Stable jurisdiction partitions under monotonically decreasing population density

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2009

Abstract

Starting from Alesina-Spolaore breakthrough publication, many papers study the problem of jurisdiction formation and spatial allocation of public goods. Bogomolnaia, Le Breton, Savvateev and Weber demonstrated that there may be no stable partitions in one-dimensional world under arbitrary population density. This work proves existence of a stable jurisdiction partition under monotonically decreasing population density.

1 Introduction

The world is divided into different scale jurisdictions: countries, regions, districts etc. Jurisdiction formation is an ongoing process: international organizations arise and develop (EU, ASEAN), new recognized and unrecognized countries emerge in different parts of the world (Eritrea, Somaliland, Montenegro, Kosovo, Abkhazia, South Ossetia), several regions in Russia merged, and municipal reform was implemented. Why do jurisdictions form? Why is their formation ongoing? Will this process converge to a stable configuration or continue eternally? In this paper we try to shed light on these questions.

Traditional conviction is that people form groups to produce some sort of public good. In case of jurisdiction formation this good is allocated at a certain point (the capital). If the cost of the good is constant then the larger the jurisdiction the less the expenses of an individual. On the other hand, since the good is localized at one point, agents living far from it spend too much on transportation. This tradeoff lies in the core of jurisdiction formation theory: regions should be neither too big nor too small.

We proceed in the tideway of traditional model presented in papers [1] and [3]. Namely, the model is one-dimensional, agents live across the line continuously according to some density. The capital of a jurisdictions locates at its median. Public good procurement costs are divided equally whereas transportation costs are beared individually. Utility is not transferable. The notion of stability is a coalitional one: a jurisdiction partition is stable if no group has incentives to secede and form a new jurisdiction, thus decreasing costs of each member.

It is shown in [1] that a stable partition always exists under uniform density. We study the case of non-uniform density. It is shown in [3] that no stable partition may exist in descrete model. We restate this result in continuous model and find sufficient conditions for existence of a stable partition. We prove that a stable partition always exists under monotonically decreasing density.

Section 2 presents the formal model. In section 3 we prove the main result. Section 4 concludes.

The author acknowledges Alexander Ivanov, Vitaliy Koshelev, Valery Makarov, Alexey Savvateev, Mikhail Raskin, Alexander Vasin, Mikhail Vyalyi and Shlomo Weber for useful comments.

2 The model

In this section we present several modifications of our formal model: from the most abstract to the most specific one. Subsections are independent and may be treated separately.

2.1 Coalitionally stable group partitions

Consider a set Ω (the world, the set of agents). Let $\mathcal{F}$ be a somehow defined set of possible coalitions along with their inner structure. I.e., for each coalition $S \subset \Omega$ the set $A(S)$ of possible inner structures is defined. Each element of $\mathcal{F}$ is a pair (S, α), where $S \subset \Omega$ and $\alpha \in A(S)$. Each agent $x \in \Omega$ has rational preferences on $\mathcal{F}$.

Definition 1. Jurisdiction partition *is the set* $\Sigma = \{(S_i, \alpha_i) \in \mathcal{F}, i \in I\}$*, such that* $\bigcup_{i \in I} S_i = \Omega$ *and* $S_i \cap S_j = \emptyset$ *if* $i \neq j$*.*

So, the whole world is split into non-intersecting[1] jurisdictions.

Definition 2. *A group* $(T, \beta) \in \mathcal{F}$ blocks Σ *if for all agents* $\theta \in T$ *it holds that* $(T, \beta) \succ_\theta (S_{i(\theta)}, \alpha_{i(\theta)})$*, where* $\theta \in S_{i(\theta)}$*.*

A partition Σ *is* stable *if it is blocked by no group.*

That is, stable jurisdiction partition is a core in certain coalition game.

2.2 Spatial allocation of public goods

Let $(\Omega, \mathcal{F}, \mu)$ be a measurable space (the world, the set of agents), ρ be a distance function on this space (transportation costs) and $p\colon \mathcal{F} \setminus \{\varnothing\} \rightrightarrows \Omega$ be a multivalued function which determines possible allocations of public good. Without loss of generality set monetary costs of public good production equal to one.

A non-empty coalition $S \in \mathcal{F}$ allocates public good at any point $m \in p(S)$ and equally divides monetary costs. Each member of the coalition pays transportation costs by himself. That is, in the case S is formed and public good is allocated at m each member $\theta \in S$ bears costs

$$C_\theta(S, m) = \frac{1}{\mu(S)} + \rho(\theta, m). \tag{1}$$

If $\mu(S) = 0$, consider costs of any member of such coalition to be infinite, making any other coalition not worse.

