Signaling an Outside Option

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May 11, 2009

Abstract

We consider the case of an upstream seller who works to improve an asset that has been specialized to a downstream buyer's needs. The buyer then makes a take it or leave it offer to the seller about how the future surplus should be split. We assume that the seller from the outset has private information about the fraction of the surplus that he can realize on his own, and show that this leads to higher investment compared to the complete information case. While a seller with a large default payoff has always strong incentives to invest, now also a seller with a low outside option can choose a large investment, trying to convey the impression of having profitable alternatives. This positive effect on investment is traded off against the occurence of inefficient separations, which result when the buyer mistakenly tries to call the seller's bluff with a low offer.

JEL classification: D23, D82

Keywords: signaling, relationship-specific investment

1 Introduction

In many industries big companies rely on the relationship-specific investments of their suppliers, yet the subcontractors are small firms compared to their customers, and potential customers are few. If the bargaining power lies entirely with the customer, how are the necessary investments induced in this environment where the customer dictates the rules and sueing for payment is unthinkable, i.e., how do the firms overcome a potential hold-up problem? It seems vital for small suppliers

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to have many customers such that in case of separation they can make up for the loss by dealing with others. Suppliers who manage not to be dependent on any one customer may be able to avoid exploitation and be compensated for their investment.

In this paper, we show that information rents resulting from asymmetric information about the position of the supplier in the market may stimulate innovation if the buyer has no other way to commit herself to adequately reward investment. Moreover, there is a signaling motive in the investment choice. If the best alternative use of the relationship-specific asset is private information to the supplier, the buyer will try to deduce this outside option from the level of investment. If the supplier is very reluctant to invest, it seems likely that he fears to be held-up because of a low outside option, and the buyer will then indeed make a low offer. If instead the supplier is very eager to invest, the buyer may belief that the private value from the investment is high, hence she has to make a high offer. Now the possibility arises that a supplier with a low outside option mimics the type with the high outside option and invests more. This effect may mitigate the hold-up problem and lead to higher investment.

The baseline model that we use in this paper is a simplified version of the property rights model developed by Grossman and Hart (1986) and Hart and Moore (1990)[1]. An upstream supplier invests into an asset, which is specific to the relationship with a downstream buyer.[2] It is not possible to write detailed long-term contracts, instead the buyer later makes a take-it-or-leave-it offer to the seller and thus determines how they share the return to investment. If the buyer knows what the seller can maximally accomplish without the cooperation of the buyer, this is what the buyer will offer to the seller. The seller's return and incentives to invest are then completely determined by this outside option.

For the seller to have any incentive to invest at all in this setting, he must own the asset. Property rights to physical assets matter for the efficiency of investment,

[1]See also Hart (1995), Farrell and Gibbons (1995) and de Meza and Lockwood (1998).

[2]An investment is called general if there is a competitive market that fully appreciates the generated return, while a specific investment loses a large part of its value if it is used outside a particular relationship. In the terminology of Klein, Crawford and Alchian (1978), general investments create no appropriable rents, while the quasi-rents that are generated by specific investments may be subject to opportunistic behavior of the other party in the relationship. Another context in which this distinction between general and specific investment is important is the acquisition of human capital (Becker (1964)). The worker, who owns the property right to his human capital, will always acquire an efficient amount of general investment, but firm-specific captial is not so easily induced.

because an owner of an asset can enhance his bargaining position by threatening to use the asset for a different purpose. Necessary for a positive effect on investment is that the alternative use requires the same kind of investment. For example, subcontractors that produce an input for a downstream firm will have some incentives to innovate if they are granted ownership of the asset that they work to improve, or a legal title to the innovation that they develop. As another example, consider an employee who may increase her human capital in the safe knowledge that it cannot be taken away from her and has a value for other employers as well. Nevertheless, if there is a large discrepancy between the asset's value in the current relationship and the next best alternative, investment incentives might be diluted for fear of opportunistic behavior of the other party.

Many investments in machines or human capital are a mix of specific and general investment. We assume in this paper that the investment may be anything in the range from entirely relationship-specific to general. In fact, how specific an investment is also depends on the characteristics of the investing party, e.g. on its access to the market for the asset or its ability to transform the asset to general use. For a worker who has the entrepreneurial ability to use his training to start his own business, all training might be considered general. In contrast, for a worker who has to rely on finding a job in a similar business the specificity of the investment depends on his cost of switching jobs. The degree of specificity determines the investing party's ex post bargaining position, but it may in fact be a hidden characteristic of the investing party. In this paper, we make the assumption that the degree of specificity, as captured by the best alternative use of the investment, is private information to the seller.

We find that this game, in which the seller tries to signal a high outside option with his investment, has a unique equilibrium (modulo out-of-equilibrium beliefs and strategies). If the seller's outside option is known to be relatively low compared to the value of the investment to the buyer, all types of sellers invest the same amount. They choose the same investment as the type with the maximum outside option does under symmetric information. Clearly, in such a pooling equilibrium investments and joint surplus are higher than in the case with complete information.

In general, the equilibrium is a hybrid, or semi-pooling, equilibrium. There is a cut-off type such that all sellers with a lower outside option pool on this type's strategy. This cut-off type, and all higher ones, mix between their own and all higher types' complete information investments. All these types hence separate in the sense that they choose different strategies. Because of the randomization, however, a chosen investment does not give away the type ex post. An observed investment could have been chosen by any type who would invest less under complete information. While the information asymmetry leads to higher investment, this effect is traded against the inefficiency generated by the non-investing party trying to appropriate part of the information rents. How the joint surplus compares to the case with complete information therefore depends on the parameters of the model.

That in our model relationship-specific investment can be used as a signal for an outside option distinguishes this paper from the rest of the literature. The idea that private information about outside options can lead to rents that foster investment has been addressed before, eg. by Malcomson (1997) and Sloof (2008). In these papers, the outside options are realized after investment decisions have been made. Although there is no signaling going on, such models yield similar qualitative predictions: in comparison to the standard hold-up model, which excludes all ex post frictions and focuses on inefficient preparations, there are now greater inefficiencies ex post and less ex ante. In particular, investment levels can be too high relative to their later use. A characteristic of the signaling model, in comparison, is a "bluffing" element that leads to an equilibrium in mixed strategies.[3]

Signaling models by now have a long tradition in economics, starting with Spence (1973), who models education as a wasteful signal of productivity.[4] It is possible to reveal private information with signals like for example warranties or high prices as signals for quality, because the cost of the signal differs across types. In contrast, in our model the cost of investment depends only indirectly on types. Because all types of sellers have the same cost of investment, types only matter if the other party uses her bargaining power and makes low offers. In particular, types of sellers completely separate under symmetric information, while in the original Spence model the wasteful signal is not used at all under symmetric information. This also means that by definition signaling cannot lead to underinvestment in that model, but this changes if one allows education to be productive (see Weiss (1983)). More related to the present paper is recent work on signaling that assumes productive investment and shows that signaling leads to higher investment and even to a Pareto improvement. This includes Hermalin (1998), in which a leader may signal a worthwhile project by exerting high effort, and Daughety and Reinganum

[3]This outcome of an equilibrium in mixed strategies due to a commitment problem is reminiscent of equilibria in hold-up problems with asymmetric information as studied in Gul (2001) and Gonzales (2004).

[4]For an excellent survey of signaling and screening models, see Riley (2001).

(2009), in which a signaling motive helps a team to overcome a free-riding problem.

