Where $V_{ND}^A(0,1) = -\frac{1}{12} - b^2$ and $V_D^A(T(b)) = -\frac{1}{12}(1-(T(b))^2) - C(T(b))$. The threshold $T(b)$ thus increases proportionally to the relative size of these two rates of increase in the Agent's loss, as his bias aments. On the one hand, the rate of increase of A's loss under Authority is simply quadratic in b, whereas under Delegation, this rate of increase grows with the marginal costs $C'(T(b))$. Thus, the higher the level of marginal costs, for every t, the lower the (positive) growth rate of $T(b)$ as a function of b. Remember that the payoff of the principal under Delegation is given by $V_D^P(T(b)) = -\frac{1}{12}(1-(T(b))^2) - b^2$. As appears in equation (12), the growth rate of the Principal's payoff under conditional Delegation is thus simply given by the difference between the rate of growth of the benefit from hiring a more biased agent investing in more information (i.e. $-\frac{1}{12}(1-(T(b))^2)$) and the rate of growth of b^2, representing the cost of hiring a more biased Agent. Now, note that the rate of growth of the benefits will be higher, the lower the marginal cost of information. If this level is sufficiently low, the superior information acquisition of the Agent will more than compensate for his biased choice in the ensuing delegated decision making stage.

Example 2: Below, we provide a graphical illustration of the payoff of the Agent under respectively Authority and Delegation, for a quadratic and a linear cost function, for respectively $k = \frac{1}{8}$ and $k = \frac{1}{11}$. The monotonously decreasing curve represents the expected payoff of A under Delegation for different values of t. The dashed u-shaped curve represents the expected payoff of A for different levels of information acquisition t. The horizontal lines represent A's expected payoff under Authority, under the hypothesis that he does not acquire any information. ($i.e. k > \frac{1}{12}$). The higher the bias of the Agent, the lower his payoff under Authority with no information acquisition (which is given by $V_{ND}^A(0,1) = -\frac{1}{12} - b^2$).

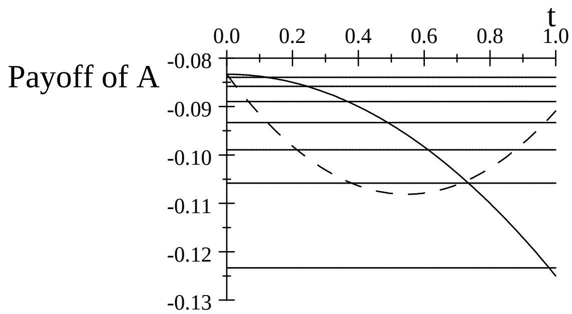


Figure 3

What appears very clearly from the above figure is that the Agent's outside option of Authority, in case he decides to acquire less than T, has a decreasing value, the larger his bias. The Agent's payoff under Delegation, on the other hand, is unaffected by his bias. There is thus an increasing discrepancy between his expected gain under the one and the other decision scheme, which the Principal may use strategically to his advantage.

Example 3: Below, we present a graphical representation of the evolution of the payoff of P under the optimal CDM, for a linear cost function given by $C(t) = kt$ and for $k = \frac{1}{8}$.

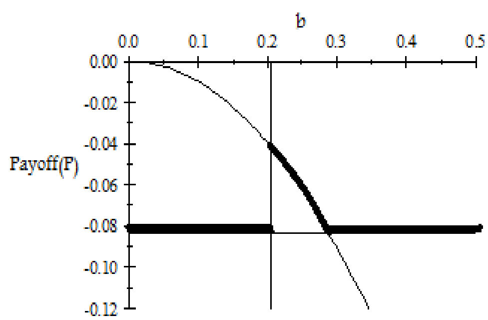
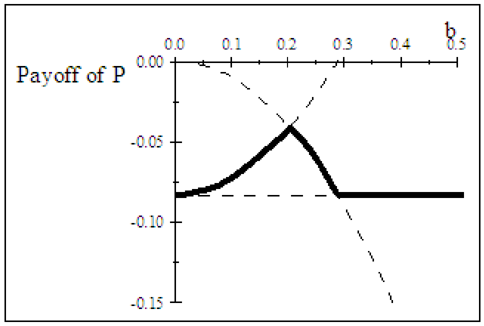


Figure 4

Example 4:Below, we provide a graphical illustration of the payoff of the Principal as a function of the bias of the Agent, for a quadratic cost function $C(t) = kt^2$ and for an intermediate cost level ($k = \frac{1}{8}$). The expected payoff function of the Principal is represented by the thick and kinked function. The dashed upwards sloping curve represents the expected payoff of P for values of b such that $T(b) \leq 1$. It represent P's payoff until the point where the curve crosses the thick downward sloping curve. From that point, the decreasing dashed curve represents the expected payoff of the agent when $T(b) = 1$. We see that the expected payoff of P is increasing in b up until $b = 0.204\,12$ in this example, while it is decreasing in b for very large values of b. Finally, the horizontal line shows the expected payoff of P $(-\frac{1}{12})$ when he chooses Authority rather than Delegation and decides without prior communication. We see that at some point, when b is superior to $\frac{1}{\sqrt{12}} = 0.288\,68$, the Principal prefers to decide alone on the basis of his prior information rather than delegating to a fully informed Agent.



Expected Loss of P as a function of b

Figure 5

■

4.2 Subcase X.1 of convex cost function

Proposition 3 *Suppose that $C \in \{X.1\}$ and that $\frac{1}{6} < C\prime(1) < \frac{1}{3}$ (implying $C(1) < \frac{1}{6}$). Let t_1 be the optimal amount of information purchased by the Agent under Authority. Then there is a $\frac{1}{\sqrt{12}} > \widetilde{b} > 0$ such that :*

- *If $b < \widetilde{b}$, P sets $T^*(b)$ such that $V^A_{ND}(T(b), N(b, T(b)) = V^A_{ND}(t)$. The decision is delegated.*

  If $\frac{1}{\sqrt{12}} \geq b \geq \widetilde{b}$, $T^*(b) = 1$ and the decision is always delegated.

