Endogenous choice of electoral rules in a multiparty system with two major dominant parties

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Abstract

In this paper we develop a basic model of endogenous choice of electoral rules in a multiparty system with two major dominant parties, that have a positive probability to win the election, in an environemnt of uncertainty about the outcome of the election (adding extra non-dominant ones does not alter the results). Using quasi-lexicographic preferences over the number of seats necessary for a party to form a single-party government (i.e win the majority of the seats in the parliament and pass legislation at will) we explore the choice of the electoral law by the parties. We show that the minor parties never agree to an electoral reform that moves away from the Proportional Representation system (PR). We also show that in the case where the electoral competition among the two dominant parties is non-trivial there exists a unique and stable equilibrium were a unique new electolar rule is being adapted by the parliament in substitution of the pre-existing PR rule. That is we show that there exist strategic incentives that drive the two dominant parties to collude in the presence of uncertainty about the outcome of the elections. The key factor that drives the result is the desire of the parties to secure the majority of the seats in the parliament and not just win the upcoming election. Hence by agreeing to distort the electoral rule in their favor not only they increase their utility (by increasing the number of seats of the potential winner) but they also increase the probability that the winner will form a single-party government. The paper in effect shows that under an uncertain political environment the two dominant parties have an incentive to collude in favor of stability (single-party governments) and practically change the political environment from multi-party to a two-party system (by eliminating the effect the third party has on the formation of the government). To conclude we also show that the equilibrium with the above characteristic does in fact exist and it is also unique and stable. In an extension of this model we relax the assumption of common priors about the strength of the dominant parties and we explore the role of electoral reform as a signalling mechanism.

1 Introduction

A crucial factor in all representative democracies is not only the outcome of any electoral process but also the possible distribution of seats in the legislative body based on the outcome of that procedure. It is obvious that the stability of any single-party government (or governmental coalition) critically depends on the number of seats allocated to the winning party according to some previously chosen electoral law (Blais, 1991). Furthermore it is this allocation of seats that matters the most: a party that wins say 51% of popular votes and receives 51% of the seats in a parliament using a proportional representation electoral law is less stable than a "minority government" that captures say 45% of the popular vote but according to some "favorably distortional" electoral law (such as mixed or plurality electoral laws) is allocated 60% of the available parliamentary seats (Blais, 1991). It can well be argued that such electoral laws could be deemed as unfair by societies and hence there is no need to worry about such incidences. Yet a closer inspection of almost all democracies in the world reveals that in some of them other electoral rules than Proportional Representation (PR hereinafter) are actually used to determine the allocation of parliamentary seats (Norris, 1997).

With this observation in mind an interesting question arises: how and which electoral laws do parties choose given their expected share of popular vote in the forthcoming electoral competition. That is to say we try to endogenize the choice of electoral rules: rather than arguing in a Duvergerian manner (Duverger, 1954) that it is some exogenous, possibly predetermined by some sort of constitutional arrangement, electoral rule that is responsible for shaping the political environment (i.e. the number of parties) we turn this assumption upside down (Colomer, 2005). In fact we construct a model where the choice of the electoral rule stems endogenously as a strategic choice of office-seeking parties in their attempt to win the majority of the seats in the parliament and form single–party governments. The intuition behind this idea is that the strategic incentives of dominant parties to control the parliament and pass legislation at will is enough to make them collude and propose electoral laws that distort the PR rule. This should also be true especially under an environment of uncertainty with respect to the outcome of the election, that is when the elections are contested by two at least parties with a positive probability of winning (Andrews-Jackman, 2005). Furthermore it is interesting to derive conditions that determine the optimal choice of an electoral law based on the expected outcome of the election. Another closely related issue is that of stability of electoral laws. Does it exist an electoral law that is stable that is once chosen by any parliament no other party will have an incentive to change that law, or even if it has it will not have the necessary majority to do so. In our 2-dominant party model we shall prove that this is in fact the case. And if such an electoral law does exist is it also the most efficient in terms of providing countries with stable, that is single-party, governments (Blais, 1991). But before entering into the specifics of the model in detail it is instructive to give a summary of previous results and some empirical observations.

2 Literature Review and Motivation

Recent literature in political science suggests that the existing variation in electoral rules and regimes across advanced democracies is due to the strategic decisions that the current ruling parties make, anticipating the coordinating consequences of different electoral rules, to maximize their representation in the legislature (or even form a single party government) according to the following conditions: If the electoral arena is considered to be fairly stable and if the political environment and the existing electoral law serves the current ruling parties (it might be the case that two parties switch turns in office regularly) then they have limited or no incentives to modify the current electoral law. However if they political arena changes due to an anticipated entry of new voters (or change in their preferences) the ruling parties will consider changing the current electoral law depending on two conditions such as the emergence of new parties and the coordinating capabilities of the old parties. If the new party is strong then the old parties shift from plurality/majority rules to PR only if they are locked into an equilibrium where no old party enjoys a dominant position (non-Duvergerian equilibrium, i.e. most of small European states), whereas they do not (they stick to majority/plurality rule) if one old party enjoys a dominant position (UK). Similarly whenever the new entrants are weak, an non-PR electoral system is maintained regardless of the structure of the old political system (USA). This argument is made in full clarity by Boix (1999), and is in accordance with the predictions of our theoretical framework were we set out to prove that it is the nature of the political competition and the structure of the political environment that determines the endogenous choice of electoral rules by the parties. To put in the most simple language possible we could say that our model predicts that in bipartisan systems or in systems with two major dominant parties the two major parties will collude, if of course their incentives align according to some conditions that we will later on address in full detail, and a majoritarian non-PR electoral rule is being chose. On the contrary if the system is characterized by the existent of many parties that have a positive chance of wining the election then PR electoral rule is chosen.

In accordance with this literature Benoit (2004) adapts the argument made for endogenous electoral rule choice and proposes a theoretical framework that examines patterns of electoral system change, that is to say they develop a model that studies the endogenous choice of electoral rules by the parties. After surveying the literature in order to group the typology of electoral system change he proposes a model that derives conditions of endogenous electoral system change my rational seat-maximizing political parties. His theoretical model predicts that electoral rule change occurs endogenously when two conditions are met: first if it exists a coalition of parties willing to agree to change the existing rule with an alternative such that each of these parties is expected to gain more seats under the newly chosen electoral rule. And second if they parties in the coalition can master enough votes in the current parliament in order for this change to take into effect. Those results are very close to the theoretical predictions of our model but we diversify ourselves in two directions. First we explicitly state the conditions that allow the two dominant parties to collude in an electoral rule change from PR to a more majoritarian one: we identify that the incentive of the dominant parties to collude stems from their desire to form single party governments and take full advantage of the "spoils of the office" and then uncertainty over the outcome of the coming election allows those incentives to align and hence gives room to collusion.

Secondly, our paper does not require both parties that participate in the coalition that votes in favour of an electoral rule change to be part of the government. In fact our paper states quite the opposite: it is the desire of the two dominant parties to form a single-party government that allows them to collude. The basic difference is that although in expected terms in both models all parties in favour of the electoral rule change must score seat (i.e. utility) gains on expected terms, in our model this is not true in real terms. Our model predicts that the if the two parties collude to change the PR rule, in the end only one of them will form a single party government and enjoy the spoils of the office.