Definition 3. Juriscdiction partition *is a system* Σ *of sets* $\{S_i, i \in I\}$ *and points* $m_i \in p(S_i)$*, such that* $S_i \in \mathcal{F} \setminus \{\varnothing\}$*,* $\bigcup_{i \in I} S_i = \Omega$ *and* $S_i \cap S_j = \varnothing$ *if* $i \neq j$*. An individual* θ *in partition* Σ *bears costs* $C_\theta(\Sigma) = C_\theta(S_{i(\theta)}, m_{i(\theta)})$*, where* $\theta \in S_{i(\theta)}$*.*

Definition 4. *A group* $T \in \mathcal{F}$ blocks *partition* Σ*, if for some point* $n \in p(T)$ *and any agent* $\theta \in T$ *it holds that* $C_\theta(T, n) < C_\theta(\Sigma)$*.*

A partition Σ *is* stable*, if no group blocks it.*

2.3 One-dimensional continuous model of jurisdiction formation

Let $\Omega \subset \mathbb{R}$ be a measurable set, μ be a measure on this set with density function $f(\theta)$, that is $\mu(S) = \int_S f(\theta) d\theta$, and $\rho(x, y) = |x - y|$ be the standard metric. We treat Ω as the world, $f(\theta)$ as population density, $\mu(S)$ as population of S and $\rho(x, y)$ as transportaion cost

[1]In concrete realizations of the model jurisdictions may intersect by “zero” sets, but general formilization of this notion seems too difficult

between points x and y. Denote by $\mathrm{med}(S)$ the set of all medians of S, i.e. $\mathrm{med}(S) = \{m \mid \mu(S \cap [-\infty, m]) = \mu(S \cap [m, +\infty))\}$. If S is nonempty then $\mathrm{med}(S)$ is also nonempty and usually consists of a single point.

A nonempty coalition $S \in \mathcal{F}$ allocates public good in a median $m \in \mathrm{med}(S)$ and equally divides monetary costs (normalized to unity). Each individual bears transportation costs by herself. That is, if coalition S is formed and allocates public good at point m then a member $\theta \in S$ bears costs

$$C_\theta(S, m) = \frac{1}{\mu(S)} + |\theta - m|. \tag{2}$$

In the case $\mu(S) = 0$ think that costs are infinite. If $\mathrm{med}(S)$ consists of a single point, we omit the second argument of C_θ.

Definition 5. *A* jurisdiction partition *is a system Σ of sets $\{S_i, i \in I\}$ and points $m_i \in \mathrm{med}(S_i)$, such that $\bigcup_{i\in I} S_i = \Omega$ and $\mu(S_i \cap S_j) = 0$ if $i \neq j$. An individual θ in partition Σ bears costs $C_\theta(\Sigma) = C_\theta(S_{i(\theta)}, m_{i(\theta)})$, where $\theta \in S_{i(\theta)}$.*

Definition 6. *A measurable group $T \subset \Omega$* blocks *partition Σ, if for some median $n \in \mathrm{med}(T)$ and each agent $\theta \in T$ it holds that $C_\theta(T, n) < C_\theta(\Sigma)$.*

A partition Σ is stable*, if no group blocks it.*

3 Main results

In this section we prove main results.

3.1 Formulations of theorems

Theorem 1 ([3])**.** *In one-dimensional continuous jurisdiction formation model for some population density $f(\cdot)$ any partition is unstable.*

In [3] a counterexample for descrete model is presented, in appendix we restate this example for continuous model.

Theorem 2. *Consider the one-dimensional continuous jurisdiction formation model. Let Ω be a half line $[0, +\infty)$ and $f(\cdot)$ be continuously differentiable and strictly decreasing. Then there exists a stable partition.*

3.2 An equivalent problem statement

In this subsection we slightly change the model to make the proof more distinct.

Consider a class of metrics $\rho_T(x, y) = |T(x) - T(y)| = \left|\int_x^y t(\theta) d\theta\right|$, where $T(\cdot)$ is a strictly increasing function and $t(\cdot)$ is its derivative. The latter may be treated as marginal transportation cost at point θ. The standard metric belongs to this class with $T(x) = x$.