The remainder of the paper is organized as follows. In Section 2, the outside option signaling game is introduced. We first solve it for a finite type space in Section 3. In Section 4 we analyze the case that the type space is a continuum. We analyze both these cases because it is much more natural to think about the problem using a finite type space, but the solution has a more tractable from in the limit of a continuous type space. We also discuss how changes in the timing or information structure would change the outcome of the game; in particular we analyze a version with commitment in Section 5. Proofs not given in the text can be found in an appendix.

2 The model

The model describes an interaction between a downstream buyer-manufacturer and an upstream supplier who has to tailor his production processes to the needs of the buyer.[5] In the game with complete information, the seller chooses an investment $i \in \mathrm{I}$, at cost $c(i)$, to improve the value of an asset/good to be traded. If seller and buyer work together, they can generate a value of $v(i)$, while the value of the good or asset to the seller without the buyer is only the fraction $\theta v(i), \theta \in \Theta \subset [0, 1]$.[6] The buyer observes the investment and the value of the asset and makes an offer about how to share the surplus with the seller. If the seller rejects the offer, he gets $\theta v(i)$ from taking his outside option, while the buyer is left with zero. If the seller accepts, they split the generated surplus as proposed by the buyer.

Assumption 1. *We assume that $\mathrm{I} = \mathbb{R}$, that the functions v and c are differentiable, increasing, and concave resp. strictly convex. Furthermore $v(0) \geq 0$, $c(0) = 0$, $c'(0) = 0$, and $\lim_{i\to\infty} c'(i) = \infty$.*

The buyer has no way to commit to a particular reaction or to write a contract that conditions on i or $v(i)$ or that specifies a particular bargaining game. Instead she makes a take-it-or-leave-it offer to the seller, which is optimal for her from an ex post perspective, but not necessarily from an ex-ante perspective. If θ is the type of the buyer, i the seller's investment, $o \in [0, 1]$ the buyer's offer, expressed

[5] As explained in the introduction, the model is very abstract and therefore fits a variety of settings, including an employer-employee relationship.

[6] There does not need to be a deterministic relationship between the value and the investment. As long as the downstream party can observe the investment and the value, the analysis remains valid if $v(i)$ represents the expected value.

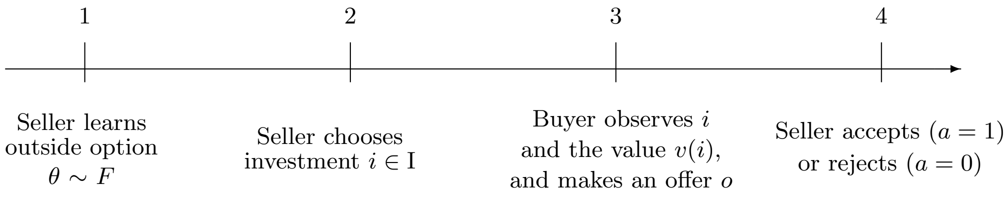

Figure 1: Timeline of the outside option signaling game.

as a share of the surplus, and $a \in \{0, 1\}$ the acceptance decision of the seller, then the seller's payoff is given by

$$(ao + (1 - a)\theta)v(i) - c(i)$$

and the buyer's payoff by

$$a(1 - o)v(i).$$

This game can be easily solved by backward induction. The seller will accept all offers $o > \theta$, and since the buyer can always offer a little bit more, we assume that the seller (except maybe if $\theta = 1$) accepts all offers $o \geq \theta$. The buyer will offer a share θ of the realized surplus, which the seller will accept, leaving him a profit of $\theta v(i) - c(i)$ from investment i. In anticipation of this return to investment the seller invests

$$i^c(\theta) = \arg\max \theta v(i) - c(i),$$

which given our assumptions always exists and is unique. Therefore also the inverse of i^c exists, which we denote by $\theta^c : i^c(\Theta) \to \Theta$. The seller's payoff under complete information, in dependence on the outside option θ, is denoted by

$$u^c(\theta) = \max_i \theta v(i) - c(i).$$

Note that the derivative of u^c is equal to $v \circ i^c$, and in particular, u^c is increasing and strictly convex. [7]

In the game with incomplete information, θ is private information of the upstream seller. The sequence of events is illustrated in Figure 1. We assume that first the seller learns his type θ, which is drawn from a type space $\Theta \subset [0, 1]$ according to a distribution function F. The buyer only knows the distribution of the outside option, but not the realized value. She observes the seller's investment, forms

[7] We could alternatively make this, or other conditions from which it follows, our assumption. That is, investment decisions are allowed to be multi-dimensional or discrete as long as the optimal investment levels lead to an increasing and strictly convex function u^c.

beliefs about the outside option and then makes a take-it-or-leave-it offer that is optimal for her given her updated beliefs about the acceptance threshold of the seller. We are interested in subgame perfect equilibria of this game, and in such an equilibrium a seller of type θ will accept an offer if and only if it is greater than the outside option. We therefore take this acceptance decision, the same as in the game with perfect information, for granted, and deal with the following payoff functions: if the seller is of type θ and invests i, and the buyer makes an offer o, then the seller gets $\max(\theta, o)v(i) - c(i)$ and the buyer gets $(1 - o)v(i)$ if $\theta \leq o$, and 0 else.

A strategy of the seller specifies an investment for each type, possibly using a randomization device to mix over a set of investments. A strategy of the seller is a function $Q : \Theta \times \mathrm{I} \to [0, 1]$ such that $Q(.|\theta) = Q(\theta, .)$ is the distribution of investments that a type θ chooses. A strategy for the buyer maps all possible investments into a share of the surplus that she offers to the seller, where she as well may randomize over a set of offers. We write a strategy of the buyer as a function $P : \mathrm{I} \times [0, 1] \to [0, 1]$, where $P_i(o) := P(i, o)$ is the probability that the buyer's offer, when observing investment i, is less or equal to o.

If the buyer's strategy is given by P, the seller's expected profit from choosing investment i is

$$U(P, i, \theta) = \int \max(\theta, o) dP_i(o)\ v(i) - c(i),$$

and given a strategy Q of the seller, the buyer's expected payoff from the pure strategy $o : \mathrm{I} \to [0, 1]$ is

$$V(Q, o) = \int \int_{[\theta \leq o(i)]} (1 - o(i))v(i)dQ(i|\theta)dF(\theta).$$

3 Finite type space

In this section, we assume that $\Theta = \{\theta_1, ..., \theta_H\}$ with $0 \leq \theta_1 < \theta_2 < ... < \theta_H < 1$.[8] We shortcut $i^c(\theta_k) =: i_k$. In the following, we fix an equilibrium of the signaling game (P, Q). We will derive properties of (P, Q), in order to eventually arrive at a characterization of all equilibra of the outside option signaling game. Let I^* be the set of investments that are chosen with positive probability in the equilibrium (P, Q), and let $\Theta^*(i)$ denote the set of all types that choose $i \in I^*$ with positive probability. We denote the equilibrium payoff to a seller of type θ by $u^*(\theta)$, i.e., with this notation we have for all $i \in \mathrm{I}^*$ and $\theta \in \Theta^*(i)$ that $u^*(\theta) = U(P, i, \theta)$.

[8]The assumption $\theta_H < 1$ is made only for simplicity. We could easily add types $\theta \geq 1$ who would always invest $i^c(\theta)$ and get no acceptable offer from the buyer. That is, a type $\theta \geq 1$ seller would neither mimic other types nor be mimicked himself.

Note that $u^*(\theta) \geq u^c(\theta)$, because a type θ can always guarantee himself the payoff $u^c(\theta)$ independent of the buyer, by investing $i^c(\theta)$ and taking his outside option. Similarly, because the seller's payoff is weakly increasing in θ for all offers and investments, $U(P, i, \theta)$ and $u^*(\theta)$ are weakly increasing in θ. A higher type could always play a lower type's strategy and get at least the same payoff as that type.