In a parallel attempt Colomer (2005) presents and tests the hypothesis that it is in fact the number of the parties that can explain the choice of electoral systems, rather than the other way around. He argues that already existing political parties tend to choose electoral system that tend to crystallize, consolidate and reinforce the existing party system rather than change it dramatically. He also concludes that political systems that are dominated by a few parties tend to establish majority rule electoral systems (or rules that distort the PR rule in general) whereas multi-party systems already existed before the introduction of PR. Our paper formalizes this idea by showing explicitly how the strategic incentives of the two dominant parties align in order to crystallize the existing 2-dominant party system and increase the probability of occurrence of stable governments (that is governments that have the majority of seats in the parliament). Furthermore our model extends to show the choice of electoral law is a strategic choice of the parties that is driven by their desire to increase their probability of forming a single-party government that has the ability to pass legislation at will (given of course some constitutional limitations). Hence our paper builds on those stylized facts and empirical evidence in order to create a simple theory of how and why parties choose the electoral laws. We also explore whether the choice of the electoral law increases the stability of the political system and whether there exists a unique electoral law (equilibrium) for a given political environment.

Finally in the opposite direction of the electoral rule change Ergun (2007) studies the change of electoral rule from plurality to PR. He finds that for office motivated rational seat maximizing parties the following conditions have to be met for the change to take place: first, the government must be formed by a coalition, that is more than one party must share the spoils of the office. And secondly the larger the number of parties and the more equitable the share of the spoils amongst the more likely is for the change to a PR rule to take place. That is Ergun (2007) starting from the opposite direction as compared to our paper (where PR is the status quo) when examining the change of electoral rules echoes the same counter-Duvergerian approach: that it is the nature of the political environment and the party structure that determine the choice of electoral rules. In this set up a fractured multi party political system where parties share the spoils of the office in an equitable way makes the adaptation of PR rule more likely. In fact this the same link observed in the literature but just going the other way around, hence confirming our intuition.

The above literature is in line with our preliminary results. First of all it is true that the decision to change the electoral law is driven by the strategic incentives to increase the representation of the dominant (leading) parties in the parliament and in fact to secure the ability of forming a single-party government by securing the majority of the seats (which under non-PR systems this does not necessarily implies that they win the majority of the popular vote). Further more it gives a theoretical reasoning to the uniqueness and the

stability of the equilibrium. Since the two leading parties decided that their incentives align and depart from the PR system then they choose one that guarantees single-party governments (utility maximization) and they do not depart from this unless the political environment changes. But even if the political environment changes this change has to be significant enough in the sense that the new entrant parties have to threaten the dominant role of the leading ones. If they fail to do so as we assume in the first part of the paper then the unique and stable equilibrium is either no change of PR rule (if they cannot coordinate) or a distortion of the PR rule that guarantees stable governments. In fact our model shows that the dominant parties use the electoral law as a strategic option in order to guarantee their dominance and maintain the current structure of the political environment (that is the 2-dominant party system). Hence our model turns Duverger upside down since his argument was that the electoral law is responsible for the shape of the party system. In our theoretical model we provide a theoretical explanation for the opposite.

In what follows we will try to develop a simple model of electoral competition that tries to endogenize the choice of electoral laws by parliamentary parties. In practice there are 3 broad categories of possible electoral laws in wide use in most democratic societies: PR rules, majority rules and mixed systems. This paper will center its comparative analysis on two of these categories of electoral systems. That is we will examine Proportional Representation (PR) and Mixed systems (which prove to be the most interesting ones). Since Mixed electoral systems are a combination of elements from the other two we will examine the most interesting variation of mixed electoral laws: the Bonus System where the largest fraction of the seats are again awarded on a PR basis and the remaining ones are given as a bonus to the party that finishes first in the electoral competition.

Here it becomes imperative to equip our analytical arsenal with a fact that stems out from empirical observation. It is widely observed[1] that PR electoral systems go hand in hand with multi-member electoral districts whereas Plurality systems require single-member districts (Norris 1999 and Blais 1991). Of course mixed systems use a combination of single-member constituencies and larger multi-member districts. The reason that this diversification becomes crucial is that the size of an electoral district is positively correlated with the cost of running an electoral campaign. It has been empirically observed and argued by many political scientists (Blais, et al. 1991) the fact that the cost of running elections increases with the size of the electoral district since larger districts implies larger constituencies and hence higher costs for candidates to reach out to citizens and gain votes for their party. Furthermore in multi-member districts apart from partisan competition with other party's candidates there is also a salient intra-partisan competition: in multi-member districts many candidates from the same party contest the election hence they must also spend additional funds not only to persuade voters to vote for their party but to also gain their first preference vote in order to get elected from the party list (with the implicit assumption that is highly unlikely with a PR system for a party to gather almost all votes in the district and hence capture all the available seats).

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[1]The only exception known to the authors is that of the US Presidential elections where plurality rule is implemented in multi-member districts and the winner of the plurality of the votes carries all the electors of that State for the Presidential election. Yet this pattern even inside the US is only observed in Presidential elections and it can be attributed to some part in the parliamentary history of the US. Even in Congressional and Senatorial elections US applies a plurality electoral law but in single-member electoral districts.

In sharp contrast with multi-member districts, under plurality electoral laws we observe single-member districts where "the winner takes it all" rule simply implies that the candidate of the winning party wins the race. Given that electoral competition in single-member districts is less fierce and hence less costly we can assume that the cost parties have to suffer in order to contest an election under a plurality system is significantly smaller than with PR systems (Blais 1991). Of course since mixed systems use a combination of multi and single member districts we can expect that the cost of running elections under such systems must lie somewhere between those other two systems and we can also hypothesize that the cost in mixed systems rises proportionally with the fraction of seats allocated under a PR rule. We abstain from making a more detailed analysis regarding the cost of running elections and for now we shall only note that the cost of running elections rises with the increase in the level of proportionality of the chosen electoral rule and vice versa.

There is also another important point to be raised. If there exist for some party or parties an electoral law that is efficient for them (in terms of allocating them the maximum possible seats given their share of popular vote) then those parties should always choose that favorable electoral law. But this in turn will send a signal to citizens and other parties about each party's expectation over the place that it will take in the election to come. Hence in an extension of the basic model where we blend signaling and electoral competition we will allow other parties and a fraction of citizens to determine their actions based on the signals the parties send with respect to their expectations of the election outcome. That is a fraction of voters will always vote for the party that is expected to win and this will add another strategic dimension on the party's choice of an electoral rule since a particular choice might be ex ante optimal given the expected seat allocation but it might not be optimal ex post given the signal that this choice will send to the public and its competitors. Hence even if each party has a distinct choice of electoral law given their expected share of vote it might be profitable to them to pool with other parties in order to avoid sending the wrong signal to the voters.

In a related but not identical analysis Palfrey (1984) considers a two dominant-party spatial competition model where parties are in rational anticipation of a third party entry. But Palfrey's results relate to the position the two dominant parties occupy in the political spectrum. Our analysis is distinct since we focus our analysis on the choice of electoral rule made by the two dominant parties given their expected vote share. This extra feature of the model combined with the signaling approach might lead to some interesting explanations concerning the prevalence of specific electoral laws in societies where there is significant political fragmentation and many old parties cease to exist and new entrants appear in the political scene.

Finally we will review the normative notion of stability, a property that is desirable for an electoral law to exhibit. Our definition of stability is slightly different from what has prevailed in current literature. We consider an electoral rule to be stable when it allocates seats in such a way that it always "produces" stable single-party governments. With respect to stability, the previous discussion hints that in settings of multi-party competition different parties might prefer distinct electoral laws given their expected vote share. But their choice might not be stable given that this choice will send a signal to voters which in turn determine their vote based on the expected winner. Hence endogenously chosen electoral rules might fail to be stable, that is some societies might exhibit substantial political fragmentation

either resulting in adapting the PR rule or resorting in frequent electoral rule changes that fail to deliver stable single-party government. And as we link the notion of efficiency with government stability it is interesting to note whether endogenously chosen electoral rules are indeed efficient.