  If $b > \frac{1}{\sqrt{12}}$, $T^*(b) > 1$ and the decision is never delegated.

- *The Principal's expected payoff is maximized for an Agent with bias $\widetilde{b} > 0$*

Proof:

Step 1: Under unconditional Delegation, the Agent chooses information t_2 such that $\frac{1}{6}t_2 = C'(t_2)$. Under unconditional Authority, he acquires a potentially positive level of information $t_1(b)$. Under Conditional Delegation, the Principal sets a conditional threshold which is of course strictly higher than t_2, as otherwise conditional Delegation would be of no use in incentivizing information acquisition by the Agent. It is therefore the case that for the threshold $T(b)$ that is chosen by the Principal $\frac{1}{6}T(b) < C'(T(b))$, given the shape of the function $C\prime(t)$. Now, it is clear that if the Agent chooses to invest in information $T(b)$ in order to obtain Delegation of the decision, he will acquire exactly information $T(b)$ as any supplementary information acquisition would decrease his payoff under Delegation. Now, we know that $V^A_D(t_2) > V^A_{ND}(t_1(b))$ so that the

Principal may incentivize maximal information acquisition by setting $V^A_{ND}(T(b), N(b, T(b)) = V^A_{ND}(t_1(b))$. Now, using the implicit function theorem, it follows that:

$$\frac{\partial T(b)}{\partial b} = \frac{\frac{\partial V^A_{ND}(t_1(b))}{\partial b}}{\frac{1}{6}T(b) - C'(T(b))} > 0 \tag{16}$$

It thus follows from the envelope theorem:

$$\frac{\partial V^P_D(T(b), b)}{\partial b} = -2b \left(\frac{\frac{1}{6}T(b)\left(1 - \frac{\partial V^A_{ND}(t_1(b))}{\partial b}\frac{1}{2b}\right) - C'(T(b))}{\frac{1}{6}T(b) - C'(T(b))} \right) \tag{17}$$

Now, note that $1 - \frac{\partial V^A_{ND}(t_1(b))}{\partial b}\frac{1}{2b} > 2$. This follows from the fact that $\frac{\partial V^A_{ND}(t_1(b))}{\partial b} < -2b$. Note indeed that when b increases infinitesimally, two scenarios may materialize. Either t_1 and the number of partitions at equilibrium change, or t_1 stays the same and so does the number of partitions in equilibrium. In the second case, note that:

$$\frac{\partial V^A_{ND}(t, N)}{\partial b} = -\frac{2b\left(N^2 + 2\right)}{3} \tag{18}$$

, which is clearly inferior or equal to $-2b$ for any $N \geq 1$. On the other hand, in the first case, suppose that the number of partitions changes from N to $N - 1$. It follows from step 2 of point b) of Proposition 1 for the case of $C \in X.1$ that :

$$\frac{\partial V^A_{ND}(t, N)}{\partial b} = -\frac{2b\left(N^2 - (N^2 - 1)\right)}{3} = -\frac{2b\,(2N - 1)}{3} \tag{19}$$

, which is obviously inferior or equal to $-2b$ for $N \geq 2$. Having now proved that $\frac{\partial V^A_{ND}(t_1(b))}{\partial b} < -2b$, it follows that:

$$\frac{1}{6}T(b)\left(1 - \frac{\partial V^A_{ND}(t_1(b))}{\partial b}\frac{1}{2b}\right) > \frac{1}{3}T(b) \tag{20}$$

Now, note that under the assumption that $C\prime(1) < \frac{1}{3}$, it follows that $\frac{1}{3}T(b) - C\prime(T(b)) \geq 0$ for any $t_2 < T(b) \leq 1$. This concludes the proof that $\frac{\partial V^P_D(T(b),b)}{\partial b} > 0$.

Step 2: We now simply wish to demonstrate that $T(b)$ is sufficiently large for the Principal to always wish to Delegate if the Agent acquires information $T(b)$. We know that $T(b) \geq t_2$. It follows that for any b such that $b \leq \frac{t_2}{\sqrt{12}}$, $T(b) \geq \sqrt{12}b$ so that the Principal wishes to delegate when the Agent acquires $T(b)$. Now, if $\frac{t_2}{\sqrt{12}} \leq \widetilde{b}$, we know that $V^A_D(T(b))$ increases until $b = \widetilde{b}$ while $V^P_{ND}(t_1(b))$ is decreasing in b. It is thus imediate that if the Principal wants to delegate for $t = T(b)$ when $b \leq \frac{t_2}{\sqrt{12}}$, he also wishes to delegate $t = T(b)$ when $\frac{1}{\sqrt{12}} \geq b > \frac{t_2}{\sqrt{12}}$. On the other hand, if $\widetilde{b} < \frac{t_2}{\sqrt{12}}$, it is trivial that P wishes to delegate for $t = T(b)$ when $\frac{1}{\sqrt{12}} \geq b > 0$.