Let $f(\theta)$ be population density and $t(\theta)$ be marginal cost of transportation. Then, by making a monotonic transformation $\theta = \theta(\eta)$ we may turn to density $f(\theta(\eta)) \cdot \theta'(\eta)$ and

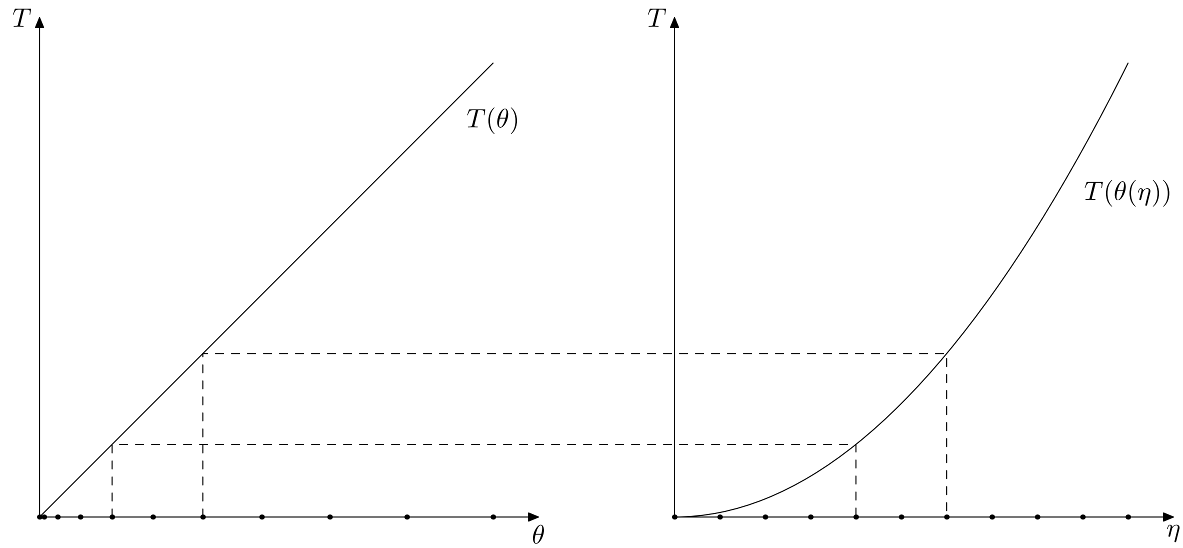

Figure 1: Transforming population density to uniform makes transportation costs convex.

marginal cost of transportation $t(\theta(\eta)) \cdot \theta'(\eta)$. Jurisdiction partition before and after transformation naturally correspond to each other and stability in one world is equivalent to that in the other.

Notice that there exists a transformation that makes population density unifrom. Namely, one should take $\eta(\theta) = \int_0^\theta f(\xi)d\xi$. Then $\eta'(\theta) = f(\theta)$ and by the inverse function theorem $\theta'(\eta) = \frac{1}{f(\theta(\eta))}$ that gives unity multiplied by $f(\theta(\eta))$. Since $f(\theta)$ increases and $\theta(\eta)$ decreases, $\theta'(\eta) = \frac{1}{f(\theta(\eta))}$ increases. If initial transportation costs are standard, i.e. $t \equiv 1$, then after transformation they would equal $\theta'(\eta)$ and then be increasing. This is equivalent to convex function T. This fact is illustrated on figure 1.

Notice that if population is finite and equals F then after transformation agents would live on the segment $[0, F]$ and transportation costs go to infinity on its right edge. If the population is infinite then agents would live on the half-line again.

Thus, an equivalent problem is stated: to prove the existence of coalitionally stable jurisdiction partitions in a world with uniform population density and convex transportation costs. Notice that in this setting the middle point of a connected coalition coincides with its median.

3.3 Construction of a stable partition

In this subsection we construct a stable jurisdiction partition for theorem 2.

Consider the agent at point θ. Different jurisdictions yield different costs to her. Restrict ourselves to jurisdictions she is the leftmost member of. Consider a jurisdiction of this class that yields minimal costs to her. We may suppose that it is connected: the connected juris-