On a separate and final note regarding equity and fairness considerations it might seem obvious that PR electoral laws are more fair in terms of allocating to each party a fair share of seats given their vote share. But there is an implicit conflict between government stability and fairness in allocating the parliamentary seats. Hence mixed electoral systems will be on the centre of focus of this paper given that the political argumentation following their introduction is based on their alleged (yet not formally proven) ability to combine those two properties. After providing readers with the basic concepts and ideas around which the paper evolves it is now appropriate to introduce the formal underpinnings of our simple model before proceeding with more extensive analysis and some basic results.

3 The Model

3.1 Basic Ingredients

Our model is one of parliamentary democracy with three parliamentary parties where two of them are the dominant ones (that is they have a positive probability of winning the election) whereas the third one is a minority party with zero probability of running first in the forthcoming election. Of course we should clarify that winning an election in our model simply means collecting more votes as compared to the other dominant party. This is something we diversify from the concept of wining enough votes to secure the majority of the seats in the next parliament. The transformation of popular vote into parliamentary seats is dependant on the implemented electoral rule which is turn chosen in advance by the parties (through a parliamentary voting procedure). In general there are three broad categories of electoral rules: proportional representation (PR), majoritarian and mixed (that combine elements of both). In the political science literature PR rules are associated with fairness whereas majoritarian ones with stability. Hence there is a trade-off between fair representation of the popular vote in the parliament and the stability of the government. Since mixed electoral systems were devised as an attempt to bridge the gap those will be the main focus of analysis in this paper. In particular we will be examining the bonus system which allocates (usually) most seat according to a PR rule and the rest (known as the bonus) are given to the party that runs first in the election (regardless of the party winning the majority of the votes). Hence by employing a bonus system it is indeed possible to have parties failing to secure the majority of the popular vote (although they do win the election by running first) yet being able to secure the majority of the seats in the new parliament.

Formally we shall consider $N = \{1, 2, 3\}$ to be the set of parties involved in the electoral competition. All parties in N are assumed to be represented in the parliament. Each party $i \in N$ holds a proportion of seats in the preceding parliament s_i^0 such that $\sum_{i=1}^{n} s_i^0 = 1$. Moreover, parties have information about the vote share that each party is about to receive in the forthcoming elections. Party 3, shall be assumed to be a "minority" party (minority

referring to ideology or ethnicity), that is, its vote share shall never exceed the vote share of party 1 or this of party 2. The expectations on future vote shares are formed by information that is commonly available to all parties. The vote share of the "minority" party 3, shall be assumed to be fixed at a level v_3, whereas, the vote shares of the other two parties will be subject to uncertainty. Formally, the vote share of party 1 in the coming elections will be modeled as a random variable:

$$v_1 \sim \Phi_1,$$
where Φ_1 is a uniform distribution in $[a_1, b_1] \subset [0, 1]$.
Equivalently,
$$v_2 \sim \Phi_2,$$
where Φ_2 is a uniform distribution in $[a_2, b_2] \subset [0, 1]$.

Notice that these expectations need to satisfy $\sum_{i=1}^{n} v_i = 1$ and $v_3 \leq v_i, \forall i \in N$. That is, both a_1 and a_2 are bigger than v_3, both b_1 and b_2 are smaller than $1 - 2v_3$ and $a_2 = 1 - b_1 - v_3$ and $b_2 = 1 - a_1 - v_3$. The proportion of seats of party $i \in N$ in the new parliament will be defined as $s_i^l(v_i)$, where l will be the applied electoral law. As stated in the previous parts of the paper, the particular electoral laws that we are interested in are the first party bonus systems. That is, l shall be the proportion of parliament seats that are allocated to the "first" party as bonus. Obviously, $l \in [0, 1]$ and, thereafter:

$$s_i^l(v_i) = v_i(1 - l) \text{ if } v_i < \tfrac{1 - v_3}{2}$$
and
$$s_i^l(v_i) = v_i(1 - l) + l \text{ if } v_i > \tfrac{1 - v_3}{2}.$$

Obviously the third party given the assumptions stated above will never be entitled to the bonus as a result of never winning the elections. The above seat allocation mechanism with respect to the popular vote is a standard application of the mixed electoral rule. The first component is the proportional allocation of the seats minus the bonus whereas the latter part is the bonus given to the winner of the election.

The utility of a party $i \in N$ shall be defined as:

$$u_i(v_1, v_2, v_3, l) = g s_i^l(v_i) + (1 - g),$$
where $g \to 0$ if $s_i^l(v_i) > 1/2$ and $g = 1$ if $s_i^l(v_i) \leq 1/2$.

That is, parties in this environment are clearly office motivated. They care about the proportion of the seats they hold, only in the case that they are in opposition or when they win but cannot form a single-party government and when they can, they mainly receive rents from being in office. In a sense this type of preferences can be described as "quasi-lexicographic" if we think of the ability of forming a single-party government - by securing the majority of parliamentary seats- as a different "good" from just "simple" parliamentary seats that are below the necessary threshold. Hence the two dominant parties mainly care to "consume" the former good (that is to form a single party government) and only if they cannot do that they care for extra parliamentary seats in order to increase their share of

seats in the parliament. Put more formally, the marginal return of an extra parliament seat is infinitely larger when a party is in the opposition or has to negotiate with other parties to form a government than when the party has a complete control of the parliament.

This formulation of the utility function can be easily justified by following the political economy and political science literature. The formation of a single-party government allows the governing party to pass legislation at will (given of course some constitutional limitations) since it holds the majority of the seats in the parliament. Hence every proposed piece of legislation by the governing party to the parliament will have secured in advance its acceptance by the majority of the parliament which supports the single-party government. Of course there are examples in recent political history where single-party governments controlling the majority of seats in parliament were unable to pass some legislation (e.g. the Blair administration in its final years). Yet it must be noted that those cases are just few exceptions from the general rule: when a party controls the majority of the seats in a parliament and subsequently is able to form a single-party government it faces no limits in passing legislation other than constitutional ones. On the other hand when a party is in opposition or when it is first but not able to form a single-party government then the number of seats that possesses in the parliament increases its bargaining position in the negotiations that follow for the formation of a coalition government. The more MP's a party has the greater its role in the future government will be (e.g. more ministerial positions being awarded to its members). So this formulation of preferences allows us to capture this very intuitive idea which is commonly observed in current state of political affairs in most modern democracies.

3.2   The Game Structure

The game shall have the following stages.

(i) $l = 0$ (pure proportional system) and the party with the largest share of seats in the current parliament (either party one or party two) shall bear the role of the "proposer" of an electoral reform. That is, it shall propose $l \in [0, 1]$.

(ii) Parties shall vote on the "proposal" l and if the votes in favor of the reform surpass a given threshold $W \in [0, 1]$ (defined exogenously by the constitution, usually requiring a "super-majority"), then the electoral reform shall pass and the following elections will be conducted by the new law. In the opposite case, that is, if the proposal does not gather the necessary parliamentary support W, the electoral reform is cancelled and future elections are conducted according to the proportional system $l = 0$.

(iii) Elections take place, and each party, according to the results of public voting and the applied electoral law takes its new seat share and computes its utility.