Step 3: Note that we know that $\widetilde{b} < \frac{1}{\sqrt{12}}$. Indeed, we know that for $b > \frac{1}{4}$ (note that $\frac{1}{\sqrt{12}} > \frac{1}{4}$), there is no communication possible under authority, even for $t = 1$. The payoff function of the Agent under Autority

is thus given by $V_{ND}^{A}(0,1) = -\frac{1}{12} - b^2$ for $b > \frac{1}{4}$. Now, note that $-\frac{1}{12} - \left(\frac{1}{\sqrt{12}}\right)^2 = \frac{1}{6}$ and remember that $C(1) < \frac{1}{6}$, as follows from the assumption that $\frac{1}{6} < C\prime(1) < \frac{1}{3}$. This implies that $\widetilde{b} < \frac{1}{\sqrt{12}}$.
■

4.3 General conclusions on Conditional Delegation with commitment

We have thus shown that for any of the cost functions that we study, whether concave, convex or linear, the Principal's payoff is increasing in the bias of the Agent when costs are intermediate and the Agent's bias is not too large. More biased Agents have more to loose from the decision not being delegated, which implies that a higher information acquisition may be obtained from such agents under the promise of delegation. It is however noteworthy that, under the chosen specification of the problem[1], the increased benefit to P proceeding from increased information acquisition by A always outweighs the direct cost following from delegating the decision to an increasingly biased agent. Expectedly, note that once the maximal information acquisition is being obtained from the Agent ($t = 1$), there is clearly no benefit from an increased agent bias. Beyond some critical value $\overline{b}$, the Principal's payoff is thus decreasing in the bias b of the Agent.

5 Conditional Delegation with no commitment

Suppose that the Principal simply delegates the decision according to what is ex post optimal, given the information acquired by the Agent.

1. The Agent chooses t
2. After observing t, The Principal chooses a decision scheme $\Omega \in \{D, ND\}$
3. An action is chosen according to the selected decision scheme

We here examine the cases where the Agent will modify his information acquisition behavior as compared to the case of unconditional Delegation. In particular, we ask whether the Agent may be willing to invest more in information than under unconditional Delegation, in order to ensure that Delegation is selected by the Principal. Note that the Principal's ability to stimulate information acquisition through the threat of not Delegating the decision is now severely restricted, as he chooses the decision scheme according to what is ex post optimal, given A's acquired information level t. Given that P already prefers Delegation to Authority for fairly low levels of t, the minimum level of extra information acquisition potentially required from the Agent in order to ensure Delegation is low.

[1] I here refer to the so called uniform-quadratic set up chosen, with quadratic utility functions for P and A and a uniform prior distribution of the state.

5.1 Linear cost function, concave cost function and subcase X.3 of convex cost function

Proposition 4 ***Delegation game without commitment***

Suppose that $C \in \{X.3, L, W\}$ *and* $\frac{1}{12} < C(1) < \frac{1}{6}$*. Define* $\widetilde{b} = \sqrt{C(1) - \frac{1}{12}}$.

- *If* $b < \widetilde{b}$*, A chooses* $t \in \{0, \sqrt{12}b\}$ *and the Principal always chooses not to delegate the decision.*

  If $\frac{1}{\sqrt{12}} \geq b \geq \widetilde{b}$*, A chooses* $t = 1$ *and the Principal chooses to Delegate the decision.*

  If $b \geq \frac{1}{\sqrt{12}}$*, A chooses* $t = 0$ *and the Principal chooses Authority.*

- *The Principal's expected payoff is maximized for an Agent with bias* $\widetilde{b} > 0$

Proof:

Note that under the above assumption, A acquires $t = 0$ under unconditional Delegation. Define as τ the value for which $V_D^A(t) = V_D^A(1)$. Note that the Agent knows that the Principal will prefer Delegation if he chooses $t \geq \sqrt{12}b$. Now, under the restriction that $t \geq \sqrt{12}b$, the optimal value of t is $t = \sqrt{12}b$ if $\sqrt{12}b < \tau$ while the optimal value is $t = 1$ if $\sqrt{12}b \geq \tau$. It follows immediately from the analysis of Proposition 2 (Part 1) that the Principal's prefered Agent has a bias $\widetilde{b} = \sqrt{C(1) - \frac{1}{12}}$.■

We see that the outcome of this Delegation game without commitment is the same as the outcome of the Delegation game with commitment, under the listed conditions on parameters. In other words, the Principal's ex ante optimal Conditional Delegation strategy also constitutes his ex post optimal strategy. The Principal's preference for Agents with a positive bias is thus a robust feature of this game.

5.2 Subcases X.2 of convex cost function (quadratic case)

Proposition 5 ***Delegation game without commitment***

Suppose that $C \in \{X.2\}$ *and* $\frac{1}{12} < C(1)$

- *The Agent chooses either* $t = 0$ *or* $t = \sqrt{12}b$*, the Principal delegates the decision if* $t = \sqrt{12}b$ *and chooses authority otherwise.*

- *The Payoff of the Principal is unaffected by the Agent"s bias b.*

Proof:

In the case of $C \in X.2$, given that $\frac{\partial V_D^A(t)}{\partial t} < 0$ for $t > 0$, the Agent will choose $t = \sqrt{12}b$ is he wants to ensure Delegation, which is the minimal information acquisition securing that the Principal chooses Delegation. On the other hand, under Authority, we know that the Agent's optimal information acquisition choice is $t = 0$. Note that with $t = \sqrt{12}b$, the payoff of the Principal is the same under Delegation as under Authority with $t = 0$ and given by $V_{ND}^A(0, 1) = V_D^A(\sqrt{12}b) = -\frac{1}{12}$. In the case of $C \in X.2$, it follows that the

Agent will indifferently choose either $t = \sqrt{12}b$ or $t = 0$, but that the Principal will always obtain a utility given by $-\frac{1}{12}$. The Principal is thus indifferent among all types of the Agent.■

5.3 Subcase X.1 of convex cost function

Proposition 6 ***Delegation game without commitment***

Suppose that $C \in \{X.1\}$ and $\frac{1}{6} < C'(1)$ (implying $C(1) < \frac{1}{6}$). Let t_1 be the information acquired under unconditional Authority and t_2 be the information acquired under unconditional Delegation.