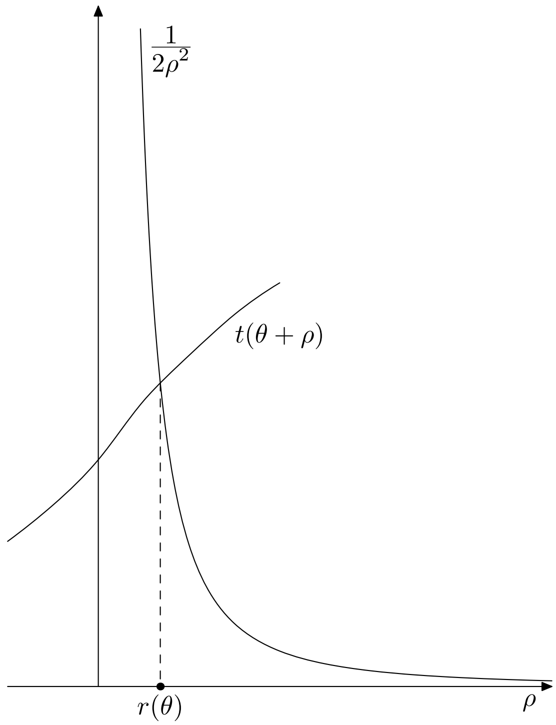

Figure 2: First order conditions for defining $r(\theta)$.

diction of the same measure yields equal monetary costs and smaller or equal transportation costs. Denote the length of such jurisdiction by $2r(\theta)$, that is, $r(\theta)$ is the distance between θ and the center of this jurisdiction. Put formally,

$$r(\theta) = \arg\min_{\rho} C_\theta([\theta, \theta + 2\rho]). \tag{3}$$

The function $r(\theta)$ may be found by solving an optimization problem

$$\frac{1}{2\rho} + T(\theta + \rho) - T(\theta) \to \min_{\rho}, \tag{4}$$

that yields first order condition

$$\frac{1}{2\rho^2} = t(\theta + \rho). \tag{5}$$

Equation (5) is illustrated on figure 2. The left part of (5) is decreasing and the right part is increasing, so it has a unique solution. Since corner points yield infinite costs, $r(\theta)$ is defined by the unique solution of (5).

Notice that if we were interested in the optimal for θ coalition where she is the *rightmost* member then we would obtain the same f.o.c. $\frac{1}{2\rho^2} = t(\theta + \rho)$ for $\rho < 0$. This f.o.c. may have several solutions but if the solution is unique then the same coalition is optimal for agents living on its left and right edges.

No we construct our stable partition. Take the leftmost agent (at point 0) and form the optimal for him jurisdiction $[0, 2r(0)]$. Then take the new leftmost agent (at point $2r(0)$) and form the optimal for him jurisdiction $[2r(0), 2r(0) + 2r(2r(0))]$, and so on. Formally speaking, we construct a sequence $y_0 = 0$, $y_i = y_{i-1} + 2r(y_{i-1})$, $i = 1, 2, 3, \ldots$ and split the workd into jurisdictions $[y_{i-1}, y_i]$, $i = 1, 2, 3, \ldots$. It is proved in Appendix that this is indeed a partition, i.e. the union of these jurisdictions is the whole world. In the next subsection we prove that this partition is stable, precisely that the leftmost agent of a seceding coalition cannot win from secession.

3.4 Proof of stability

In this subsection we show that the constructed partition (denote it Σ) is stable. The main idea is the following: prove that the leftmost agent of the seceding coalition blocks secession. For each agent θ define a function $C^r(\theta) = C_\theta([\theta, \theta + 2r(\theta)])$ equal to her minimal costs in a jurisdiction she is the leftmost member of. Let T be a seceding coalition and y be its leftmost member. Then costs beared by y after secession equal $C_y(T)$ that is not less than $C^r(y)$. We prove that the latter is not less than costs beared by y before secession, i.e. $C_y(T) \geq C^r(y) \geq C_y(\Sigma)$. Thus, y would not secede with coalition T.

The proof is based on two lemmas:

Lemma 3. *The function $C^r(y)$ is increasing.*

Lemma 4. *For any agent y on the right half of jurisdicrion $[y_{i-1}, y_i]$ it holds that $C^r(y) > C_y(\Sigma)$.*

If agent y lives on the left half of jurisdiction $[y_{i-1}, y_i]$ then $C_y(\Sigma) \leqslant C_{y_{i-1}}(\Sigma) = C^r(y_{i-1}) \leqslant C^r(y) \leqslant C_y(T)$, where the last but one inequality uses lemma 3. Thus we have $C_y(\Sigma) \leqslant C_y(T)$. Otherwise, if agent y lives on the right half of jurisdiction $[y_{i-1}, y_i]$ then lemma 4 implies $C_y(\Sigma) < C^r(y) \leqslant C_y(T)$. In both cases $C_y(\Sigma) \leqslant C_y(T)$, that necessitates stability.