3.3   Understanding the Proposer's Problem

For simplicity, let us for the rest of the paper assume that the proposer is always party 1. That is, we assume that $s_1^0 > s_2^0 > s_3^0$. Since the electoral law influences the seat shares of the parties and, thus, their utility after the coming elections, party 1 shall propose the

electoral law $l \in [0, 1]$ that given the threshold $W \in [0, 1]$ maximizes its expected utility. That is, if $W \leq s_1^0$ party 1 will propose $l \in [0, 1]$ such that $l = \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$. In other words, if the current seat shares of the proposer exceed the necessary amount of votes for an electoral reform, the proposer faces an unconstrained maximization program.

On the contrary, if $W > s_1^0$ the proposer needs the support of one (or more) of the other parties to proceed with a possible electoral reform. An obvious, but, nonetheless, useful corollary that can be stated at this point, demonstrates the behavior of the minority party 3 in such cases.

Lemma 1 *The minority party never consents to any electoral reform proposal*

Since the minority party expects to receive the bonus l with probability zero, it just expects utility losses from any distortion in the proportionality of the electoral system. Its expected utility from any electoral law is $Eu_3(v_1, v_2, v_3, l) = v_3(1 - l)$, which is, obviously decreasing in l.

Given the above observation, party 1 will have to secure party's 2 support in order to proceed with an electoral reform. That is it shall have to propose $l = \arg\max\{Eu_1(v_1, v_2, v_3, l)\}$ s.t. $Eu_2(v_1, v_2, v_3, l) \geq Eu_2(v_1, v_2, v_3, 0)$, facing, this time, a constrained (participation constrain of party 2) maximization program.

In general, since we have assumed that v_3 is fixed and common knowledge, and that $v_1 = 1 - v_2 - v_3$ the proposer faces one source of uncertainty (information about v_1 is equivalent to information about v_2). Thus, the proposer's expected utility shall be:

$$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1 - a_1)}\left[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1\right]$$

and, equivalently, party's 2 expected utility shall be:

$$Eu_2(v_1, v_2, v_3, l) = \frac{1}{(b_2 - a_2)}\left[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_2(1-l)+l]dv_2 + \int_{\frac{1/2-l}{1-l}}^{b_2} dv_2\right].$$

Notice that there exist two critical vote shares. The first one, $(1 - v_3)/2$, defines the necessary vote share so as for one of the two parties to be first (and get the bonus l). The second, $\frac{1/2-l}{1-l}$, is the vote share that that first party needs to have a one party majority in the parliament given an electoral law l ($s_i^l(v_i) > 1/2$). Obviously, if $(1 - v_3)/2 \geq \frac{1/2-l}{1-l}$ the first party will have the a majority of seats in the parliament independently of the exact vote share and, thus, the expected utility of party 1 shall be:

$$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1 - a_1)}\left[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1\right]$$

and party's 2:

$$Eu_2(v_1, v_2, v_3, l) = \frac{1}{(b_2 - a_2)}\left[\int_{a_2}^{(1-v_3)/2} v_2(1-l)dv_2 + \int_{(1-v_3)/2}^{b_2} dv_2\right]$$

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As $\frac{1/2-l}{1-l}$ is decreasing in l, an increasing distortion in the proportionality of the electoral law (an increasing l) does not only affect the potential seat gains of a party in case it runs first in the elections but as well, increases the probability of this party gathering a majority of seats in the parliament and hence forming a single-party government.

3.4 Definitions

We will classify the results given the following definitions.

Definition 1 *The electoral reform process is trivial if $W \leq s_1^0$*

In case the proposer (Party 1) has at present a large enough proportion of parliament seats so as decide the electoral reform at will, then we shall consider that the reform process is a trivial one, as it will just depend only on the preferences of the proposer.

Definition 2 *The electoral reform is possible if $W \leq 1 - v_3$*

This comes directly from the implications of corollary 1. The minority party 3 shall never consent to an electoral reform, thus, if a reform is to take place, W must be such that the minority party cannot block the reform.

Definition 3 *The electoral competition is trivial if $a_1 > \frac{1-v_3}{2}$ or if $b_1 < \frac{1-v_3}{2}$*

The above definition just describes the case in which the probability of party 1 running first in the coming elections is either 1 or 0. In such cases the "winner" of the elections, and, thus, the party that will take the bonus l is not subject to uncertainty. On the other hand, when electoral competition is non-trivial, both parties 1 & 2 have a positive probability of running first in the elections and grabbing the "bonus" parliament seats.

Definition 4 *The proposer (Party 1) is the "leading" party if and only if $\frac{a_1+b_1}{2} > \frac{1-v_3}{2}$*

If a party is expected to run first in the elections, then it shall be called the leading party. Since $E(v_1) = \frac{a_1+b_1}{2}$ and $\frac{1-v_3}{2}$ the vote share threshold above which a party 1 is the winner of the elections, Party 1 is the leading party if and only if the above inequality holds. Alternatively, Party 2 is the leading one.

4 Results

Assuming that an electoral reform is possible ($W \leq 1 - v_3$) we can state the following.

Proposition 1 *When both the electoral reform process and the electoral competition are trivial and the proposer is: (i)the leading party, then $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$, (ii) not the leading party, then $l^* = 0$*

Here, the leading party is a sure winner and the proposer can implement any electoral reform without the consent of any other party. That is, if the leading party is the proposer of the electoral reform, the proportionality distortion introduced by the l^* bonus electoral law will be such that it will guarantee to the winner, a majority of seats in the new parliament, otherwise if the proposer is not the leading party (i.e. expected to lose the election) it proposes that the proportional representation rule is not amended. And since it holds the necessary majority in the parliament the electoral rule does not change. This result can be viewed as the simplest case scenario.

The idea behind this proposition is very simple. Since the electoral reform process is trivial the proposer holds enough seats in the current parliament to pass any electoral rule that is it faces an unconstrained maximization program where there is no need to satisfy the participation constraint of party 2. Hence the proposer just chooses l^* in order to maximize expected utility. Given that the electoral competition is trivial party 1 proposes $l^* = 0$ when it is not the expected winner (i.e. not the leading party) and proposes $l^* \geq \max\{0, \frac{1/2-a_1}{1-a_1}\}$ otherwise. That is it proposes such a bonus that secures with certainty the majority of the seats in the new parliament when it is the leading party and expected to win the election. To see this consider that given that the electoral competition is trivial set $v_1 = a_1$ (that is the lowest possible outcome for party 1) and then solve for l^* such that $s_1^{l^*} = a_1 (1-l^*)+l^* > 1/2$. The solution to this unconstrained maximization problem yields to the leading party with certainty the majority of the seats in the parliament for every realization of the distribution Φ_1. Hence by choosing this level of l^* the leading party guarantees it self the ability to form a single-party government and get the highest possible level of utility. On the other hand if it is not the leading party any distortion of the PR rule will decrease its utility since it will never get the bonus l^* since its utility is strictly decreasing on l^*. This completes the argument.

Proposition 2 *When the electoral competition is trivial but the electoral reform process is non-trivial, then $l^* = 0$*

In this case, the two parties have to agree on the electoral reform and the proposer, is either a certain winner or a certain loser in the coming elections. In the first case, it is impossible to propose a bonus electoral law such that party 2 could consent in its implementation (since it just expect utility loss from proportionality distortion given that its utility is strictly decreasing in l^*) and in the later, the proposer is completely unwilling to distort proportionality because now the proposers utility is strictly decreasing on l^*. The idea that drives the result is that in this case there is no room for collusion since strategic incentives of the two dominant parties do not align because there is no uncertainty for the outcome of the electoral competition. The leading party will always prefer a value of $l^* > 0$ but the other party will always reject this proposal because its utility is strictly decreasing in l^*. Hence since the electoral reform is not trivial and requires the consent of both dominant parties it is obvious why no electoral rule reform is ever going to be accepted by this parliament. That is we are stuck to the status quo ante and the proportional rule persists as the electoral rule (i.e. $l^* = 0$).