- *If $b \leq \frac{t_2}{\sqrt{12}}$, A chooses $t = t_2$ and the decision is delegated*
  If $b > \frac{t_2}{\sqrt{12}}$, A either chooses $t = \sqrt{12}b$ or $t = t_1$
- *The Payoff of the Principal is weakly decreasing in the Agent's bias b*

Proof:

Under unconditional Delegation, the Agent chooses an amount of information t_2 such that $\frac{t_2}{6} = C'(t_2)$. Now, either $b \leq \frac{t_2}{\sqrt{12}}$ in which case the decision is always delegated. If, on the other hand, $b \geq \frac{t_2}{\sqrt{12}}$, the Agent may choose to acquire the minimum $\sqrt{12}b$ necessary to ensure delegation, but never more, as this would be wasteful. Thus, for $b > \frac{t_2}{\sqrt{12}}$, P never obtains a payoff superior to $V_D^P(\sqrt{12}b) = V_{ND}^P(0,1) = -\frac{1}{12}$. On the other hand, for $b \leq \frac{t_2}{\sqrt{12}}$, P obtains a payoff given by $V_{ND}^P(t_2, N(b, t_2))$, which is superior or equal to $-\frac{1}{12}$. The payoff of the Principal is thus weakly decreasing in the bias of the Principal.■

Example 5: Suppose a cost function given by $C(t) = kt^3$, with $k = \frac{1}{18}$. The Agent acquires information $t = \frac{10}{18}$ for $b \lesssim 0.160$. For any superior bias, either the Agent acquires information $t = \sqrt{12}b$ and the decision is delegated or he acquires no information and the decision is not delegated.

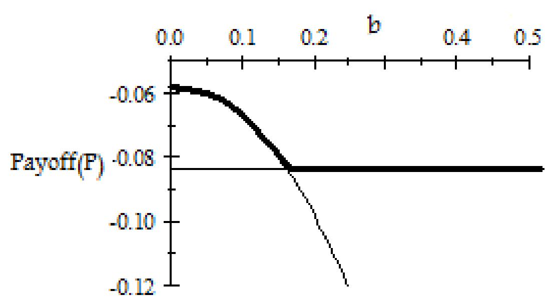


Figure 6

5.4 General conclusions on Conditional Delegation with no commitment

The Principal's preference for Agents with a strictly positive bias is thus a robust finding, as it holds even when P cannot commit to a conditional Delegation rule ex ante, whenever costs are intermediate and the cost function is either concave or linear (or convex with a concave first derivative).

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6 Appendix

6.1 Derivation of the partitional equilibrium and of equilibrium payoffs under communication

We here partially reproduce the analysis proposed in Ottaviani (2000).

Upon receival of message s$\in [s_i, s_{i+1}]$, the posterior mean of x is given by

$$E\left[x\left|s_i, s_{i+1}\right.\right] = \frac{t(s_i + s_{i-1}) + 1 - t}{2} \tag{21}$$

Therefore, supposing that partitioning arises in equilibrium communication, the action chosen by the Agent upon receival of a message indicating a signal belonging to $[s_i, s_{i+1}]$, is given by the $E\left[x\left|s_i, s_{i+1}\right.\right]$. Now, the equality condition that must be solved by each i $\in \{1, .., N\}$ is given by as follows. Define

$$L_i = -t\left(\frac{t(s_i+s_{i-1})+1-t}{2}-s_i\right)^2-(1-t)\int_0^1\left(\frac{t(s_i+s_{i-1})+1-t}{2}-x\right)^2dx \tag{22}$$

Then, the following condition must be respected:

$$L_i = L_{i+1} \tag{23}$$

Developing the abvove given condition, one obtains:

$$s_{i-1}(2ts_i - ts_{i-1}+4b) = s_{i+1}(2ts_i - ts_{i+1}+4b) \tag{24}$$

$$\Leftrightarrow \quad s_{i+1}-s_i = s_i - s_{i-1} + \frac{4b}{t} \tag{25}$$

Which, according to the same method as exposed in Crawford and Sobel (1981), yields:

$$s_i = \frac{i}{N} + \frac{2bi(i-N)}{t} \tag{26}$$

The expected payoff of the Principal is thus given by:

$$V^P_{CT}(t) = -t\sum_{i=1}^{N}\int_{s_{i-1}}^{s_i}\left(\frac{t(s_i+s_{i-1})+1-t}{2}-x\right)^2dx-(1-t)\sum_{i=1}^{N}(s_i-s_{i-1})\int_0^1\left(\frac{t(s_i+s_{i-1})+1-t}{2}-x\right)^2dx \tag{27}$$

While the expected payoff of the Agent is given by:

$$V^A_{CT}(t) = -t\sum_{i=1}^{N}\int_{s_{i-1}}^{s_i}\left(\frac{t(s_i+s_{i-1})+1-t}{2}-x-b\right)^2dx-(1-t)\sum_{i=1}^{N}(s_i-s_{i-1})\int_0^1\left(\frac{t(s_i+s_{i-1})+1-t}{2}-x-b\right)^2dx \tag{28}$$

Developing these Expressions yields the formulas given in the main text.

6.2 Proposition 1 for the case of C(t)$\in \{W, L, X.3, X.2\}$

A.0. Authority

Note that $\frac{\partial V^A_{ND}(N,t)}{\partial t} = \frac{1}{6}t\left(\frac{N^2-1}{N^2}\right) - C'(t)$.