Proof of lemma 3. Increasing marginal transportation costs t implies that $C_\theta([\theta, \theta + 2\rho])$ is increasing in θ for any fixed ρ. Then function $C^r(\theta) = \min_\rho\{C_\theta([\theta, \theta+2\rho])\}$ is also increasing in θ. □

Recall that typically juriscdiction $[y_{i-1}, y_i]$ is optimal not only for y_{i-1} but also for y_i. In this case y_i bears smaller costs in $[y_{i-1}, y_i]$ than in $[y_i, y_{i=1}]$. Indeed, we may rearrange the proof of lemma 3: for any fixed ρ it holds that $C_\theta([\theta - 2\rho, \theta]) < C_\theta([\theta, \theta + 2\rho])$. The inequality still holds for optimal ρ's. Now the statement of lemma 4 may be deduced from an inequality $\frac{d}{d\theta}C^r(\theta) < t(\theta)$. Unfortunately, the latter inequality is not generally true. Yet, lemma 4 can be proved in general and we proceed by presenting this proof.

We need a technical lemma proved in the Appendix:

Lemma 5. *Consider a graph of the function $y = \frac{1}{x^2}$. Let A and B be two points on the same connected component of this graph. Denote their projections to x- and y-coordinate lines by A_x, B_x and A_y, B_y respectively. Then the area of curvilinear trapezoid A_yABB_y is twice as large as that of A_xABB_x.*

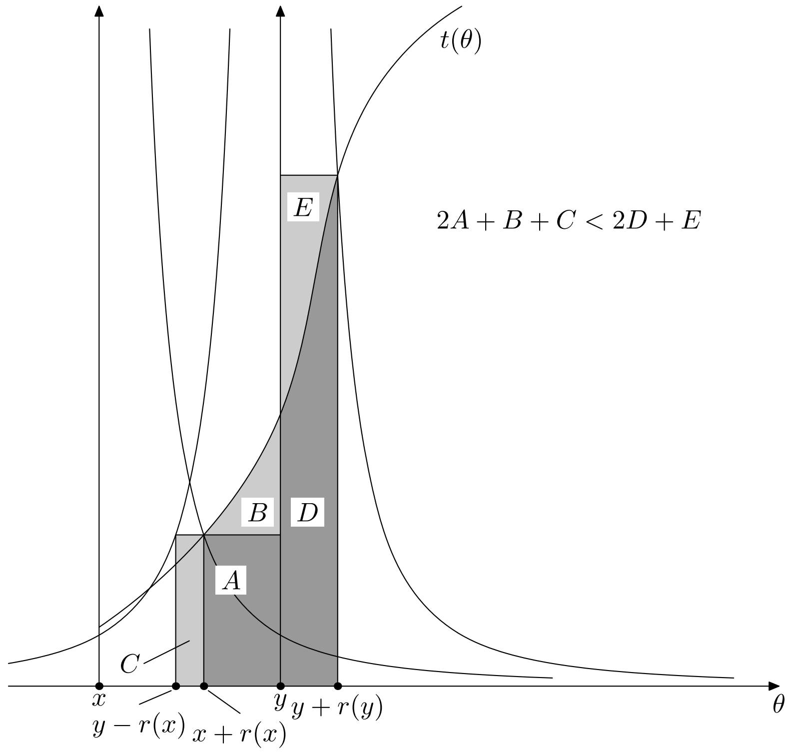

Figure 3: Graphical representation of inequality (8).

Proof of lemma 4. Denote by $[x, x+2r(x)]$ the segment $[y_{i-1}, y_i]$ containing y on its right half. According to f.o.c. (5) it holds for all θ that

$$\frac{1}{2r(\theta)} = r(\theta)t(\theta + r(\theta)).$$

This implies

$$C^r(y) = r(y)t(y + r(y)) + T(y + r(y)) - T(y) = r(y)t(y + r(y)) + \int_{y}^{y+r(y)} t(\theta)d\theta; \qquad (6)$$

$$C_y(\Sigma) = \frac{1}{2r(x)} + T(y) - T(x + r(x)) = r(x)t(x + r(x)) + \int_{x+r(x)}^{y} t(\theta)d\theta. \qquad (7)$$

Thus, the initial inequality converts to

$$r(x)t(x + r(x)) + \int_{x+r(x)}^{y} t(\theta)d\theta < r(y)t(y + r(y)) + \int_{y}^{y+r(y)} t(\theta)d\theta. \qquad (8)$$