Proposition 3 *When the electoral competition is non-trivial then $l^* \in \{0, \frac{v_3}{1+v_3}\}^2$.*

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[2]Proof in the Appendix.

This is the main result of the analysis. In an environment of imperfect information about the winning party of the coming elections, the distortion that the proposer might introduce in the proportionality of the electoral by the means of a bonus system l, shall be such that it will guarantee that the party that will run first in the elections will have a majority of seats in the parliament. That is, in case party 1, alone or with the support of party 2, promotes a bonus electoral rule, it will be at this extent so as to create an artificial "two party system" and solidify the current shape of the political system. This result is a summary of the results in the next proposition hence we will restrict further analysis of this main result in the section that follows to combine both propositions 3 and 4 together.

Proposition 4 *In specific, when the electoral competition is non-trivial and: (i) the electoral reform process is trivial, then $l^* = \frac{v_3}{1+v_3}$ if $(2 - b_1 - \frac{1-v_3}{2})(b_1 - \frac{1-v_3}{2}) \geq (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_1\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_1)$ (ii) the electoral reform process is trivial, then $l^* = 0$ if $(2 - b_1 - \frac{1-v_3}{2})(b_1 - \frac{1-v_3}{2}) < (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_1\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_1)$, (iii) the electoral reform process is non-trivial then $l^* = \frac{v_3}{1+v_3}$ if the proposer is the leading party and $(2 - b_2 - \frac{1-v_3}{2})(b_2 - \frac{1-v_3}{2}) \geq (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_2\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_2)$, (iv) the electoral reform process is non-trivial then $l^* = 0$ if the proposer is the leading party and $(2 - b_2 - \frac{1-v_3}{2})(b_2 - \frac{1-v_3}{2}) < (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_2\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_2)$, (v) the electoral reform process is non-trivial then $l^* = \frac{v_3}{1+v_3}$ if the proposer is not the leading party and $(2 - b_1 - \frac{1-v_3}{2})(b_1 - \frac{1-v_3}{2}) \geq (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_1\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_1)$, (vi) Finally the electoral reform process is non-trivial then $l^* = 0$ if the proposer is not the leading party and $(2 - b_1 - \frac{1-v_3}{2})(b_1 - \frac{1-v_3}{2}) < (\frac{1-v_3}{2}\frac{v_3}{1+v_3} + a_1\frac{v_3}{1+v_3})(\frac{1-v_3}{2} - a_1)$.*

Remark 1 *Here in presenting this result we have implicitely assumed that both b_1 and b_2 are less than $1/2$. In case that either b_1 and/or b_2 are greater than $1/2$ then one wanting to reproduce the above result of Proposition 4 simply substitutes b_1 and/or b_2 with the value $1/2$ and the result then follows in the same way as shown above.*

[Insert Graph 1 about here]

We will provide here an idea of the proof and a discussion for each case separately since this is the main result of the paper. Here the electoral competition is never trivial that is the following conditions always hold: $a_1 < \frac{1-v_3}{2}$ and $b_1 > \frac{1-v_3}{2}$. First of all we note that formally the proof is derived from the maximization program of the proposer. As shown in the appendix $Eu_i(v_1, v_2, v_3, l)$ for $i = \{1, 2\}$ is convex with respect to l (strictly convex for some values) for $l \in .(0, \frac{v_3}{1+v_3}]$ and decreasing for $l \in (\frac{v_3}{1+v_3}, 1]$. Hence the only two candidates for an optimum, as we argued in the proof of proposition 3 are either $l = 0$ or $l = \frac{v_3}{1+v_3}$. In both cases i) and ii) since the electoral reform process is trivial the proposer faces the unconstrained version of the maximization program, that is it maximizes its expected utility $EU_i(v_1, v_2, v_3, l)$ by choosing l^* s.t. $l^* = \arg\max EU_i(v_1, v_2, v_3, l)$ without having to satisfy the participation constraint of party 2 (recall that by corollary 1 the third party never agrees to accept any l^* other than the one that corresponds to the PR rule- that is $l^* = 0$). To continue with our proof we will first argue why the two candidate equilibrium strategies (values of l^*) are either $l^* = 0$ or $l^* = \frac{v_3}{1+v_3}$. Technically speaking those two equilibrium values of l^* are derived from the maximization program of the proposer (in fact they are corner solutions, see appendix). Nevertheless the intuitive argument seems more clear if we refer to those values of l^* as equilibrium strategies. First consider the case that $l \in (\frac{v_3}{1+v_3}, 1]$:

in this case whenever $v_1 > \frac{1-v_3}{2}$ then the increase in the bonus does not give the proposer any extra utility since its utility is already 1 (this corresponds to the constant part of the Eu expression). That is the value of l^* is such that $s_1^{l^*}(v_1) > \frac{1}{2}$. Hence any further increase in the size of the bonus does not give the proposer any greater level of utility. On the contrary consider the case when $v_1 < \frac{1-v_3}{2}$: in this case the strategy (proposal) with higher bonus than $l^* = \frac{v_3}{1+v_3}$ is strictly dominated by l^*. The reason is that in this case the proposer runs second hence does not receive the bonus hence its utility is strictly decreasing in l. This is exactly what we have shown in the proof, that for values of $l > \frac{v_3}{1+v_3}$ expected utility is decreasing. As a result we have argued that any strategy to propose $l > l^*$ is weakly dominated by l^* for any value of v_1 and in particular strictly dominated for values of $v_1 < \frac{1-v_3}{2}$. Hence to conclude strategy $l = 0$ and strategy $l^* = \frac{v_3}{1+v_3}$ by convexity of the Expected Utility with respect to l, as argued above, are the only two possible strategy proposals that maximize the proposer's expected utility. That is in equilibrium the proposer proposes (plays) one of those two strategies (notice that in each case the equilibrium is unique).

Ideally the proposer would prefer to propose the PR rule (i.e. $l = 0$) whenever is expected to run second and propose l^* otherwise yet in our environment there exist uncertainty about the outcome of the election. Hence the proposer has to compare the expected loss from proposing l^* and running second (or put it differently the expected gain in utility from running first in the election and forming a single party government) with the expected loss in utility of winning the election and not being able to form a single party government because it proposed $l = 0$ (or equivalently the expected gain from running second and winning more seats). Those two equivalent statements are mathematically expressed by the preceding inequality. Given that parties 1 and 2 are symmetric the same analysis applies for the receiver of the proposal in deciding whether to accept or not the proposal. Now the results of the proposition follow: in case (i) given that the electoral reform process is trivial the proposer proposes l^* whenever the expected gain exceeds the expected loss and proposes $l = 0$ whenever the opposite is true [case (ii)]. In cases (iii) and (iv) the electoral reform is non-trivial but the proposer is the leading party which implies that he always prefers to propose l^* since for party 1 expected benefit always exceeds the expected loss. Yet now the proposer faces a constrained maximization program and therefore it has to satisfy the participation constraint of party 2 (which is analogous and symmetric to the previous inequality for party 1). Whenever the participation constraint of party 2 is satisfied the proposer (party1) proposes l^* as in case (iii) otherwise party 2 will reject any proposal of l^* other than $l = 0$ and as result the reform will not pass, hence party one proposes $l = 0$. Cases (v) and (vi) is the other side of the same coin. Now party 2 is the leading party hence its participation constraint is always satisfied and it always accept a proposal of l^*. But now it is party 1 that proposes l^* even if it is not the leading party whenever its expected gain exceeds its expected loss and $l = 0$ otherwise. This completes the argument.