In the following, we will first show that if an investment i may occur at all in equilibrium, then it is chosen with positive probability by the type $\theta^c(i)$ that chooses i under symmetric information, and by none of the higher types. Then, in Lemma 2, we show that investing i is optimal for all types from θ_1 to $\theta^c(i)$. Finally, in Prop. 5 we will answer the question which investments will be chosen in equilibrium. The reader who is not interested in the proofs may skip the lemmas leading to Prop. 5 which contains the main result of this section.

When the buyer observes an investment $i \in \mathrm{I}^*$, she updates that the seller must have an outside option in $\Theta^*(i)$. The share she offers will therefore also lie in $\Theta^*(i) \subset \{\theta_1, ..., \theta_H\}$, and it will never be more than the highest possible type would accept, i.e. the offer is not higher than $\theta_m = \max \Theta^*(i)$. The profit to type θ_m from choosing i is therefore equal to $\theta_m v(i) - c(i)$, which would be strictly smaller than $u^c(\theta_m)$ if $i \neq i_m$. Therefore $i = i_m$, which means that if an investment i occurs in the signaling equilibrium, then $\theta^c(i)$ the highest type to choose this investment. In particular, only investments $i_k, k = 1, ..., H$ occur in equilibrium.

We will sometimes use the one-to-one relationship between θ_k and i_k and express everything in types. We can also identify the buyer's offer with the type that just accepts it, and then write the equilibrium strategies P and Q as matrices. An entry p_{kl} of P stands for the probability of offer θ_l when investment i_k is observed, and an entry q_{kl} in Q is the probability of type k investing i_l, or "mimicking" type l. Since we have shown that in any equilibrium the mixed strategy of type θ_k has support $\{i_k, ..., i_H\}$ and the buyer's random offer following investment i_k takes on values in $\{\theta_1, ..., \theta_k\}$, equilibrium strategies P and Q are triangular matrices. Equilibrium conditions for strategies (P, Q) in matrix form then look as follows:

(i) $q_{kl} > 0$ implies that

$$l \in \arg\max_m \sum_{j=1}^{m} p_{mj} \max(\theta_j, \theta_k) v(i_m) - c(i_m),$$

(ii) for each l with $i_l \in \mathrm{I}^*$, $p_{lj} > 0$ implies that

$$j \in \arg\max_k (1 - \theta_k) \sum_{j=1}^{k} f_j q_{jl}.$$

We will show next that the set of best responses to P of a given type θ_k includes all investments that are greater or equal than i_k and are chosen at all in the equilibrium. In other words, if an investment i_k is chosen at all, then it is optimal for every type smaller or equal to the corresponding type θ_k.

Lemma 2. *For all $i_k \in \mathrm{I}^*$ it holds that $U(P, i_k, \theta) = u^*(\theta)$ for all $\theta = \theta_1, ..., \theta_k$.*

Proof. We know already that $U(P, i_k, \theta_k) = u^*(\theta_k)$. First, we show that this also holds for the lowest type, i.e. that $U(P, i_k, \theta_1) = u^*(\theta_1)$. To this end, let θ_l be the lowest type with this property, i.e., $U(P, i_k, \theta_l) = u^*(\theta_l)$ and $U(P, i_k, \theta) < u^*(\theta)$ for all $\theta < \theta_l$. Since no type below θ_l chooses i_k, the offer following it cannot be lower than θ_l. Type l's expected payoff then does not depend on him being type θ_l, but every lower type would get the same payoff when investing i_k :

$$U(P, i_k, \theta_l) = \int odP_{i_k}(o)v(i_k) - c(i_k) = U(P, i_k, \theta) \text{ for all } \theta \leq \theta_l.$$

Payoff monotonicity then implies that $U(P, i_k, \theta) = u^*(\theta)$ for any type $\theta \leq \theta_l$, hence $l = 1$.

Second, we show that for a seller of type θ_l the investments that are best responses to P can be found by maximizing $P_i(\theta_{l-1})v(i)$ over all $i \in \mathrm{I}^*$, where we define $P_i(\theta_0) = 0$. More precisely, the claim is

$$\arg\max_{i \in \mathrm{I}^*} U(P, i, \theta_l) = \arg\max_{i \in \mathrm{I}^*} P_i(\theta_{l-1})v(i) \subset \arg\max_{i \in \mathrm{I}^*} U(P, i, \theta_{l-1}). \tag{1}$$

It is clear that the claim implies the lemma, since then

$$i_k \in \arg\max_{i \in \mathrm{I}^*} U(P, i, \theta_k) \subset ... \subset \arg\max_{i \in \mathrm{I}^*} U(P, i, \theta_1).$$

It remains to prove the claim, which we will do by induction. Since we know that $U(P, i, \theta_1) = u^*(\theta_1)$ for all $i \in \mathrm{I}^*$, it holds for $l = 1$ for the appropriate definitions. Assume the claim is true for type $l - 1 \geq 1$. For all $i \in \mathrm{I}^*$ with $u^*(\theta_{l-1}) = U(P, i, \theta_{l-1})$ type θ_l's payoff is

$$U(P, i, \theta_l) = u^*(\theta_{l-1}) + (\theta_l - \theta_{l-1})P_i(\theta_{l-1})v(i). \tag{2}$$

while for any $i' \in \mathrm{I}^*$ with $U(P, i', \theta_{l-1}) < u^*(\theta_{l-1})$ it holds that

$$U(P, i', \theta_l) < u^*(\theta_{l-1}) + (\theta_l - \theta_{l-1})P_{i'}(\theta_{l-1})v(i'). \tag{3}$$

Using the induction hypothesis, we have that for any such i and i'

$$P_{i'}(\theta_{l-1})v(i') = P_{i'}(\theta_{l-2})v(i') \leq P_i(\theta_{l-2})v(i) \leq P_i(\theta_{l-1})v(i),$$

hence we have shown that $U(P, i', \theta_l) < U(P, i, \theta_l)$. The remainder of the claim follows easily. □

To summarize, we have shown so far that in any equilibrium, while there may be investments that do not occur at all, every investment that does occur is chosen by the type that would invest the same amount with symmetric information. Furthermore, all lower types' payoff from choosing this investment equals their equilibrium payoff. In order to be consistent with this structure, the buyer's strategy must induce all these indifferences. This observation gives rise to the following lemma.

Lemma 3. *For all k and $i \in \mathrm{I}^*$ with $m > k$ it holds that*

$$P_{i_m}(\theta_k)v(i_m) = \frac{u^*(\theta_{k+1}) - u^*(\theta_k)}{\theta_{k+1} - \theta_k}. \tag{4}$$

Moreover, for all $i_m, i_k \in \mathrm{I}^*$ with $m \geq k$ it holds that $p_{mk} > 0$.