Point a). We first treat the case of $C(t) = kt^2$. In this case, either $N \geq \sqrt{\frac{1}{1-12k}}$ and $\frac{\partial V^A_{ND}(N,t)}{\partial t} \geq 0$ for any t or $N < \sqrt{\frac{1}{1-12k}}$ and $\frac{\partial V^A_{ND}(N,t)}{\partial t} < 0$ for any t. So $V^A_{ND}(N,t)$ is maximized either for $t=0$ or for $t=1$. Call t_i the optimal informational investment for a number of partitions i, i.e. $t_i = \underset{t\in[0,1]}{\arg\max} V^A_{ND}(t,i)$. Note furthermore that $V^A_{ND}(1,i)$ is increasing in i for $i \leq N(b,1)$. On the other hand, when the Agent chooses $t=0$, he always receives $V^A_{ND}(0,1) = -\frac{1}{12}$. It is clear that for a given b, the Agent will thus choose either $t=0$ or $t=1$ depending on the relative sizes of $V^A_{ND}(1,N(b,1))$ and $V^A_{ND}(0,1)$. We now examine the remaining cases, i.e. C(.) $\in \{X.3, L, V\}$. Here, denote by $\bar{t}(N)$ the value of t such that $C'(t(N)) \geq \frac{1}{6}t\left(\frac{N^2-1}{N^2}\right)$ for $t \leq \bar{t}(N)$ and $C'(t) < \frac{1}{6}t$ for $t > \bar{t}(N)$. Note that under Authority, for every N, $\frac{\partial V_{ND}^A(N,t)}{\partial t} < 0$ for $t < \bar{t}(N)$ while $\frac{\partial V_{ND}^A(N,t)}{\partial t} \geq 0$ for $t \geq \bar{t}(N)$. Note that either $\bar{t}(N) \leq 1$ or $\bar{t}(N) > 1$ given the assumptions made on the cost function. It follows, that for every N, the optimal value of t is either 0 or 1. It follows from the same argument as in the case of $C(t) = kt^2$ that ther Agent always either chooses $t = 0$ or $t = 1$. Note that for $V_{ND}^A(1, N(b,1)) < C(1)$. Therefore, if $V_{ND}^A(0,1) > C(1)$, implying.$C(1) > \frac{1}{12}$, the Agent never acquires any information under Authority. This is thus a sufficient condition for the Agent not to acquire any information under Authority.

Points b) and c). Given that the information acquisition choice of the Agent is binary, we simply need to study the evolution of $V_{ND}^A(1, N(b,1)) - V_{ND}^A(0,1)$ as b decreases. Now, note that:

$$\Delta(b) = V_{ND}^A(1, N(b,1)) - V_{ND}^A(0,1) = \frac{b^2\left(1 - (N(b,1))^2\right)}{3} + \frac{(N(b,1))^2 - 1}{12\,(N(b,1))^2} - C(1) \tag{29}$$

Now, suppose $b' < b$. If $N(b',1) = N(b,1)$, then it follows automatically that $\Delta(b') > \Delta(b)$. On the other, suppose that $N(b',1) > N(b,1)$, then note that:

$$\partial\left(\frac{b^2\left(1-N^2\right)}{3} + \frac{N^2-1}{12N^2}\right)\Big/\partial N > 0 \text{ for } N < \frac{1}{2}\frac{\sqrt{2}}{\sqrt{b}}, \text{ i.e for } N < N(b,1) \tag{30}$$

It follows automatically that $\Delta(b') > \Delta(b)$. Given that $\Delta(b)$ is decreasing in b, it follows that if A chooses $t = 1$ for b, he must also choose $t = 1$ for bias b'. The reverse may however not be true. It may also be, if costs are sufficiently high, that there exists a $\bar{b}$ such that if and only if $b > \bar{b}$, $\Delta(b) < 0$, which implies that the Agent acquires no information for $b > \bar{b}$.

Point d). This follows trivially from the proofs of point a), b) and c).

A.1. Delegation

This follows trivially from arguments similar to those invoked in the proof for the case of Authority.

A.2. Comparing authority and delegation

Under Authority, $t_1 = 1$ iff $V_{ND}^A(1, N(b,1)) \geq -\frac{1}{12}$. Under Delegation, $t_1 = 1$ iff $C(1) < \frac{1}{12}$. Now, note that $V_{ND}^A(1, N(b,1)) < -C(1)$. The result follows. Note furthermore that $V_D^A(t_2) > V_D^A(t_1)$ and $V_D^A(t_1) > V_{ND}^A(t_1)$. It follows by transitivity that $V_D^A(t_2) > V_{ND}^A(t_1)$.

6.3 Proposition 1 for the case of C(t)$\in \{W, L, X.3, X.2\}$

A.0. Delegation

Note that $\frac{\partial V_D^A(t)}{\partial t} = \frac{1}{6}t - C'(t) \geq 0$ for t s.t. $\frac{t}{6} \geq C'(t)$ while otherwise $\frac{\partial V_D^A(t)}{\partial t} < 0$. The maximum of $V_D^A(t)$ is thus characterized.

A 1. Some details on the Expected payoff functions under authority

Note that

$$\frac{\partial V_{ND}^i(t,N)}{\partial N} = \frac{1}{6N^3}\left(t^2 - 4N^4b^2\right) > 0 \text{ for } N < \frac{1}{2}\frac{\sqrt{2}}{b}\sqrt{bt} \text{ for i} \in \{A, P\} \tag{31}$$

And note that $\frac{1}{2}\frac{\sqrt{2}}{b}\sqrt{bt} > N(b,t) = -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2t}{b}}$. It follows that both the Agent and the Principal prefer the equilibrium with the largest possible number of partitions. Now, note that $N(b,t)$ is logically increasing in t and decreasing in b, as seen from the figure below. Defining the function $M(X,t)$ as the maximal value of b allowing for an equilibrium with a number X of partitions, given information acquisition t, one obtains:

$$M(X,t) = 2\frac{t}{(2X+1)^2 - 1} \tag{32}$$

Which is represented below. The higher the line, the lower the value of X that it corresponds to:

