Dynamics of neighborhood formation and segregation by income

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Work in Progress

Abstract

This paper analyzes some determinant conditions under which neighborhood formation gives rise to segregation by income. In contrast to the literature, we explore the *sequential* arrival of poor and rich individuals to neighborhoods exploited by oligopolistic land-developers. These developers try to maximize a discounted flow of lot prices during neighborhood formation, taking advantage of the local externalities generated by the rich and the poor. Under a speedy arrival of new potential inhabitants and / or low discount rates, competing developers are more likely to concentrate rich people in the same neighborhood. This happens because the benefits from early agglomeration are outweighed by a more profitable matching of rich neighbors within nearby lots.

1 Introduction

The choice of neighborhood is, together with the choice of marriage couple, one of the most determinant decisions in the social and economic life of any individual. Therefore, segregation by income at the neighborhood level has long-lasting effects in the dynastic trajectories of those favored and disfavored by the environment they live in. In this paper we propose a framework in which, from a purely positive - as opposed to normative - point of view, we can analyze the *dynamic* conditions in which *new* integrated or segregated neighborhoods arise.

As it is well-known, the endogenous processes of social sorting or scrambling have been already extensively studied by the profession. Early pioneers, like Schelling (1978), were able to vividly describe the conditions for integrated equilibria to become unstable. The main difference between

Schelling (1978)'s study and Becker and Murphy (2000)'s is that the former mainly analyzes a process by which the members of two (or more) groups want to congregate with other members of their own kind; whereas in the latter case there is a unique desirable type ('the rich'), and everybody wants to be close to them. In principle, we may think that segregated equilibria are more plausible in Schelling's environment, though De Bartolome (1990), Benabou (1993) or Becker and Murphy (1994) show that - under reasonable conditions - segregation is excessive in the second case from a normative point of view.

To the best of our knowledge, Becker and Murphy (2000)'s model is closest in spirit to ours. As in the case of Schelling (1978), externalities within neighborhoods are the driving force of poor and rich individuals' welfare and willingness to pay. For instance, any person would like to live close to well reputed judges or doctors who could help you in case of need. Therefore, people will tend to receive higher positive externalities from rich (rather than poor) neighbors. In their model (like in ours) people derive utility both from local externalities and from housing facilities (amenities)[1].

However, their starting point is a fully integrated neighborhood where everybody has already a residence. In that context, they analyze the conditions under which the initially integrated configuration evolves towards a different one (partially or fully segregated). On the contrary, in our model we portray a new neighborhood that is filled step by step with the *sequential arrival* of poor and rich residents. Also unlike Becker and Murphy (2000), who include a *single* developer and *competitive bidding* by households (those with the highest willingness to pay obtain the desired residence), we consider a situation of duopolistic competition between developers. That competition (together with the fact that people arrive sequentially and henceforth can not directly compete with each other) prevents the extraction of the whole willingness to pay from buyers.

The main features of our model are its *dynamic nature* (because of the sequential arrivals) and the *strategic interaction* between developers. What does the dynamic (sequential) nature of our model add? It adds the possibility of understanding how the residents' higher (or lower) arrival rates determine the final degree of neighborhood segregation. Why? Because in this case, even when the rich had a uniformly higher marginal willingness to pay to live with other rich people, this does not guarantee segregation. Since there are *agglomeration gains* from accepting immediately the poor who come, under a very slow arrival rate it may be in the developers' interest to offer them a cheap lot. Therefore, we can observe the implications of lower interest rates (when waiting for the rich is not costly) and higher / lower arrival rates of the rich (dependent on income distribution) for the degree of segregation.[2]

[1] In order to focus better on the main point, the quality of amenities is usually assumed to be identical for rich and poor households.

[2] Our model is also similar to Henderson and Thisse (2001), in the sense that they model the competition between

2 The Model: main assumptions and basic structure

There is a town with two separate negighbourhoods, each of which is run by a developer (duopolist). For simplicity, each neighborhood consists of two lots of land, where each lot can host only one person. There are also four potential inhavitants, which can be of two types: *low* or *high* (l or h). By a high (low) type we mean an individual who is very (not so much) desirable as a neighbour because he produces high (low) positive externalities. For instance, an example of a high type would be a neighbour who builds a nice house, keeps the external lights on at night, does not through rubbish where he must not, he is a good contact in order to get good jobs etc. The four potential inhavitants sequentially arrive to the town. In particular, in a discrete time framework a type h arrives in periods 1 and 3 and a type l arrives in periods 2 and 4.[3]

At the time of arrival each potential inhavitant has to chose whether to buy a lot of land in one of the two neighbourhoods or not to buy any at all. The per-period *total utility*, U, that each individual gets from living in a particular lot of land is the addition of a per-period *residential utility* and a per-period *within-neighborhood externalities* (hearafter refered to as just externalities). The inexistence of between-neighbourhood externalities is assumed. The residential utility is the part of the total utility that is obtained because a particular lot of land offers: a) the possibility of phisically living there; b) the enjoyment of the nearby amenities; c) the convenience of being close to good jobs etc. For simplicity we assume that each resident lives in the lot of land without building any house.