Proof. The first claim follows from the proof of Lemma 2, because there we had that for all $i_m \in \mathrm{I}^*$ with $m > k$ it holds that

$$u^*(\theta_{k+1}) = u^*(\theta_k) + (\theta_{k+1} - \theta_k)P_{i_m}(\theta_k)v(i_m).$$

To show the last claim of the lemma, note first that for any type θ_k with $i_k \in \mathrm{I}^*$ it must be true that $p_{kk} > 0$, because else $U(P, i_k, \theta_{k-1})$ is too low: if $p_{kk} = 0$, this payoff is equal to

$$U(P, i_k, \theta_{k-1}) = ((1-p_{kk})\theta_{k-1} + p_{kk}\theta_k)v(i_k) - c(i_k) = \theta_{k-1}v(i_k) - c(i_k) < u^c(\theta_{k-1}).$$

Second, assume that for $m > k$ as in the lemma we have $p_{mk} = 0$. Then

$$0 = P_{i_m}(\theta_k)v(i_m) - P_{i_m}(\theta_{k-1})v(i_m) = \frac{u^*(\theta_{k+1}) - u^c(\theta_k)}{\theta_{k+1} - \theta_k} - \frac{u^c(\theta_k) - u^*(\theta_{k-1})}{\theta_k - \theta_{k-1}},$$

whence

$$u^c(\theta_k) = u^*(\theta_{k+1})\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + u^*(\theta_{k-1})\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}}$$

As mentioned before, the function u^c is strictly convex. Therefore, and because

$$\theta_k = \theta_{k+1}\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + \theta_{k-1}\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}},$$

we have that

$$u^c(\theta_k) < u^c(\theta_{k+1})\frac{\theta_k - \theta_{k-1}}{\theta_{k+1} - \theta_{k-1}} + u^c(\theta_{k-1})\frac{\theta_{k+1} - \theta_k}{\theta_{k+1} - \theta_{k-1}}.$$

Hence, $p_{mk} > 0$. □

Now that we have some idea about the offers that the buyer must be willing to make, we turn to a description of the buyer's behavior, in order to pin down the seller's equilibrium strategy. The details can be found in the proof of the following proposition that describes the structure of an equilibrium. But first we need more notation and an assumption:

Assumption 4. *Let* $R(\theta) := (1-\theta)F(\theta)$ *and* $\bar{k} := \min\{k : \ R(\theta_k) > R(\theta_{k+1})\}$.[9] *We assume that* R *is strictly concave on* $\{\theta_{\bar{k}}, ..., \theta_H\}$.

Assume for a moment that all types choose the same investment i. Then $R(\theta)$ describes the buyer's expected share of the surplus $v(i)$ if she makes a take it or leave it offer of θ. The maximum $\bar{\theta}$ of this function is the offer that she would make in a pooling equilibrium. Can a pooling equilibrium exist? Since the highest type θ_H chooses i_H in any equilibrium, if all types pool on the same investment, this must be i_H. It follows that there is such a pooling equilibrium if and only if $\bar{\theta} = \theta_H$. This suggests that complete pooling is only possible for types lower than $\bar{\theta}$, and since a separating type could easily be mimicked by a lower type, equilibria must be hybrids between separating and pooling.

Proposition 5. *If Assumption 4 holds, then an equilibrium of the signaling game must have the following form: No investment below* $i_{\bar{k}}$ *is chosen. A type* θ_k *with* $k \geq \bar{k}$ *mixes between all investments in* $\{i_k, ..., i_H\}$*, with expected payoff equal to* $u^c(\theta_k)$*. All types* θ_k *with* $k \leq \bar{k}$ *mix over* $\{i_{\bar{k}}, ..., i_H\}$ *with payoff* $u^c(\theta_{\bar{k}})$*. When observing investment* i_k*, the buyer mixes between offers in* $\{\theta_{\bar{k}}, ..., \theta_k\}$*, and her expected payoff from any such offer is* $(1-\theta_k)v(i_k)$.

Proof. See the Appendix. □

All equilibria of the outside option signaling game lead to the same payoffs. Refinements to pin down beliefs following zero probability events are not needed for this result. The reason is that even if an investment i_k does not trigger an acceptable offer from the buyer, type θ_k can still get $u^c(\theta_k)$ by himself.

From all the indifference conditions that have to be met in an equilibrium we are able to obtain an equilibrium candidate. Combining Prop. 5 and Lemma 3 yields for all $k \geq \bar{k}$ and $m > k$

$$P_{i_m}(\theta_k) = \frac{u^c(\theta_{k+1}) - u^c(\theta_k)}{(\theta_{k+1} - \theta_k)v(i_m)} \quad \text{and } P_{i_k}(\theta_k) = 1, \tag{5}$$

[9] Let $\theta_{H+1} = 1$.

as well as for $k < \bar{k}$

$$P_{i_m}(\theta_k) = 0. \tag{6}$$

The equilibrium conditions for the seller's strategy are

$$(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} = (1-\theta_k)\sum_{j=1}^{k} f_j q_{jk} \quad \text{for all } k \geq l \geq \bar{k} \tag{7}$$

and

$$(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} \leq (1-\theta_k)\sum_{j=1}^{k} f_j q_{jk} \quad \text{for all } l < \bar{k}. \tag{8}$$

Due to the definition of $\bar{k}$, the latter condition can be fulfilled by defining

$$q_{jk} = q_{\bar{k}k} \qquad \text{for all } j < \bar{k}. \tag{9}$$

Let us further define $\lambda_k := \frac{f_k(1-\theta_k)(1-\theta_{k-1})}{\theta_k-\theta_{k-1}}$ and $\lambda_{H+1} := 0$. Possible values for the q_{jk} are:

$$\begin{aligned}
q_{\bar{k}k} &= \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} && \text{for all } k > \bar{k} && (10)\\
q_{\bar{k}\bar{k}} &= 1 - \frac{\lambda_{\bar{k}+1}}{R(\theta_{\bar{k}})} && && (11)\\
q_{jk} &= \frac{\lambda_k - \lambda_{k+1}}{\lambda_j} && \text{for all } k \geq j > \bar{k} && (12)
\end{aligned}$$

Proposition 6. *The strategies described in equations (5), (6), (9), (10), (11) and (12) form an equilibrium of the outside option signaling game.*

Proof. See the Appendix. □

Example

We look at an example with three types to illustrate the different kinds of equilibrium and the uniqueness issue. First, since $R(\theta)$ is the buyer's expected share of $v(i)$ if all types choose the same investment i and the buyer offers θ, pooling on the investment i_3 is an equilibrium if and only if $(1-\theta_3) = \max_\theta R(\theta)$. We write this equilibrium in the matrix form described at the beginning of this section:

$$\mathrm{Q} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \ \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Note that beliefs out of equilibrium, i.e. after observing an investment $i \neq i_3$, are not pinned down uniquely. Consequently also the first two rows in P are not uniquely determined.

In case $(1-\theta_2)F(\theta_2) = \max_\theta (1-\theta)F(\theta)$ an equilibrium is of the following form:

$$\mathrm{Q} = \begin{pmatrix} 0 & q_{12} & 1-q_{12} \\ 0 & q_{22} & 1-q_{22} \\ 0 & 0 & 1 \end{pmatrix}, \ \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & p_{32} & 1-p_{32} \end{pmatrix}$$

Again, the first row of P does not have to be the unit vector. To see how the structure of Q translates into the condition for R, let $\mu_2 := q_{22}f_2 + q_{12}f_1$ be the probability of i_2 being chosen (here the same as the probability of any lower investment being chosen). The conditions for the buyer are

- $(1-\theta_3)(1-\mu_2) = (1-\theta_2)(F(\theta_2)-\mu_2)$ which is equivalent to $\mu_2 = \frac{R(\theta_2)-R(\theta_3)}{\theta_3-\theta_2}$. This expression is always less or equal to 1, and it is nonnegative iff $R(\theta_2) \geq R(\theta_3)$.
- $(1-\theta_2)(F(\theta_2)-\mu_2) \geq (1-\theta_1)(F(\theta_1)-q_{12}f_1)$ which is equivalent to $q_{12}f_1 \geq \frac{R(\theta_1)-R(\theta_2)}{1-\theta_1} + \frac{(1-\theta_2)\mu_2}{(1-\theta_1)}$
- $(1-\theta_2)\mu_2 \geq (1-\theta_1)q_{12}f_1$ which is equivalent to $q_{12}f_1 \leq \frac{(1-\theta_2)}{(1-\theta_1)}\mu_2$