4.1 Discussion of the Results

Here in this section we will discuss the implications of the main result of the paper that are stated in Propositions 3 and 4. The main implication of the result of proposition 3 is that in our model the two dominant parties endogenously choose an electoral rule such that

they crystallize and solidify the two-party system. In fact they optimal level of l^* is such that it completely eliminates the effect of the third party in the partisan competition. Of course the third party never stood any chance of winning the election (i.e. running first) not to mention forming a single-party government. Yet in the absence of a distorted electoral rule (such as the bonus system) the third party played a role in the formation of coalition governments. It many instances it had the necessary amount of seats in the parliament in order to influence the formation of the government. Now in our model we have shown by Proposition 3 (which is a summary of 4) that the two dominant parties have strategic incentives that align (under mild assumptions) in order to eliminate the political impact of the third party. Hence they solidify the two-dominant party political environment by endogenously choosing the level of the bonus l^*. Our model therefore provides a theoretical expression for the reverse statement of Duverger's law according to which the electoral rule affects and shapes the political competition the political environment and the number of parties and the nature of the political competition. Here starting upside down we construct a model that shows the inverse that is: it is parties that endogenously choose the electoral rule in such a way that they solidify the pre-existing political system (in our case one with two dominant parties). The drivers of these result are the strategic incentives of the dominant parties to collude in an environment of uncertainty about the outcome of the election (i.e. the electoral competition is non-trivial) and of course the desire of both dominant parties to form single-party governments once they win the election. The latter is what it creates the incentive to collude: they collude by proposing an electoral rule that distorts the PR in order to eliminate the impact of the third party. The former (uncertainty over the outcome) is what it gives room for this collusion to take place. If there was no uncertainty over the outcome of the election then there would have been no room for collusion. It is exactly this combination that allows the incentives to align under some mild conditions. Those conditions are the participation constraints of the two dominant parties. Regardless of being the leading party or not each of the two will consent in a change of the electoral rule only if in expected terms the benefits of the electoral reform (that distorts the PR into their favour as the bonus system does) are greater than the potential losses. In such a case we can see that not only incentives align but also there is room for collusion. Otherwise as expected no electoral rule reform takes place and hence we are stuck with the PR rule. However it must be noted that in the next section of paper we explore into greater depth the implication of those conditions by performing a comparative statics analysis in order to study in greater depth the transition process from the PR rule to the bonus one.

Another equally important implication of Propositions 3 and 4 is the fact that the equilibrium outcome is unique and stable. Once as stated before the two dominant parties decide to depart from the PR rule there is a unique value l^* that is proposed and accepted. In fact the value is such that it guarantees that the winner of the election will always form a single-party government. This can be viewed as a minimax behavior of the two dominant parties. Given that the expected gains from distorting the PR rule by adapting the bonus one exceed the expected loss the two parties are faced with the problem which value of l to propose. The answer is unique and it is the value l^* such that in the case each one of them is the winner of the election it would be able to form a government on its own but in the case it loses it would have minimized its maximum loss of seats (observe that the seat loss is minimized when we have the PR rule). That is they answer the following question: "which

electoral rule guarantees me a single-party government in case I win but on the same time it minimizes my loss of parliamentary seats in the worst case scenario that I run second?". The answer in this question is a unique value of l, namely l^* and this is an important feature of the model. Its implication is that not only parties have an incentive to agree and distort the PR rule in order to solidify the two-party political system but they also agree on a unique new electoral rule. In the next section we will also examine the stability of the electoral reform process and check whether and under which conditions the change in the electoral rule is permanent or whether we observe cyclical behavior.

4.2   Two step electoral reform

We shall now explore a particular environment, in which the electoral reform may be applied in the coming elections only if it gathers a support W_2 and may be applied in the first elections after the coming ones if it gathers a support $W_1 < W_2$. That is, now, there is an extra strategic consideration for both dominant parties apart from the particular bonus magnitude; the exact time of the electoral reform application. If an electoral reform proposal gathers more than W_2 votes in the parliament, then we shall call it an "immediate electoral reform" and if it gathers more than W_1 votes but less than W_2 we shall call it a "two-step electoral reform"

To simplify the analysis, we shall assume that (i) the current electoral rule is $l = 0$, (ii) unless a single-party government is formed, elections are immediately repeated, (iii) the proposer either proposes $l = l^*$ or makes no electoral reform proposal (elections take place according to $l = 0$) and that (iv) $u_i(v_1, v_2, v_3, l) = 1$ if $s_i^l(v_i) > 1/2$ and $u_i(v_1, v_2, v_3, l) = 0$ if $s_i^l(v_i) \leq 1/2$.

To concentrate only in the non-trivial cases we shall furthermore assume that both b_1 and b_2 are larger than $1/2$.

Notice that in this framework the expected utility of parties not only depends on the exact electoral reform, but also on the implementation time of the reform. This is why we need to define a set of possible electoral reform scenarios $\Psi = \{0, L^*, \hat{L}^*\}$. The first element of the set Ψ indicates that no electoral reform takes place, the second element that an immediate electoral reform l^* is implemented and the last element of the set that a two step electoral reform is decided (first elections are performed according to rule $l = 0$ and those that follow according to rule $l = l^*$).

So, the expected utility of party one is formally defined as $E[u_1(v_1, v_2, v_3, l)|\psi]$, where $\psi \in \Psi$. In specific:

$$E[u_1(v_1, v_2, v_3, l)|0] =$$
$$\frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})[\frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})[\frac{b_1-1/2}{b_1-a_1} + ... =$$
$$= \frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})\frac{b_1-1/2}{b_1-a_1} + \beta^2(1 - \frac{b_1-1/2}{b_1-a_1})^2(1 - \frac{b_2-1/2}{b_2-a_2})^2\frac{b_1-1/2}{b_1-a_1} + ... =$$
$$= \frac{b_1-1/2}{b_1-a_1}\frac{1}{1-\beta(1-\frac{b_1-1/2}{b_1-a_1})(1-\frac{b_2-1/2}{b_2-a_2})} =$$
$$= 2(a_2 - b_2)\frac{2b_1-1}{\beta-2\beta a_1-2\beta a_2-4a_1a_2+4a_1b_2+4a_2b_1-4b_1b_2+4\beta a_1a_2}$$

and
$$E[u_1(v_1, v_2, v_3, l)|L^*] = \frac{b_1-\frac{1-v_3}{2}}{b_1-a_1}$$

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and

$$E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] = \frac{b_1-1/2}{b_1-a_1} + \beta(1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_2-1/2}{b_2-a_2})\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1}$$

where $\frac{b_1-1/2}{b_1-a_1}$ is the probability that party one gathers more than half of the votes in the elections, $\beta \in [0, 1]$ is the discount factor, $(1-\frac{b_1-1/2}{b_1-a_1})(1-\frac{b_2-1/2}{b_2-a_2})$ is the probability that none of the two dominant parties controls the parliament (and thus no single party government can be formed and we proceed to new elections) and $\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1}$ is the probability of party one being the winner of elections.

For simplicity and without loss of generality we shall consider that $\beta = 1$.