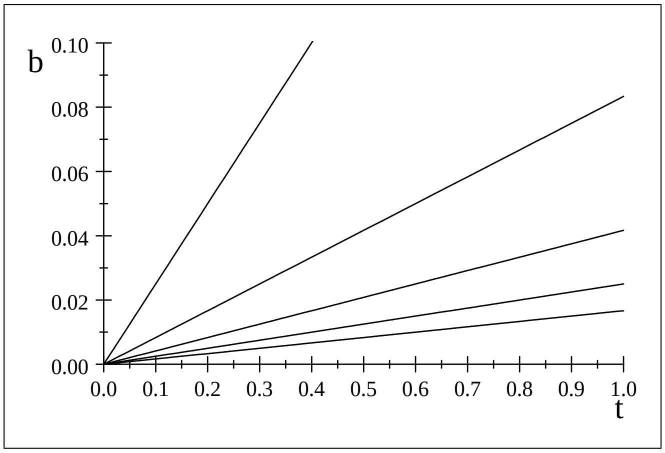

Figure 7

A.2. Approximations of the Expected Payoff function of the Agent under authority

The characterization of the Agent's optimal information acquisition under Authority is difficult given the complec nature of the payoff function for this case. This follows from the fact that the discrete number of partitions N depends on t, and is a discontinuous function of t. We however here present closed form formulas for approximations of the payoff function of the Agent under Authority and examine the optimal value of t that is chosen under these payoff functions. Note that the value of $V_{ND}^{P}(t,N)$ is very well approximated by the following two functions:

$$\overline{V}_{ND}^{A}(t) = V_{ND}^{A}(t, \frac{1}{2}\frac{\sqrt{2}}{b}\sqrt{bt}) = -\frac{2}{3}b^2 - \frac{1}{3}bt + \frac{1}{12}t^2 - \frac{1}{12} - C(t) \tag{33}$$

$$\underline{V}_{ND}^{A}(t) = V_{ND}^{A}(t, -\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2t}{b}}) = -b^2 - \frac{1}{3}bt + \frac{1}{12}t^2 - \frac{1}{12} - C(t) \tag{34}$$

Where $\overline{V}_{ND}^{A}(t,.)$ represents an upper bound on the loss of the Agent given an informational investment of t, while $\underline{V}_{ND}^{A}(t)$ is a lower bound. Indeed, $\frac{1}{2}\frac{\sqrt{2}}{b}\sqrt{bt}$ is the Agent's prefered number of partitions while $-\frac{1}{2} + \frac{1}{2}\sqrt{1+\frac{2t}{b}}$ is a lower bound on the maximum number of possible partitions at equilibrium, given t and b. Note that the Agent's utility is continuously increasing in N until $N = \frac{1}{2}\frac{\sqrt{2}}{b}\sqrt{bt}$.

Note that $\overline{V}^{A}_{ND}(t) - \underline{V}^{A}_{ND}(t) = \frac{1}{3}b^2$, so that the true value of the Agent's payoff for information t is no more than $\frac{1}{3}b^2$ from the values $\overline{V}^{A}_{ND}(t)$ and $\underline{V}^{A}_{ND}(t)$. Note furthermore that the loss function $-Ib^2 - \frac{1}{3}bt + \frac{1}{12}t^2 - \frac{1}{12}$, for $I \in \left\{-\frac{2}{3}, -1\right\}$ is convex and first decreasing in t and subsequently increasing in t.

And note now that:

$$\frac{\partial \overline{V}^{A}_{ND}(.)}{\partial t} = \frac{\partial \underline{V}^{A}_{ND}(.)}{\partial t} = -\frac{1}{3}b + \frac{1}{6}t - C'(t) \tag{35}$$

The two bounds on $V^{P}_{ND}(t)$ thus have the same derivative and differ only by very little. Now, note that $\frac{1}{6}t - C'(t)$ is first increasing up till $\widetilde{t}$ such that $\frac{1}{6}\widetilde{t} - C'(\widetilde{t})$ and then decreasing in t. Furthermore, $\frac{1}{6}(0) - C'(0)$ so that $\frac{\partial \overline{V}^{A}_{ND}(.)}{\partial t}$ is first negative. To summarize, $\overline{V}^{A}_{ND}(.)$ and $\underline{V}^{A}_{ND}(.)$ can be expected to either be continuously decreasing or alternatively be first decreasing, then increasing, and finally decreasing again. Thus, the two potential maxima of the function are given by:

$$t = 0 \text{ or t} = \max_{t \in [0,1]} t \text{ s.t.} -\frac{1}{3}b + \frac{1}{6}t - C'(t) = 0 \tag{36}$$

A.3 Some properties of the Expected cost function

Suppose a given b. This bias defines a partition $\left\{\tau_0, \tau_1, .., \tau_{N(b,1)}\right\}$ over the interval $[0,1]$, such that for $t \in [\tau_{z-1}, \tau_z]$, $N(b,t) = z$, where z is an integer. Note that τ_z solves:

$$z = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2\tau_z}{b}} \tag{37}$$

Now, note that:

$$\text{Property 1:} \qquad V^{A}_{ND}(\tau_{z-1}, z-1) = V^{A}_{ND}(\tau_{z-1}, z) \tag{38}$$

$$\text{Property 2:} \qquad V^{A}_{ND}(t, z) > V^{A}_{ND}(t, k), \text{ if } t \in [\tau_{z-1}, \tau_z] \text{ and for every integer } k \neq z \tag{39}$$

A.4 A graphical representation of the payoff functions of A and P

Below, we represent the payoff function of the Agent (which is a concatenation of different payoff functions $V^{A}_{ND}(t, N)$ for different values of N) and its approximations for a cost function given by $C(t) = k(\ln(1-t)+t)$, $k = 0.03$ and $b = 0.05$. For each interval, marked by the vertical dashed curves, a different maximal number of partitions may be sustained, i.e. 1, 2 or 3. In each interval, the payoff function $\mathrm{V}^{A}_{ND}(\mathrm{t})$ corresponds to the highest of the three thick continuous curves. Finally the approximations of $\mathrm{V}^{A}_{ND}(\mathrm{t})$ are given by the continous dashed curves. Note that in this example, these approximations are very accurate.