Obviously, the last two conditions can only be fulfilled if $R(\theta_1) \leq R(\theta_2)$. If this holds, the solutions are $q_{12} = \frac{(1-\theta_2)\mu_2}{R(\theta_1)} - \Delta$ for any $0 \leq \Delta \leq \frac{R(\theta_2)-R(\theta_1)}{R(\theta_1)}$. Thus, in this case the solution is typically not unique. If we make the restriction $q_{12} = q_{22}$, the last two conditions, which state that the buyer prefers offering θ_2 to offering θ_1, read

- $q_{12}f_1 \geq \frac{R(\theta_1)-R(\theta_2)}{1-\theta_1} + \frac{R(\theta_2)q_{12}}{(1-\theta_1)} \iff 1 \geq q_{12}$
- $f_1 \leq \frac{(1-\theta_2)}{(1-\theta_1)}F(\theta_2) \iff R(\theta_1) \leq R(\theta_2)$

That is, we immediately have a solution, given by $q_{12} = q_{22} = \frac{R(\theta_2)-R(\theta_3)}{F(\theta_2)(\theta_3-\theta_2)}$. This is not surprising, because here the pooling condition (R increasing) holds up to θ_2. The proposed equilibrium in Prop. 6 also uses this fact. The buyer's expected profit does not depend on the values of q_{12} and q_{22}, only on μ_2.

If $(1-\theta_1)F(\theta_1) = \max_\theta (1-\theta)F(\theta)$, then the equilibrium is unique:

$$\mathrm{Q} = \begin{pmatrix} q_{11} & q_{12} & 1-q_{11}-q_{12} \\ 0 & q_{22} & 1-q_{22} \\ 0 & 0 & 1 \end{pmatrix}, \mathrm{P} = \begin{pmatrix} 1 & 0 & 0 \\ p_{21} & 1-p_{21} & 0 \\ p_{31} & p_{32} & 1-p_{31}-p_{32} \end{pmatrix}$$

For the values of the entries, see Proposition 6. The expressions may become complex, that is why we look at a continuous strategy space in the next section.

We know from Prop. 5 that a strategy of the form

$$\mathrm{Q} = \begin{pmatrix} q_{11} & 0 & 1 - q_{11} \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

cannot be part of an equilibrium. This can be checked explicitly here, showing that for this to be an equilibrium it must be true that $R(\theta_1) = \max_\theta R(\theta)$ and R convex, contradicting our assumption that R is concave. While it might be possible to relax this assumption and still say something about the resulting equilibria, we do not address this question in this paper.

4 Continuous type space.

The expressions for equilibrium strategies will have a simpler form in this section, which treats the continuous type space as the limit case. Hence, in this section $\Theta = [\theta_L, \theta_H]$. We assume that F is an atomless distribution on Θ with density $f > 0$, for which the derivative f' exists.

Assumption 7. *F is log-concave.*

Analogous to the previous section, we define $\bar{\theta} = \theta_H$ if $R'(\theta) \geq 0$ on Θ, and else

(13) $$\bar{\theta} = \inf\{\theta \in \Theta : \; R'(\theta) < 0\},$$

and have

Remark 8. *Given Assumption 7, R is concave on $[\bar{\theta}, \theta_H]$ and $\bar{\theta} = \arg\max_\theta R(\theta)$.*

Proof. To show that F log-concave (which is implied by f log-concave) is sufficient for this property of R, we will show first that

$$R''(\theta) \geq 0 \Rightarrow R'(\theta) > 0.$$

The second derivative of R is

$$R''(\theta) = (1 - \theta) f'(\theta) - 2f(\theta)$$

such that $R''(\theta) \geq 0$ implies that $f'(\theta) \geq 0$ and

$$(1 - \theta) \geq \frac{2f(\theta)}{f'(\theta)}.$$

Hence,

$$R'(\theta) = (1-\theta)f(\theta) - F(\theta) \geq \frac{2f(\theta)^2 - F(\theta)f'(\theta)}{f'(\theta)} \geq \frac{f(\theta)^2}{f'(\theta)} > 0.$$

From the definition of $\bar{\theta}$, where we have a local maximum, the claim easily follows.

□

Proposition 9. *Given Assumption 7, an equilibrium of the signaling game is given by*

$$P_i(\theta) = \begin{cases} 0 & \theta < \bar{\theta} \\ \frac{v(i^c(\theta))}{v(i)} & \bar{\theta} \leq \theta \leq \theta^c(i) \\ 1 & \theta \geq \theta^c(i) \end{cases} \tag{14}$$

and $Q(i|\theta) = Q(i|\bar{\theta})$ for all $\theta < \bar{\theta}$, and for all $\theta \geq \bar{\theta}$

$$Q(i|\theta) = \begin{cases} 0 & i < i^c(\theta) \\ 1 - \frac{(1-\theta^c(i))^2 f(\theta^c(i))}{(1-\theta)^2 f(\theta)} & i^c(\theta) \leq i < i^c(\theta_H) \\ 1 & i = i^c(\theta_H) \end{cases} \tag{15}$$

The proof is straightforward and therefore omitted. It can also be shown that this equilibrium is the limit of the equilibrium found in the previous section (Prop. 6) as the partition becomes finer.

4.1 Surplus Comparison

In the following paragraphs, we compare different timings and information regimes with respect to the payoff that is generated for the seller and the buyer as well as the joint surplus. In some applications as for example the mobility of a worker, it seems realistic that the worker knows his mobility but the employer never learns it until it is too late. Alternatively, it may be the case that the worker learns his outside option only after making the firm-specific investment. In a market setting, it may be that the outside option is known to both sides from the start, or that both sides learn it after investment decisions have been made. We always evaluate payoffs and surplus with respect to the distribution F, and that is also how the expectations in the following expressions should be understood.

First we look at the case of complete information. The difference to the signaling model is that in this case the outside option is common knowledge even before investment is undertaken. The seller's expected profit is $E[u^c(\theta)]$ and the buyer gets $E[(1-\theta)v(i^c(\theta))]$. The expected joint surplus is $E[S(i^c(\theta))]$ with $S := v - c$.

If the outside option becomes common knowledge only after the investment is sunk, and is not known before to any party, the expected social surplus is $S(i^c(E[\theta]))$. If we assume that $S(i^c(\theta))$ is concave function in θ (eg. $v''' - c''' \leq 0$) then this surplus is higher than the one before. The seller gets $u^c(E[\theta])$ and is therefore worse off than in the complete information case, because he cannot prepare for his outside option. The buyer is better off with $(1 - E[\theta])v(i^c(E[\theta]))$, capturing the quasi-rent from low types who invest too much.

A third timing and information structure of the game is that the seller, and only the seller, learns the outside option later. In this case, there is no signaling motive. The buyer makes an offer of $\bar{\theta}$ and the seller invests $i^c(E[\theta \vee \bar{\theta}])$. While the investment is higher than in the two cases above, it is not always put to its best use, as all types above $\bar{\theta}$ reject the offer. The seller gets $u^c(E[\bar{\theta} \vee \theta])$ which is more than in the previous case, as he enjoys some informations rents. The buyer gets $R(\bar{\theta})v(i^c(E[\theta \vee \bar{\theta}]))$.