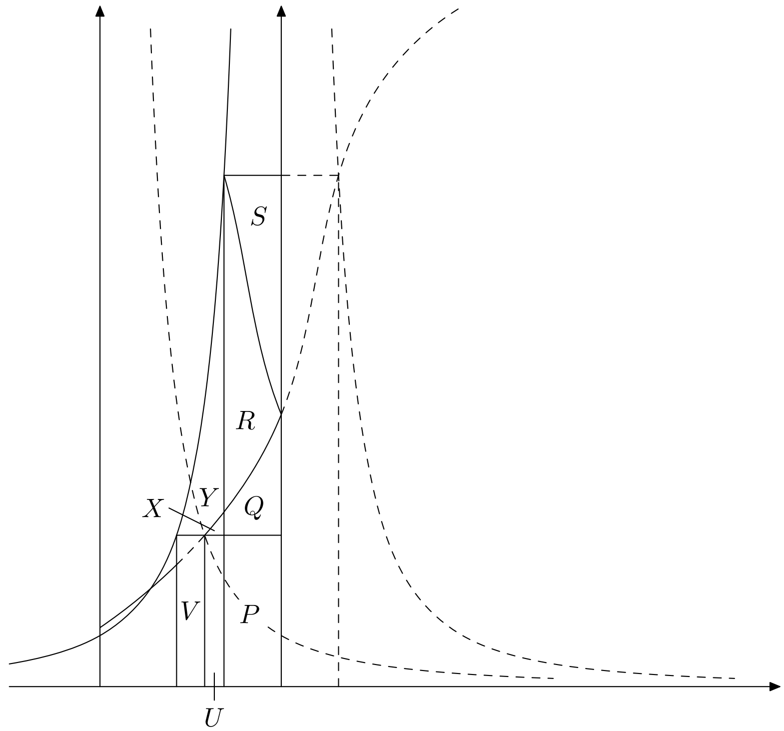

Figure 4: Illustration to the proof of inequality (8).

The latter inequality may be interpreted graphically. Namely, $r(x)t(x+r(x)$ equals to the sum of areas of rectangulars A and C on figure 3. (We exploit the fact that y lies on the right half of $[x, x+2r(x)]$ and thus $y - r(x) < x + r(x) < y$.) Concerning the integral $\int\limits_{x+r(x)}^{y} t(\theta)d\theta$, it equals to the area under the graph of t, i.e. to the sum of areas A and B. Thus, the left part of the inequality equals $2A + B + C$. Similarly the right part equals to $2D + E$ and we need to prove that $2A + B + C < 2D + E$.

Now we present a graphical proof. Reflect the right part of the figure about a vertical line passing through y. There may be two cases depending on what value is greater: $r(y)$ or $y - x - r(x)$. In the first case there are 8 shapes: P, Q, R, S, U, V, X, Y, as shown on figure 4. The second case is studied in the Appendix. It may be easily seen that $A = P + U$, $B = Q + X$, $C = V$, $D = P + Q + R$ and $E = S$. Inequality (8) is equivalent to the following:

$$2P + 2U + Q + X + V < 2P + 2Q + 2R + S \Leftrightarrow 2U + X + V < Q + 2R + S.$$

According to technical lemma, it holds that $2U + 2V + 2X + 2Y = X + Y + Q + R + S$, thus $2U + X + V = Q + R + S - Y - V$. Substituting this to the inequality, we get $Q + R + S - Y - V < Q + 2R + S \Leftrightarrow -Y - V < R$. The latter is true since Y, V and R are positive numbers. Thus, inequality (8), lemma 4 and the main result are proved. □

Finally we illustrate the proof by a graph of agents' costs in the constructed partition.