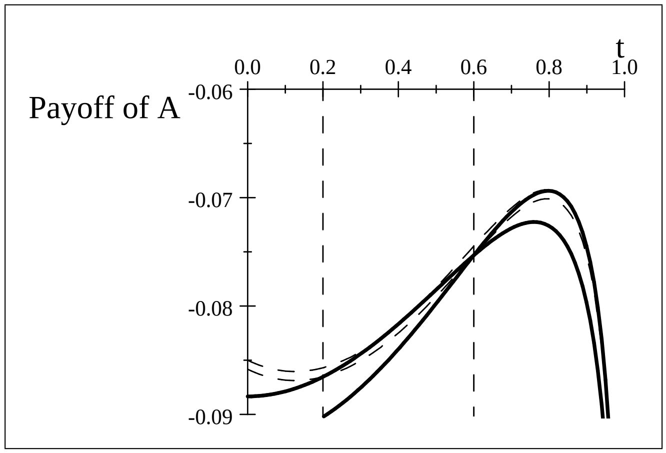

Figure 8

A.4. Proof of point a), showing that t_1 satisifies $\frac{t_1}{6}\left(\frac{N(b,t_1)^2-1}{N(b,t_1)^2}\right)-C'(t_1)$ for some $N\leq N\left(b,1\right)$

We prove this by contradiction. Suppose that the global maximum t_1 does not satisfy this condition. Note that the function $\frac{t_1}{6}\left(\frac{N(b,t_1)^2-1}{N(b,t_1)^2}\right)-C'(t_1)$ is humpshaped and thus has a unique maximum solving the condition given above. Thus, the condition may not be respected if, for a given N, t s.t $\frac{t}{6}\left(\frac{N^2-1}{N^2}\right)-C'(t)$ is such that $N(b,t)\neq N$. There are thus two possibilities, which we call case A and case B: either t_1 is maximal, given $N(b,t_1)$, or it is minimal. In the first case, $V(t,N(b,t_1))$ is upwards sloping at the point t_1, while in the second case, it is upwards sloping at that point. In the first case, thie implies that $t_1=\max\{t\in[0,1]$ s.t $N(b,t)=N(b,t_1)\}$ while in the second case, it must thus be that $t_1=\min\{t\in[0,1]$ s.t $N(b,t)=N(b,t_1)\}$. In case A, either $N(b,t_1)=N(b,1)$ (subcase 1) or $N(b,t_1)<N(b,1)$ (subcase 2).In subcase 1, it must be the case t hat $t_1=1$. In subcase 2, we however know that $V_{ND}^A(t,N(b,t_1)+1)\geq V_{ND}^A(t,N(b,t_1))$ for $t\geq t_1$. This contradicts the fact that t_1 is a global maximum. In case B, either $N(b,t_1)=2$ (subcase 1) or $N(b,t_1)>2$ (subcase 2). In subcase 1, we however know that $V_{ND}^A(t,1)\geq V_{ND}^A(t,2)$ for $t\leq t_1$ which contradicts the fact that t_1 is a maximum. Similarly, in subcase 2, we know that $V_{ND}^A(t,N(b,t_1)-1)\geq V_{ND}^A(t,N(b,t_1))$ for $t\leq t$, which contradicts the fact that t_1 is a global maximum. This concludes the proof.

A.5. Proof of point b)

Step 1: Suppose a bias b of the Agent. This bias defines a partition $\left\{\tau_0,\tau_1,..,\tau_{N(b,1)}\right\}$ over the interval $[0,1]$, such that for t$\in[\tau_{z-1},\tau_z]$, $N(b,t)=z$, where z is an integer. Now, for every integer z, we examine the function $V_{ND}^A(t,z)$. Note that for each value of z, $V_{ND}^A(t,z)$ has a unique maximum for $t=t_z$ such that $\frac{t_z}{6}\left(\frac{z^2-1}{z^2}\right)=C\prime\left(t_z\right)$. Note also, that given the convexity of $C(t)$, $t_{z\prime}>t_z$, for $z\prime>z$.

In what follows we use $V(t,z)$ instead of $V_{ND}^A(t,z)$ for notational simplicity. Now, suppose that t_k, for some $k\leq N(b,1)$, is such that the Agent decides to acquire an amount of information t_k in equilibrium, ensuring k partitions in the ensuing communication equilibrium. This means that t_k ensures the highest possible payoff to the Agent. We now show that this implies $V(t_k,k) > V(t_z,z)$, $\forall z \neq k$, $z \leq N(b,1)$. Suppose that $z < k$. This implies that $t_z < t_k$. Now, if $t_z \in [\tau_{k-r-1}, \tau_{k-r}]$, for some integer r such that $0 \leq r < k$, we know that $V(t_z,z) < V(t_z, k-r) < V(t_k,k)$. The first inequality follows from Property 2 and the second inequality follows from the definition of $V(t_k,k)$. Finally, suppose that $t_z \in [\tau_{z-1}, \tau_z]$. It is then immediate that $V(t_z,z) < V(t_k,k)$. We may reproduce the same arguments for any $z > k$ such that $z < N(b,1)$. We may thus conclude that $V(t_k,k) > V(t_z,z)$, $\forall z \neq k$, $z \leq N(b,1)$.

Step 2: Now, suppose that the Agent's bias decreases from b to $b\prime < b$. In what follows, we adopt the notation $V(t,z,b)$ for the function $V^A_{ND}(t,z)$, when the Agent has a bias b. Note that for every $z < N(b,1)$, $V(t,z,b\prime)$ is maximized at the same point as $V(t,z,b)$, respecting the condition $\frac{t_z}{6}\left(\frac{z^2-1}{z^2}\right) = C\prime(t_z)$. Furthermore, note that:

$$V(t_z,z,b\prime) = V(t_z,z,b) + \frac{(b^2-(b')^2)z^2}{3} \tag{40}$$

Suppose now that for some $k \leq N(b,1)$, t_k was the equilibrium information acquisition of the Agent under Authority, giving rise to k partitions. Define furthermore the following objects:

$$\Delta_{kz}(b) = V(t_k,k,b) - V(t_z,z,b) \tag{41}$$

$$\Delta_{kz}(b\prime) = V(t_k,k,b\prime) - V(t_z,z,b\prime) = \Delta_{kz}(b) + \frac{(b^2-(b)^2)(k^2-z^2)}{3} \tag{42}$$