Finally, in the signaling equilibrium (Prop. 9), a seller with outside option θ gets $\max(u^c(\theta), u^c(\bar{\theta}))$, i.e. the seller's expected profit is

$$F(\bar{\theta})u^c(\bar{\theta}) + \int_{\bar{\theta}}^{\theta_H} u^c(\theta)f(\theta)d\theta.$$

To find the buyer's surplus in the signaling equilibrium, note first that $-R''(\hat{\theta})$ is the probability density of investment on $[\bar{\theta}, \theta_H)$. Therefore, the buyer's expected payoff is

$$\int_{\bar{\theta}}^{\theta_H} -R''(\hat{\theta})(1 - \hat{\theta})v(i^c(\hat{\theta}))d\hat{\theta} + (1 - \theta_H)^2 f(\theta_H)v(i(\theta_H))$$

We see that the seller has an incentive to learn the outside option early, because in the signaling equilibrium his expected payoff is $E[u^c(\theta \vee \bar{\theta})] > u^c(E[\theta \vee \bar{\theta}])$. It is also better for him in expected terms if it is common knowledge that he is aware of his outside option, but its value is secret.

If sellers came from two different distinguishable groups, such that the distribution of outside options for one group first order dominates the distribution of the other group, then all sellers in the group with higher outside options are better off if the cut-off value is also shifted to the right. In addition to being directly beneficial to the seller, higher outside options also have the effect that lower types can hide behind the higher average bargaining power in their group. In the cases where the cut-off value is not increased, the sellers may be worse off. Consider for example the case of only two possible outside options and a pooling equilibrium. If the higher of the possible values increases, the equilibrium may become the semi-pooling one, and this may make the seller worse off in expected terms.

The effects of a FOSD shift in the distribution of types on the buyer's profit is ambiguous. While the presence of higher types leads to higher investment, it also means that the buyer has to make higher offers. This reflects the nonmonotonic relationship between the buyer's profit and the type of the seller with symmetric information: low types generate low surplus and high types get a large share of the pie, so that typically the buyer prefers sellers of intermediate bargaining power. In the next section we consider the case that the investment decision is contractible. It is clear that in that case, lower seller types are unambiguously better for the buyer.

5 The case with commitment

In the game that is studied in the main part of this paper, all the buyer can do is make a take it or leave it offer based on her updated beliefs. In this section we shall explore the consequences of full commitment and ask what would happen if the buyer could offer a binding contract conditional on investment before the seller moves. We assume that she still cannot observe the seller's type and characterize the optimal screening contract. While in the signaling model the seller moves first, now the buyer can act before the seller takes the investment decision. [10]

Proposition 10. *If the buyer can write a contract on the investment decision, the outcome involves investment of $i^c(1)$ and inefficient separation if $\theta \geq \bar{\theta}$: Such a type θ takes the outside option with probability $x(\theta) = \frac{v(i^c(\theta))}{v(i^c(1))}$. Each seller type is left with the same payoff as in the case without commitment,* $\max(u^c(\theta), u^c(\hat{\theta}))$*. The buyer gets $S(i^c(1)) - u^c(\bar{\theta}) - \int_{\bar{\theta}}^{\theta_H} S(i^c(\theta))dF(\theta)$.*

Proof. We use the revelation principle and let a general contract be a map from types into outcomes that satisfies the incentive compatibility constraints of each type of seller telling the truth. The buyer also has to take into account that the seller can go for his outside option, then getting a payoff of $\theta v(i)$ after having invested an amount i, and $u^c(\theta)$ ex ante.

All that matters for truth telling and participation of the seller is his expected payoff, and the buyer in addition cares for the surplus created by the contract. Therefore, it is sufficient to concentrate on contracts of the form $(t(\theta), i(\theta), x(\theta))$,

[10] Adverse selection problems with type-dependent reservation utilities have been addressed before, but in different frameworks (Moore (1985), Jullien (2000)). Our problem has a much simpler structure, but is not a special case of these results.

where $t(\theta)$ is an up-front payment from the seller to the buyer, $i(\theta)$ is the investment that an announced type θ is required to make, and $x(\theta)$ is the probability of separation. With probability $1 - x(\theta)$, buyer and seller collaborate and the seller gets the whole ex post surplus $v(i(\theta))$. There is no loss of generality in assuming this form of contracts, because all payoff transfers from the seller to the buyer can be handled by the up-front payment $t(\theta)$. Given such a contract, the expected payoff to a seller of type θ who pretends to be of type $\tilde{\theta}$ is

$$(1 - x(\tilde{\theta}))v(i(\tilde{\theta})) + x(\tilde{\theta})\theta v(i(\tilde{\theta})) - c(i(\tilde{\theta})) - t(\tilde{\theta}).$$

A truth-telling seller creates the joint surplus $S(i(\theta)) - x(\theta)(1-\theta)v(i(\theta))$, and gets $u_S(\theta) = S(i(\theta)) - x(\theta)(1-\theta)v(i(\theta)) - t(\theta)$ for himself. The buyer's optimization problem is the following:

$$\max \int_{\theta_L}^{\theta_H} t(y)dF(y),$$

subject to the incentive compatibility constraint

$$\text{(IC)} \qquad u_S(\theta) \geq u_S(\tilde{\theta}) + (\theta - \tilde{\theta})x(\tilde{\theta})v(i(\tilde{\theta}))$$

and the ex ante participation constraint

$$\text{(PC)} \qquad u_S(\theta) \geq u^c(\theta),$$

which have to hold for all $\theta, \tilde{\theta} \in [\theta_L, \theta_H]$.

It may seem intuitive that an optimal contract specifies efficient investment, because the seller types do not differ with respect to the cost of investment, only with respect to the outside option. The screening device therefore is the probability of separation, not the investment. However, since in order to separate the seller's types this probability must be positive, it is not obvious that $i(\theta) = i^c(1)$, because $i^c(1)$ is not the optimal preparation for every type (which would be $i^c(1-(1-\theta)x(\theta))$). In particular, so far the formulation also allows for some types not participating and choosing $x = 1, i = i^c(\theta), t = 0$.

To see that setting $i(\theta) = i^c(1)$ is without loss of generality, consider any contract $(t(\theta), i(\theta), x(\theta))$. The contract $(\tilde{t}(\theta), \tilde{i}(\theta), \tilde{x}(\theta))$ defined by

$$\tilde{t}(\theta) = t(\theta) + S(i^c(1)) - S(i(\theta) \geq t(\theta),$$

$\tilde{i}(\theta) = i^c(1)$, and

$$\tilde{x}(\theta) = x(\theta)\frac{v(i(\theta))}{v(i^c(1))} \in [0, 1]$$

leads to the same IC and PC constraints and weakly higher expected profit for the buyer. In particular, this means that excluding types is not a good idea for the buyer.

For any $x : [\theta_L, \theta_H] \to [0, 1]$ that is part of an IC contract, if $x(\tilde{\theta}) = 0$ for some type $\tilde{\theta}$, then we know that lower types pool on this type, i.e. $u_S(\tilde{\theta}) = u_S(\theta)$ for all types $\theta \le \tilde{\theta}$. In the buyer's optimal contract it will then hold that $x(\theta) = 0$ and $t(\theta) = S(i^c(1)) - u^c(\tilde{\theta})$ for all $\theta \le \tilde{\theta}$. We therefore now take a threshold $\theta^0 \in \Theta$ as given and replace the IC constraints by the requirement that x is nondecreasing and

$$u_S(\theta) = \int_{\theta_0}^{\theta} x(\tilde{\theta}) v(i^c(1)) d\tilde{\theta} + u^c(\theta^0).$$

We define $X^0 := \{x : [\theta^0, \theta_H] \to (0, 1], \text{ nondecreasing}\}$. Following the standard method of finding an optimal screening contract we write the problem as

$$\max_{x \in X^0} S(i^c(1)) - u^c(\theta^0) - \int_{\theta^0}^{\theta_H} (R'(\theta) + 1) x(\theta) v(i^c(1)) d\theta$$

s.t. $\int_{\theta^0}^{\theta} x(\tilde{\theta}) v(i^c(1)) d\tilde{\theta} \ge u^c(\theta) - u^c(\theta^0)$.