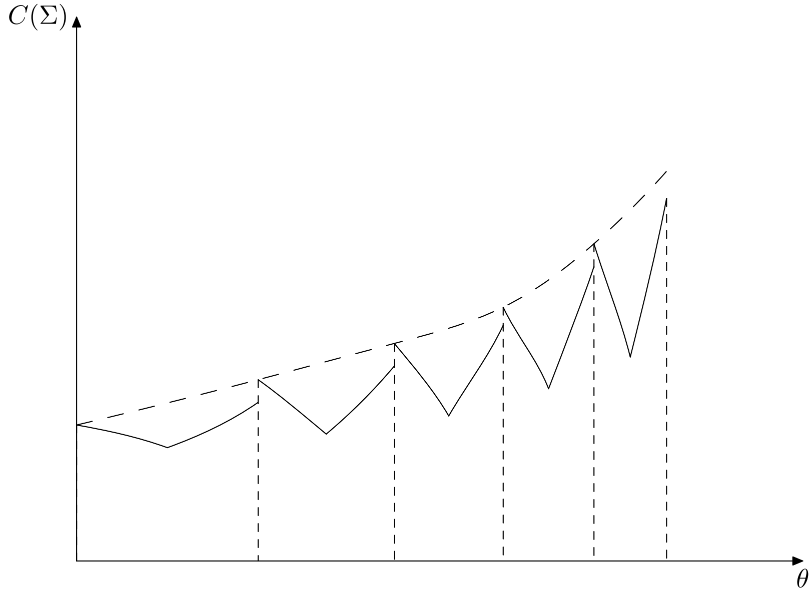

Figure 5: The constructed partition.

The solid line on figure 5 shows agents' costs in psrtition Σ and the dashed line graphs $C^r(y)$. The above proof guarabtees that the dashed line is always above the solid one.

4 Conclusion

The problem of jurisdiction formation is still of current interest. We proved that under monotonically decreasing population density a stable jurisdiction partition always exists. Apparently, a stable partition exists for much broader class of densities, that might approximate the actual housing density. If it is so, real jurisdiction formation may converge to some stable partition as usually occurs within a country. Unfortunately, separatist movements often relegate economic concerns to the background, and polarized human convictions come to the front. In this case there may be no stable configuration and jurisdiction formation should be constrained.

A Proof of theorem 3.1

Consider the following "two-peaked" density: all population is concentrated on ϵ-vicinities of points 0 $\frac{1}{6}$. The population of the first "peak" is 4 and that of the second is 5. It is clear that for all $\epsilon > 0$ there exists a density function with such properties. All calculations in what follows are made to within 2ϵ for sufficiently small ϵ. Show that any partition is unstable.

Denote by L the coalition of all left-peak agents and by R the coalition of all right-peak agents. Suppose that there is a single jurisdiction. Then its capital is on the right peak, right-peak agents bear costs $\frac{1}{9}$ and left-peak agents bear costs $\frac{1}{9}+\frac{1}{6}>\frac{1}{4}$. Thus, L secedes and single-jurisdiction partition is unstable.

Suppose that there are two jurisdictions A and B. By Dirichlet principle at least one jurisdiction (say, A) contains more agents from the right peak then from the left. Then its capital is somewhere on the right peak. If the capital of B is also on the right then everyone gains from uniting jurisdictions ($A \cup B$ is blocking). Suppose that the capital of B is on the left peak. Denote by C the coalition with population 8 that equally contains agents from both peaks and has capital in the middle. Each member of C bears costs $\frac{1}{8}+\frac{1}{12}=\frac{5}{24}<\frac{2}{9}$. If C is not blocking then population of either A or B is greater than $\frac{9}{2}$. If population of A is greater than $\frac{9}{2}$ then it contains all right-peak agents (otherwise remaining right-peak agents want to join A). If it also contains some left-peak agents then L is blocking, since $\frac{1}{4}<\frac{1}{9}+\frac{1}{6}<\frac{1}{|A|}+\frac{1}{6}$. If population of B is greater then $\frac{9}{2}$ then R is blocking by similar reason. Finally, if $A=R$ and $B=L$ then coalition C with capital at point $\frac{1}{10}$ is blocking, since $\frac{1}{8}+\frac{1}{10}=\frac{9}{40}<\frac{1}{4}$ and $\frac{1}{8}+\frac{1}{15}=\frac{23}{120}<\frac{1}{5}$. The remaining case is when the capital of A is on the right peak and the capital of B is somewhere between two peaks. Denote the population of B by $2a$ and the coordinate of its capital by y. If $a<\frac{5}{2}$ then right-peak members of B together with A form a blocking coalition. Thus, population of A is less than 4 and its members bear costs greater than $\frac{1}{4}$. For L and R not being blocking coalitions it is necessary that $\frac{1}{2a}+y<\frac{1}{4}$ and $\frac{1}{2a}+\frac{1}{6}-y<\frac{1}{5}$. After summarizing it turns out that $\frac{1}{a}<\frac{1}{4}+\frac{1}{5}-\frac{1}{6}=\frac{17}{60}$, that implies $a>3$. Thus, members of A bear costs greater than $\frac{1}{3}$ and members of B — not greater than $\frac{1}{6}+\frac{1}{6}<\frac{1}{3}$. Thus, members of A have incentives to join B, and B allows it if new members come equally from both peaks to keep the capital at y. Thus, the only possible situation is when $B=C$ and A has population 1 and is situated on the right peak. If R is not blocking then $\frac{1}{8}+\frac{1}{6}-y<\frac{1}{5}$. If $L \cup A$ is not blocking then $\frac{1}{8}+y<\frac{1}{5}$. These two inequalities imply that $\frac{1}{4}+\frac{1}{6}<\frac{2}{5}$ which is not true.