Lemma 2 *In this case dominant parties never agree on an electoral reform. Or in other words, if $s_1^0 < W_2$, no "immediate electoral reform" is ever observed.*

The validity of the above lemma is obvious if we consider the nature of the above expected utilities of the parties. Given that $\beta = 1$, then $E[u_1(v_1, v_2, v_3, l)|0]$ is just the probability that party one will at some point form a single-party government if no electoral reform takes place. Equivalently, $E[u_1(v_1, v_2, v_3, l)|L^*]$ is the probability that party one will form a single party government if an immediate electoral reform takes place. That is, $E[u_1(v_1, v_2, v_3, l)|0] + E[u_2(v_1, v_2, v_3, l)|0] = E[u_1(v_1, v_2, v_3, l)|L^*] + E[u_2(v_1, v_2, v_3, l)|L^*] = 1$. We can rewrite this as $E[u_1(v_1, v_2, v_3, l)|0] - E[u_1(v_1, v_2, v_3, l)|L^*] = E[u_2(v_1, v_2, v_3, l)|L^*] - E[u_2(v_1, v_2, v_3, l)|0]$. It thus becomes clear that if $s_1^0 < W_2$ and if party ones is for the reform $(E[u_1(v_1, v_2, v_3, l)|0] - E[u_1(v_1, v_2, v_3, l)|L^*] < 0)$ then party two will be against the reform $(E[u_2(v_1, v_2, v_3, l)|L^*] - E[u_2(v_1, v_2, v_3, l)|0] < 0)$. The immediate electoral reform cannot be an equilibrium of the present game. But what about the two-step electoral reform?

Proposition 5 *Consider that, $W_1 \leq s_1^0 < W_2$. If the proposer is the leading party then no electoral reform takes place ($l = 0$). If the proposer is not the leading party then a two-step electoral reform is applied (the proposer suggests $l = l^*$ and is the only party to support it)[3].*

We are implicitly assuming that parties act as a whole; that is, all the members of a party's parliamentary delegation vote the same. In this way, even if the current seats of a party exceed the supermajority threshold (W_2) and the party, say, prefers a two-step electoral reform to an immediate one, is compelled to act in favour of the later. This will not be of major significance, because, as the above proposition clearly indicates, we are especially interested in the equilibrium of a political environment in which no party has the adequate share of seats to apply an immediate electoral reform at will.

Unlike the main part of the analysis, where the proposer has a continuum of strategies to choose from, here the choice set is constrained to a binary choice (no proposal or proposal l^*). Yet this does not alter the set up dramatically since in Proposition 3 we have proven that in fact in equilibrium the optimal choice is a binary one (either $l = 0$ or $l = \frac{v_3}{1+v_3}$). Hence if the proposer offers l^* and $W_1 \leq s_1^0 < W_2$, then by lemma 2 party two shall vote against the

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[3]Proof in the Appendix.

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reform. But since the seats of the proposer exceed the low threshold, the electoral reform will pass and will be applied in the elections after the coming ones. The intuition behind this result can be found in what we call the "confidence of the leader". The leading party knows that it is impossible to have an immediate electoral rule reform given that the second party will vote against it. But it also knows that if it proposes a bonus then in the unlikely case that in the end loses the elections it would have helped its opponent to win majority of seats in the parliament and form a single party government. So in expected terms it is worse off. Whereas if it sticks with the PR rule chances are that at some point after elections being repeated many times it would manage to secure the majority in the parliament and form a single party government. The crucial thing to note is that in when the proposal is being made the party who is considered to be the leading one for the upcoming election is also considered to be leading party on the continuation of the game should no party gain control of the parliament and hence new elections are called. This result can also be verified by actual election results. When a party is closer to the threshold of forming a single-party government but unable to do so the first time then as elections are repeated until some party takes control over the majority of the parliament, empirical observation says that citizens at some point after being tired from consecutive electoral procedures choose to give the majority of votes (and seats) to the leading party. This electoral behavior is what we call the "confidence of the leader". The opposite is true when the proposer is not the leading party. Then by reversing the previous argument we see that the proposer proposes l^* and by Lemma 2 is the only party that votes for it.

The best way to illustrate Proposition 5 is with an example. Imagine that the leading party is the well-prepared student whereas the other party is the ill-prepared one. On absolute terms it is better if the exam questions are easy (or favorable) but in relative terms even if it sounds counter-intuitive the well-prepared student prefers a hard exam because this will help her diversify herself from the ill-prepared one and be the only to excel. That is by choosing a hard exam it decreases the chances of the other student to pool with her and also excel. In general in games of competition (such as elections) where the relative position matters the most this argument makes much sense. The leading party prefers a "harder" or less favorable electoral law in order to decrease the other party's chances of winning the majority of seats in parliament and hence forming a single-party government, being sure that even with the PR rule at some point after consecutive electoral procedures it will finally secure the majority in the parliament. This is exactly the same with the confidence of the well-prepared student that no matter hard the exam is in the end she will be the best performing in the whole class.

5 Conclusions

To summarize this paper is an attempt to shed some light in the way that parties choose the electoral rules with which elections take place. In this analytically tractable model we try to explore the endogenous choice of electoral rules by the parties in a setting where there exist two dominant parties. That is we explore how the nature of the political competition under an environment of uncertainty with respect to the outcome of the electoral process affects the endogenous choice of the electoral rule. In recent political economy literature

Duverger's law that the electoral rule determines the nature of the political competition and the structure of the political system in general has be reversed (that is the link between PR rules and multiparty systems as opposed to majority rules and bipartisanships). Here we present an analytical model that turns Duverger upside down in a more formal way than previous literature. This is not to say that we have proven Duverger wrong: rather we argue that if the converse hypothesis is true then our model provides a formal expression of this. That is to say in our paper we argue that the nature of political competition drives parties to choose endogenously the appropriate electoral rules in order to solidify and maintain the current political system. Hence someone would expect that our model will predict that bipartisan systems will tend to choose majority rules as opposes to multiparty systems in which PR rule prevails.

This is exactly what our model theoretically predicts. Our main result states that in our setting we have two dominant parties which under some mild assumptions and conditions have aligned incentives to collude and choose a majority electoral rule (a mixed system that uses PR and a bonus to the first party). The important feature of our model is the uniqueness and stability of the equilibrium: once the strategic incentives of the two dominant parties align there is a unique proposal to reform the electoral rule and adapt a less proportional one which is accepted by the two dominant parties. The key result of our paper is that when the electoral competition is non-trivial regardless if the reform process is trivial or not there is a unique proposal of a bonus electoral law that it gets accepted by the parliament and is introduced as the new electoral rule. In fact the value of the bonus l* is such that it completely eliminates the political impact of the third party. If the PR rule was to remain then in some instances the third minority party would have an important role to play when either of the two major dominant parties was not able to occupy the majority of the seats in the parliament. In such instances the third party would have an important role to play in the formation of a coalition government.