Because $R'(\theta) + 1 \ge 0$, $x(\theta)$ must be as small as possible. This suggests that the PC should bind everywhere, which we will indeed show next. First, because the objective function can also be written as

$$S(i^c(1)) - u_S(\theta_H) - \int_{\theta^0}^{\theta_H} R'(\theta) x(\theta) v(i^c(1)) d\theta$$

it is clear that $\theta^0 \ge \bar{\theta}$. Furthermore, for the part that depends on x we can use integration by parts to get

$$\begin{aligned}
&u_S(\theta_H) + \int_{\theta^0}^{\theta_H} R'(\theta) x(\theta) v(i^c(1)) d\theta \\
=&(1 - \theta_H) f(\theta_H) u_S(\theta_H) - R'(\theta^0) u^c(\theta^0) - \int_{\theta^0}^{\theta_H} R''(\theta) u_S(\theta) d\theta \\
\ge&(1 - \theta_H) f(\theta_H) u^c(\theta_H) - R'(\theta^0) u^c(\theta^0) - \int_{\theta^0}^{\theta_H} R''(\theta) u^c(\theta) d\theta
\end{aligned}$$

This shows that the objective function is maximized if the PC is binding everwhere. For this to be true, the buyer would have to set

$$x(\theta) = \frac{v(i^c(\theta))}{v(i^c(1))},$$

which is indeed increasing, hence must be the solution to the optimization problem. Finally, we find the optimal θ_0: Solving

$$\max_{\theta_0} S(i^c(1)) - u^c(\theta_0) - \int_{\theta_0}^{\theta_H} (R'(\theta) + 1) v(i^c(\theta)) d\theta$$

yields $\bar{\theta}$ as the optimal cut-off value. $\square$

The optimal contract induces higher investment now that i is verifiable, as there should be no hold-up problem. That the buyer offers no contract for any investment other than $i^c(1)$ means that there is overinvestment relative to the investment's later use. The buyer promises a contract over the full surplus $v(i^c(1))$ with some probability, in exchange for an up-front payment. The seller can choose among a menu of contracts consisting of combinations of separation probabilities and up-front payments

$$\left(\frac{v(i)}{v(i^c(1))}, S(i^c(1)) - S(i)\right), \quad i \in [i^c(\bar{\theta}), i^c(\theta_H)],$$

or trade for sure and pay $S(i^c(1)) - u^c(\bar{\theta})$ up-front. This contract excludes higher types with positive probability, which would be impossible without a form of commitment.

6 Conclusion

In the present paper, we introduced private information about the reservation value in a simple property rights model.[11]The simplicity of the model allowed us to fully characterize the resulting equilibrium payoffs, which are uniquely determined. The equilibrium involves pooling up to a certain type of outside option, such that all lower types get the same payoff and because they accept all offers in equilibrium, these types are not distinguishable, even ex post. Higher types follow a mixed strategy and on average obtain the same payoff as with complete information. The seller has to mix between the investments because there is a strong force against a separating equilibrium in this model: if only high types choose a certain investment and get high offers, they will be mimicked by lower types.

In the outside option signaling game, there is a gap between the chosen investment and the investment that would result if the seller would get the full return to his investment. We have shown that this gap vanishes if investment is verifiable. This gap would also shrink if the seller had greater bargaining power than in the

[11]In fact, the model is essentially an ultimatum game with a prior investment stage, in which the receiver invests to increase the pie that they can share. The receiver then has private information about the payoff he gets when rejecting an (unfair) offer of the proposer. Simply obtaining a proportion of the pie as payoff when rejecting an offer is not a good model of human behavior, and therefore for this application the reader is referred to more realistic models of behavior like the game in von Siemens (2007), which then leads to a more complex signaling structure.

game that was analyzed. For example, if the bargaining game was modeled as the seller making a tioli offer with probability α and the buyer only with probability $1-\alpha$, then a higher α would increase the surplus and the seller's payoff. Since there is more investment on average, the buyer's payoff is non-monotonic in α. It would also be interesting to allow for more complex, and maybe more realistic, bargaining games at the ex post stage. One game that should leave the results unchanged is repeated offers by the buyer, but if both players can make offers, results will change and become difficult to obtain (compare Skryzpazc (2004)).

There are a couple of other extensions of the model that present themselves. One interesting task for future work is to allow the payoff that the buyer gets when the seller takes the outside option to be dependent on the seller's type. This would admit a greater set of applications, in particular the interpretation of the outside option as suing the buyer for payment, with private information about the probability of winning.[12] Another possible extension is the case of pure rent-seeking, in which the investment increases the outside value but is of little use inside the relationship. Investment can still be used as a signal for profitable outside opportunities, but higher investment is no longer more efficient.

[12]See Chonné and Linnemer (2008) for a related model in the context of pretrial bargaining and investment in trial preparation.

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Proofs

Proof of Proposition 5. Let $i_k \in \mathrm{I}^*$. When observing i_k, the buyer's expected profit from offering θ_l is $G(\theta_l|i_k)(1-\theta_l)$, where

$$G(\theta_l|i_k) = \frac{\sum_{j=1}^{l} f_j q_{jk}}{\sum_{j=1}^{k} f_j q_{jk}}.$$

We know from Lemma 3 that to be consistent with the seller's behavior, the buyer, when observing i_k, has to offer all θ_j, $i_j \in \mathrm{I}^*, j \le k$ with positive probability. She will offer θ_k if

$$\sum_{j=1}^{k} f_j q_{jk}(1-\theta_k) \ge \sum_{j=1}^{l} f_j q_{jk}(1-\theta_l) \text{ for all } l,$$

and θ_l if

$$\sum_{j=1}^{k} f_j q_{jk}(1-\theta_k) = \sum_{j=1}^{l} f_j q_{jk}(1-\theta_l).$$

As a first step, we write down all inequalities that define the buyer's behavior in an equilibrium (P, Q). Denote by

$$K := \{k: \ i_k \in \mathrm{I}^* \backslash \{i_H\}\}$$

all chosen investments that are strictly smaller than i_H. We treat H separately because we have to account for the fact that Q is a stochastic matrix, i.e., that the row entries add up to one. For all $j, l \le k, l, k \in K$ the following inequalites must hold:

$$\begin{aligned}
\sum_{i=1}^{j} f_i(\theta_k - \theta_j) q_{ik} + \sum_{i=j+1}^{k} f_i(\theta_k - 1) q_{ik} &\le 0 \\
-\left(\sum_{i=1}^{l} f_i(\theta_k - \theta_l) q_{ik} + \sum_{i=l+1}^{k} f_i(\theta_k - 1) q_{ik}\right) &\le 0 \\
-q_{jk} &\le 0
\end{aligned}$$

plus (straightforward calculation) for all $l < H, i \in K$

$$\begin{aligned}
R(\theta_H) - R(\theta_l) &\ge \sum_{j=1}^{l} \sum_{j \le k \in K} f_j(\theta_l - \theta_H) q_{jk} + \sum_{j=l+1}^{H-1} \sum_{k \in K} f_j(1-\theta_H) q_{jk} \\
R(\theta_i) - R(\theta_H) &\ge \sum_{j=1}^{i} \sum_{j \le k \in K} f_j(\theta_H - \theta_i) q_{jk} + \sum_{j=k+1}^{H-1} \sum_{j \le k \in K} f_j(\theta_H - 1) q_{jk} \\
1 &\ge \sum_{j \le l \in K} q_{ji}
\end{aligned}$$

We are going to treat the variables we are looking for as one big vector, denoted by q. That is, the entries in q are indexed by $jk, 1 \leq j \leq k, k \in K$. Similarly, we define a vector μ^{jk} by $\mu^{jk}_{ik} = f_i(\theta_k - \theta_j)$ for all $i \leq j$ and $\mu^{jk}_{ik} = f_i(\theta_k - 1)$ for all $i > j$ and zero else. Furthermore, define a vector μ^l by $\mu^l_{jk} = f_j(\theta_l - \theta_H)$ for all $j \leq l$ and $\mu^l_{jk} = f_j(1 - \theta_H)$ for all $j > l$. Last, let 1^j denote a vector with $1^j_{jk} = 1$ for $j \leq k \in K$ and 0 else. And let e^{jk} be a vector with $e^{jk}_{jk} = 1$ and 0 else.