The last case is the case of three or more jurisdictions. Two jurisdictions cannot have capitals at the same point (and even at the same peak), otherwise they gain from uniting. Thus, at least one of the jurisdictions has capital between two peaks. Further reasoning repeats that of the last case of two-jurisdiction situation.

B Details of the proof of the main result

B.1 Correctness of the partition

Show that constructed in section 3.3 jurisdictions give the whole world in union. Equivalently, $T(y_i)$ must converge to infinity as $i \to \infty$. Equivalently, either y_i or $t(y_i)$ must converge to infinity as $i \to \infty$. Suppose that the latter is not true: $y_i \to y$ and $t(y_i) \to t(y)$. (both sequences have limit since they are increasing and by assumption boundered.) Define $\hat{y}=y+2r(y)>y$. It turns that $y_i+2r(y_i) \to \hat{y}$ as $i \to \infty$. Thus, for sufficiently large i it holds that $y_{i+1}=y_i+2r(y_i)>y$. This contradicts to the definition of y and proves the initial statement.

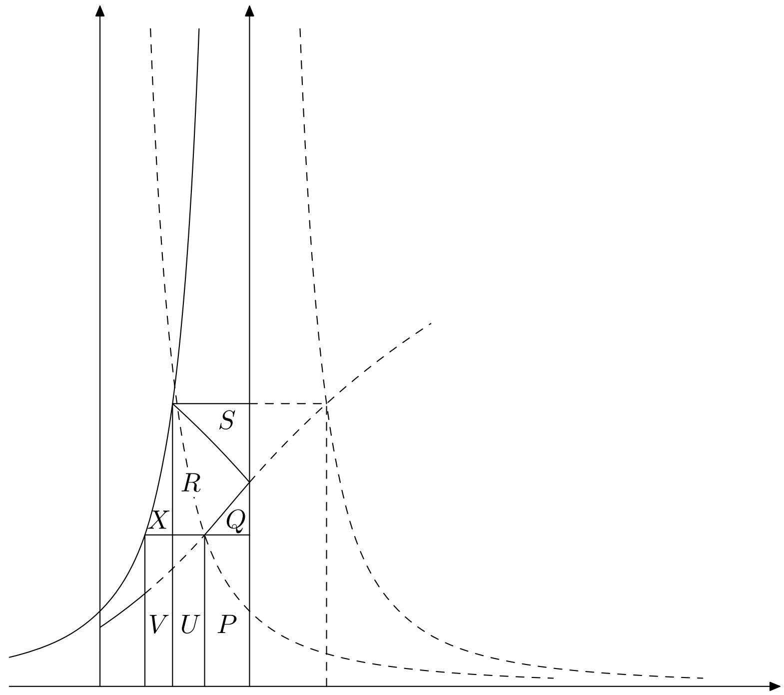

Figure 6: The second case in the proof of inequality (8).

B.2 Proof of technical lemma 5

Let coordintes of A be (x_0, y_0) and those of B be (x_1, y_1). Without loss of generality $0 < x_0 < x_1$. Then area of curvilinear trapezoid A_xABB_x equals $\int_{x_0}^{x_1} \frac{1}{t^2} dt = \frac{1}{x_0} - \frac{1}{x_1}$ and area of curvilinear trapezoid A_yABB_y equals $\int_{y_1}^{y_0} \frac{1}{\sqrt{t}} dt = 2\sqrt{y_0} - 2\sqrt{y_1} = 2\left(\frac{1}{x_0} - \frac{1}{x_1}\right)$, which is twice as large as the first area, q.e.d.

B.3 The second case in lemma 4

A diagram for the second case is presented on figure 6. In this case in terms of figure 3 it holds that $A = P$, $B = Q$, $C = U + V$, $D = P + Q + R + U$ and $E = S$. The desired inequality $2A+B+C < 2D+E$ is equivalent to $2P+Q+U+V < 2P+2Q+2R+2U+S \Leftrightarrow V < Q + 2R + U + S$. According to lemma 5 it turns that $2V + 2X = X + Q + R + S$, from which $V = Q + R + S - X - V$. By substituting this to the last inequality we obtain $-X - V < R + U$. This is true since all values are positive.

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