It follows immediately that $\Delta_{kz}(b\prime) > \Delta_{kz}(b)$ if $k > z$ while $\Delta_{kz}(b\prime) < \Delta_{kz}(b)$ if $k > z$. This implies that if $V(t_k,k,b) > V(t_z,z,b)$, then $V(t_k,k,b\prime) > V(t_z,z,b\prime)$, if $z < k$. On the other hand, if $k < z$, it may be that $V(t_k,k,b) > V(t_z,z,b)$ while $V(t_k,k,b\prime) < V(t_z,z,b\prime)$. Call t_l the optimal information amount acquired under bias $b\prime$ and let l be the associated number of partitions. Now, we know from step 1 that the optimal information amount t_l chosen under bias $b\prime$ will be such that $V(t_l,l,b\prime) > V(t_z,z,b\prime), \forall z \neq l$ s.t $z \leq N(b\prime,1)$. Knowing that $V(t_k,k,b\prime) - V(t_z,z,b\prime)$, if $z < k$, while it may be that $V(t_k,k,b\prime) < V(t_z,z,b\prime)$ for $k < z$, it thus follows that $t_l \geq t_k$.

A.6. Proof of point c)

This follows directly from point b).

A.6. Proof of point d)

This follows immediately from the proof of point b)

A.7.Comparing Authority and Delegation:

Note that t_1 satisfies the condition that $\frac{t_1}{6}\left(\frac{N^2-1}{N^2}\right) = C'(t_1)$ while t_2 satisfies $\frac{t_2}{6} = C'(t_2)$. It follows automatically, given that $\left(\frac{N^2-1}{N^2}\right) < 1$ for any $N > 0$, that $t_2 > t_1$, under the convexity assumption made about the cost function. Note furthermore that $V^A_D(t_2) > V^A_D(t_1)$ and $V^A_D(t_1) > V^A_{ND}(t_1)$. It follows by transitivity that $V^A_D(t_2) > V^A_{ND}(t_1)$.

6.4 Notes on example 1

Note that the optimal information level acquired under Authority given the approximation functions that we use solve $\frac{t}{6} - 3kt^2 - \frac{b}{3} = 0$, yielding $\widetilde{t}_1 = \frac{1}{36k}\left(\sqrt{-144bk+1}+1\right)$. Now, inserting this formula into these approximation functions, we obtain the following:

$$\overline{V}^A_{ND}(\widetilde{t}_1) - (-\frac{1}{12} - b^2) = \frac{1}{46\,656k^2}\left(15\,552b^2k^2 + 3\sqrt{1-144bk} - (1-144bk)^{\frac{3}{2}} - 432bk - 432bk\sqrt{1-144bk} + 1\right) \quad (43)$$

$$\underline{V}^A_{ND}(\widetilde{t}_1) - (-\frac{1}{12} - b^2) = \overline{V}^A_{ND}(\widetilde{t}_1)\frac{1}{46\,656k^2}\left(\sqrt{1-144bk}+1\right)\left(\sqrt{1-144bk} - 288bk + 1\right) \quad (44)$$

Obtaining the following conditions for these functions being nill:

$$-1492\,992b^3k^3\,(162bk-1) = 0 \Leftrightarrow b = \frac{1}{162k} \quad (45)$$

$$-432bk\,(192bk-1) = 0 \Leftrightarrow b = \frac{1}{192k} \quad (46)$$

Which yields the bounds on $\overline{b}$ given in example 2.

6.5 A remark for a particular class of concave cost functions

We here prove a particular feature of this game for a particular class of polynomial concave functions. For functions of the form $C(t) = kt^{\alpha}$, for $\alpha \leq 1$, the optimal CDM leads the Principal to choose Authority for any $b < \widetilde{b}$. This contrasts with the case of quadratic loss functions of the form $C(t) = kt^2$, as shown in the above given proof.

Remark 2 ***A particular class of concave cost functions***

Suppose $\frac{1}{6} > C(1) > \frac{1}{12}$ and $C(t) = kt^{\alpha}$, for $\alpha \leq 1$. In the Conditional Delegation game with commitment, the Principal always chooses Authority for any $b < \widetilde{b}$, where $\widetilde{b} = \sqrt{C(1) - \frac{1}{12}}$.

Proof: First, note that when choosing among $V^P_D(b,t)$ and $V^P_{ND}(0,1)$, the Principal only chooses Delegation if $t \geq \sqrt{12}b$. We thus simply show that $V^A_D(\sqrt{12}b) < V^A_{ND}(0,1)$ given that $C(t) = kt^{\alpha}$, for $\alpha \leq 1$. The inequality condition is equivalent to the condition that:

$$2b^2 < C(\sqrt{12}b) \Leftrightarrow b < \left(\frac{k(12)^{\frac{\alpha}{2}}}{2}\right)^{\frac{1}{2-\alpha}} \quad (47)$$

Now, note that $\left(\frac{k(12)^{\frac{\alpha}{2}}}{2}\right)^{\frac{1}{2-\alpha}}$ is increasing in k and α. Note finally that $k = \frac{1}{6}$ and $\alpha = 1$, $\left(\frac{k(12)^{\frac{\alpha}{2}}}{2}\right)^{\frac{1}{2-\alpha}} = \frac{1}{\sqrt{12}}$. This proves that for $b < \frac{1}{\sqrt{12}}$, $V^A_D(\sqrt{12}b) < V^A_{ND}(0,1)$. Remembering that $\widetilde{b} < \frac{1}{\sqrt{12}}$ under the assumption that C(1)<$\frac{1}{6}$, implying k<$\frac{1}{6}$, the conclusion follows immediately.■