Our inequalities now read

$$\begin{aligned} -e^{jk}q &\leq 0 && 1 \leq j \leq k, k \in K \\ 1^j q &\leq 1 && j = 1, ..., H-1 \\ \mu^{jk} q &\leq 0 \text{ for all } k \in K, j < k \text{ and} \geq 0 \text{ for } j \in K \\ \mu^l q &\leq R(\theta_H) - R(\theta_l) \text{ for all } l < H \text{ and } \geq 0 \text{ for } l \in K \end{aligned}$$

As the second step, we find a system of inequalities that is an alternative of this system, i.e. that has a solution if and only if this one has none. We use Theorem 22.1 of Rockafellar (1970) to get the following alternative system:

(*i*) $\sum_{j=1}^{H-1} \beta_j + \sum_{l=1}^{H-1} \delta_l (R(\theta_H) - R(\theta_l)) < 0$

(*ii*) $\sum_{j=1}^{H-1} 1^j \beta_j + \sum_{jk} \mu^{jk} \gamma_{jk} + \sum_{l=1}^{H-1} \mu^l \delta_l \geq 0$

where we are looking for coefficients $\beta_j \geq 0$, $j = 1, ..H-1$, γ_{jk} (≥ 0 if $j \notin K$), δ_l, (≥ 0 if $l \notin K$). For the analysis, it is convenient to write the second equation as an equation in each coefficient jk with $k \in K$ and $j \leq k$

$$\beta_j + \sum_{i=1}^{j-1} \gamma_{ik} f_j(\theta_k - 1) + \sum_{i=j}^{k-1} \gamma_{ik} f_j(\theta_k - \theta_i) + \sum_{l=1}^{j-1} \delta_l f_j(1 - \theta_H) + \sum_{l=j}^{H-1} \delta_l f_j(\theta_l - \theta_H) \geq 0$$

Let $\hat{k} = \min K$. We claim that $\bar{k} = \hat{k}$ and first show that $R(\theta_l) \leq R(\theta_{\hat{k}})$ for $l < \hat{k}$. Assume not. Then there is a solution with $\delta_l = \gamma_{lk} = 1$ and $\delta_{\hat{k}} = \gamma_{\hat{k}k} = -1$ and all other coefficients equal to zero: The first inequality is obviously satisfied, and for the second, since $k \geq \hat{k} > l$ always holds, there are only three cases to distinguish, $j > \hat{k}$, $l < j \leq \hat{k}$, and $j \leq l$.

Similarly, one can show that $R(\theta_{\hat{k}+1}) \leq R(\theta_{\hat{k}})$ is also necessary, because else there is a solution with $\delta_{\hat{k}+1} = \gamma_{\hat{k}+1k} = 1$ and $\delta_{\hat{k}} = \gamma_{\hat{k}k} = -1$. The easy case distinctions are again left to the reader. Hence, $\hat{k} = \bar{k}$. Note that we could have shown more generally that $K \subset \{k \text{ with } R(\theta_k) \geq R(\theta_{k+1})\}$.

Next we show that K is an interval. Assume to the contrary that there is a gap in K, i.e, there exist $l < m < h$ with $m \notin K$, $l = \max\{k \in K, k \leq m\}$ and $h = \min\{k \in K, k \geq m\}$. There is a $\lambda \in (0,1)$ with $(1-\lambda)\theta_h + \lambda\theta_l = \theta_m$. Define $\delta_l = \gamma_{lk} = -\lambda, \delta_m = \gamma_{mk} = 1, \delta_h = \gamma_{hk} = -(1-\lambda)$ for all relevant $k \in K$. Then the first condition holds because R is concave on K: $\lambda R(\theta_l) + (1-\lambda)R(\theta_h) - R(\theta_m) < 0$. That the second condition always holds with equality is seen immediately if $k \leq l$, for which this condition takes the form $\theta_m - \theta_H - \lambda(\theta_h - \theta_H) - (1-\lambda)(\theta_l - \theta_H) = 0$. For the remaining case $k \geq h$ there has to be again a case distinction regarding j, each case leading to the same result. Thus concavity of R implies that there are no gaps in chosen investment, $K = \{\bar{k}, ..., H-1\}$.

□

Proof of Prop. 6. First, we check that the strategies fulfill equation 7:

$$\begin{aligned}
(1-\theta_l)\sum_{j=1}^{l} f_j q_{jk} &= (1-\theta_l)\left(\sum_{j=1}^{\bar{k}} f_j \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} + \sum_{j=\bar{k}+1}^{l} \frac{\lambda_k - \lambda_{k+1}}{\lambda_j}\right) \\
&= (1-\theta_l)\left(\frac{1}{1-\theta_{\bar{k}}} + \sum_{j=\bar{k}+1}^{l}\left(\frac{1}{1-\theta_j} - \frac{1}{1-\theta_{j-1}}\right)\right)(\lambda_k - \lambda_{k+1}) \\
&= \lambda_k - \lambda_{k+1},
\end{aligned}$$

which is independent of l. Similarly for the remaining cases.

Next, note that

$$\frac{R(\theta_k) - R(\theta_{k-1})}{\theta_k - \theta_{k-1}} = \frac{f_k(1-\theta_{k-1})}{(\theta_k - \theta_{k-1})} - F(\theta_k) = \frac{f_k(1-\theta_k)}{(\theta_k - \theta_{k-1})} - F(\theta_{k-1})$$

and therefore

$$\lambda_k - \lambda_{k+1} = (1-\theta_k)\left(\frac{R(\theta_k) - R(\theta_{k-1})}{\theta_k - \theta_{k-1}} - \frac{R(\theta_{k+1}) - R(\theta_k)}{\theta_{k+1} - \theta_k}\right) \geq 0.$$

Also,

$$R(\theta_{\bar{k}}) - \lambda_{\bar{k}+1} \geq 0 \iff (\theta_{\bar{k}+1} - \theta_{\bar{k}})F(\theta_{\bar{k}}) - f_{\bar{k}+1}(1-\theta_{\bar{k}+1}) \geq 0 \iff R(\theta_{\bar{k}}) - R(\theta_{\bar{k}+1}) \geq 0.$$

These conditions imply that all $q_{jk} \geq 0$. We still need to show that they add up to one:

$$\begin{aligned}
\sum_{k=j}^{H} q_{jk} &= \sum_{k=j}^{H} \frac{\lambda_k - \lambda_{k+1}}{\lambda_j} = 1 \qquad \text{for all } j > \bar{k} \\
\sum_{k=\bar{k}}^{H} q_{j\bar{k}} &= 1 - \frac{\lambda_{\bar{k}+1}}{R(\theta_{\bar{k}})} + \sum_{k=\bar{k}+1}^{H} \frac{\lambda_k - \lambda_{k+1}}{R(\theta_{\bar{k}})} = 1
\end{aligned}$$

Here we have that all low types follow the same strategy. If such a restriction is not imposed, there may be more possible values for the strategies. $\square$