But in our model the two dominant parties have incentives to eliminate this role of the third party. The strategic incentives to collude and reform the PR rule into a more majoritarian one come from the desire of the dominant parties to form a single-party government once they win the elections. That is they are office motivated. Hence they have a strong preference of occupying the majority of the seats in parliament once they win elections. On the other hand if there was no uncertainty about the outcome of the election (that is the electoral game was trivial) then the room for agreement would be non existent. Hence the second factor that allows the two dominant parties to agree upon a reform of the electoral rule into a more majoritarian one is the existence of uncertainty about the outcome of the electoral game. In such a case each party tries to guarantee itself by choosing / proposing the appropriate electoral rule reform the majority of the seats in the forthcoming parliament and hence the ability to govern alone and pass legislation at will. On the other hand it tries to satisfy the participation constraint of the second party so that for both parties the expected gain of the reform will exceed the expected loss. Otherwise the second party will not accept the proposed change. This is not the same as saying that both parties agree on a distortion of the PR rule when they are the expected winners (leading parties as we refer to them in the main part of the analysis) because in that case they would have been no room for an agreement. Rather we prove that the two parties have incentives to collude and propose a unique electoral law reform that gets accepted by the parliament if the

above condition is satisfied. The uniqueness of this equilibrium appears to have a very nice intuitive explanation: under the presence of electoral uncertainty if it is the case that both parties would agree to distort the PR electoral rule into a more majoritarian one then they will choose one that guarantees the formation of a single party government if the are the winners of the electoral game but on the other hand it will minimize their maximum seat loss in the parliament (in comparison with the seats they would have occupied were the PR rule was not to be changed) if they are to run second. Hence this minimax behavior of the two dominant parties is the third critical factor that combined with the other two (electoral uncertainty and desire for single-party governments) is responsible for the uniqueness of the equilibrium in our model.

In the model we also examine a particular case where a supermajority is needed in order for the electoral rule reform to be accepted by the parliament. In this case our results give an explanation as to why some times even when a party is the expected leader is opposed to an electoral rule reform when in opposition. In particular our result shows that when the necessary majority needed in order to change the electoral rule is more than three fifths of the total seats in the parliament then the opposition party (one of the two dominant ones) has incentives to block the proposed reform. Finally in the last chapter of the paper we explore into greater depth the stability of the equilibrium of the game and we conduct some comparative statics exercises in order to determine the conditions under which this equilibrium occurs. We conclude our paper with a discussion on future extensions of the model on endogenous choice of electoral rules in multiparty systems with more than two parties that contest the elections and we provide a model that generalizes the result into a broader setting. Still the basic results confirm the intuition and the main result of the paper. It is the nature of political competition and the structure of the political environment with the rules of the electoral game that provide an adequate reasoning and explanation on how and why parties choose endogenously the electoral rule in order to solidify and crystallize the current status quo.

6   References

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7 Appendix (Proofs)

Proposition 3. Given the structure of the game, to show that when the electoral competition is non-trivial then $l^* \in \{0, \frac{v_3}{1+v_3}\}$, is quite easy. In the first part of the proof we shall demonstrate that, for both parties the exact bonus l that maximizes their expected utility is either 0 or $\frac{v_3}{1+v_3}$. Then given this result, we will offer a trivial argument to show that if both parties maximize their expected utility with a bonus $\frac{v_3}{1+v_3}$, this specific electoral reform takes place, and in case at least one maximizes its expected utility with $l = 0$ no electoral reform takes place.

For the first part of the proof we need to prove that the expected utility of party 1 is convex in $l \in [0, \frac{v_3}{1+v_3}]$, strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The arguments are equivalent for party 2. Since the electoral competition is non-trivial, we have that $b_1 > \frac{1-v_3}{2}$. If $b_1 < 1/2$ then there exist $\hat{l} \in (0,1)$ s.t. $\frac{1/2-\hat{l}}{1-\hat{l}} = b_1$. For $l \in [0, \hat{l}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} [v_1(1-l)+l]dv_1]$, for

$l \in (\hat{l}, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that $Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$. One may observe that $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} = 0$ for $l \in [0, \hat{l}]$, $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} > 0$ for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. Moreover, if $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} \geq 0$ for $l \in [0, \hat{l}]$ then $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} > 0$ for $l \in (\hat{l}, \frac{v_3}{1+v_3}]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is convex in $[0, \frac{v_3}{1+v_3}]$, strictly convex in a subset of $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for maximum are $\{0, \frac{v_3}{1+v_3}\}$. If $b_1 > 1/2$, for $l \in [0, \frac{v_3}{1+v_3}]$ we have that $Eu_1(v_1, v_2, v_3, l) =$

$\frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{\frac{1/2-l}{1-l}} [v_1(1-l)+l]dv_1 + \int_{\frac{1/2-l}{1-l}}^{b_1} dv_1]$ and for $l \in (\frac{v_3}{1+v_3}, 1]$ we have that

$Eu_1(v_1, v_2, v_3, l) = \frac{1}{(b_1-a_1)}[\int_{a_1}^{(1-v_3)/2} v_1(1-l)dv_1 + \int_{(1-v_3)/2}^{b_1} dv_1]$. Just, as before $\frac{\partial^2 Eu_1(v_1,v_2,v_3,l)}{\partial l^2} >$

0 for $l \in [0, \frac{v_3}{1+v_3}]$, and $\frac{\partial Eu_1(v_1,v_2,v_3,l)}{\partial l} < 0$ for $l \in (\frac{v_3}{1+v_3}, 1]$. That is, $Eu_1(v_1, v_2, v_3, l)$ is strictly convex in $[0, \frac{v_3}{1+v_3}]$ and decreasing in $(\frac{v_3}{1+v_3}, 1]$. The only candidates for maximum are $\{0, \frac{v_3}{1+v_3}\}$. This concludes the first part of the proof.

If both parties maximize their expected utility with $l = \frac{v_3}{1+v_3}$ then party one proposes this electoral reform and party 2 votes for it. If the optimal bonus for the proposing party 1 is $l = 0$ then it does not propose any electoral reform. And when $l = 0$ maximizes the expected utility of party 2 then it always votes against any electoral reform. ∎

Proposition 5.   Since $\beta = 1$, then $E[u_1(v_1, v_2, v_3, l)|0] = \frac{2(2b_1-1)(b_2-a_2)}{2a_1+2a_2-4a_1b_2-4a_2b_1+4b_1b_2-1}$ and $E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] = \frac{b_1-1/2}{b_1-a_1} + (1 - \frac{b_1-1/2}{b_1-a_1})(1 - \frac{b_1-1/2}{b_1-a_1})\frac{b_1-\frac{1-v_3}{2}}{b_1-a_1}$. It is obvious that if $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|L^*]$ then $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] >$

$E[u_1(v_1, v_2, v_3, l)|L^*]$ and that if $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|L^*]$ then $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|\hat{L}^*] < E[u_1(v_1, v_2, v_3, l)|L^*]$. This holds for the second party as well. In other words if a party prefers an immediate reform to no-reform, then it also prefers a two step reform to no reform and the opposite. If the proposer (party one) is the leading party ($\frac{b_1 - \frac{1-v_3}{2}}{b_1 - a_1} > 1/2$) then $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|L^*]$ which means that $E[u_1(v_1, v_2, v_3, l)|0] > E[u_1(v_1, v_2, v_3, l)|\hat{L}^*]$ and that $E[u_2(v_1, v_2, v_3, l)|0] < E[u_2(v_1, v_2, v_3, l)|\hat{L}^*] < E[u_2(v_1, v_2, v_3, l)|L^*]$. Equivalently, if the proposer (party one) is not the leading party ($\frac{b_1 - \frac{1-v_3}{2}}{b_1 - a_1} < 1/2$) then $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|L^*]$ which means that $E[u_1(v_1, v_2, v_3, l)|0] < E[u_1(v_1, v_2, v_3, l)|\hat{L}^*]$ and that $E[u_2(v_1, v_2, v_3, l)|0] > E[u_2(v_1, v_2, v_3, l)|\hat{L}^*] > E[u_2(v_1, v_2, v_3, l)|L^*]$.

∎

Utility function for a dominant party for different values of l

Legend:
- $l = 0$ (black)
- $l = l^*/4$ (red)
- $l = l^*$ (green)

Y-axis: Utility

X-axis: Vote share

a_1 b_1