On the one hand, the per-period residential utility, u, is assumed to be identical for h and l individuals and also independent of the neihbourhood an individual lives. For simplicity we also assume that an individual gets zero per-period total utility when he does not have a place to live (i.e. its outside option is zero). On the other hand, the per-period externality enjoyed by individual i from his neighbour j is $x_{ij} \forall i, j \in (l, h)$. For simplicity we assume that $x_{hh} > x_{hl} > x_{lh} = x_{ll} \geq 0$ and that an individual gets zero externality in the case of not having any neighbourgh.

developers to attract high-income segments of the population. However, they endogenize the number and size of developments and do not consider externalities as the main determinant of the willingness to pay. Furthermore, their model is static and focuses in the number of communities and the degree of internal heterogeneity within them.

[3]We recognise that a random arrival rate would be more appropriate than a deterministic one. However, the handling of the former type becomes extremelly complicated. Also notice that, because of lack of space, the alternative sequence (i.e. 'low-high-low-high') would not be explored. Later on we argue that this alternative sequence would lead to similar results. Finally, It is clear that in our model we are considering a fixed proportion of types h and l; perhaps an interesting extension for future research would be one where a parameter is added to account for the different proportions between types h and l.

A resident can in principle move from one neighbourhood to the other. In order to do that, in the case that both neighbourhoods are fully ocupied, the particular resident would need to contact a resident from the other neighbourhood and agree on a swap, which may include a side payment. On the contrary, in the case there is an available lot in the other neighbourhood, he should contact the developer of the other neighbourhood and negotiate a deal.

However, in many cases there is a switching cost that prevents people from changing neighbourhoods. A switching cost may exists for the following reasons: the individual likes his current job which is close to his current neighbourhood, he has good friends in this neihbourhood etc. Indeed, this switching cost would be certainly higher if families instead of individuals were considered. In this paper we will assume that this switching cost is high enough and later on we will provide the particular parameter values under which this is the case.[4]

A resident's total willingness to pay, WP, to live in a particular neighbourhood is determined by the present value of all the expected per-period total utilities enjoyed by him since his arrival to the neighborhood till the end of his infinite life.

A higher arrival rate of potential residents to the town is equivalent to a lower length of the period between the arrival of one individual and the next one. This at the time implies that the lower is the discount rate needed to be applied to discount the future expected per-period total utilities.

When a particular resident shows up each developer would bid a particular price in order to attract it. In making this bid each developer will have to consider the outside option of not attracting the current potential resident in order to attract the potential resident coming in the next immediat period. Both developers compete a la Bertrand and we assume that they cannot differentiate the product to target any specific population group. However, they are allowed to price-discriminate over time to extract as much of the residents' cumulative total utility as possible. A result of this discrimination is that the price of the land will go up with the time and so it would not be optimal for any particular potencial resident to strategically postpone the purchasing of a land.

Finally, let us emphasize that our analysis is going to be purely *positive* instead of normative. This means that we will not try to determine the optimal allocation of residents for a given arrival

[4]The assumption of a high enough swithching cost allows us to focuse on an interesting result; i.e. the fact that a slower arrival rate is associated with a higher level of segregation. We understand that a more interesting modeling approach would have been the introduction of a switching cost's parameter. Presumably, this would have allowed us to obtain the additional result that a higher switching cost is associated with a lower long run neighbourghood segregation (at least for fast enough arrival rates) plus an additional interaction effect between swithching cost and arrival rate. These obviously are very interesting testable hypothesis. However, our choice was a consequence of the trade off between the optention of these two additional results and the simplicty of the paper.

rate, but just the predictable pattern of segregation / integration.

3 Integrated and segregated equilibria

3.1 Solving backwards: periods 3 and 4

Since neighborhoods just have two lots, the only possible configurations are fully integrated (with a high-type and a low-type in each neighborhood) or fully segregated (with two highs with one of the developers and two lows with the other). Buyers will arrive sequentially to the neighborhoods, where both duopolists will compete for them. The sequence begins with a high-type (h_1) and continues with a low-type (l_2), with both types alternating subsequently. The developer who attracts h_1 in period 1 will be called "rich developer" (**RD**), whereas the other duopolist will be called "poor developer" (PD).

The game that determines the nature of the equilibrium must be solved backwards, starting by the last arrival (l_4). When either the poor or the rich developer receive l_4 there will be no competition between them, since there will be just one empty lot in one of the neighborhoods. Therefore, both neighborhoods would charge a monopoly price over the buyer l_4, i.e. $p^{PD}_{4(h,l|h,l)} = p^{\mathbf{RD}}_{4(h,l|h,l)} = WP^{l_{4(h,l|h,l)}}_{\mathbf{RD}} = WP^{l_{4(h,l|h,l)}}_{PD}$. That is, they will be able to extract the whole cumulative externality plus the residential utility received by l_4. However, we still do not know which of the two developers will receive l_4. Therefore, now we must find out which prices both developers would set on h_3. The minimum price that the PD would set on h_3 is one which leaves him indifferent between attracting h_3 or not attracting it. This price is:

$$p^{\mathbf{RD}\min}_{3(l|h)} = \frac{p^{\mathbf{RD}}_{4(l,h|h,l)}}{(1+r)} = \frac{WP^{l_{4(l,h|h,l)}}_{\mathbf{RD}}}{(1+r)} \qquad (1)$$

The previous expression implies that the **RD** is indifferent between capturing h_3 at a price $p^{\mathbf{RD}\min}_{3(l|h)}$ or waiting until period 4 to extract a monopoly price from buyer l_4. On the other hand, the minimum price for the poor developer will be:

$$p^{PD\min}_{3(l|h)} = \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)} \qquad (2)$$

Who will attract h_3 in case both developers are competing for it? As we have seen, each developer has a different minimum price that can be charged to the buyer. The winner of the competition will be the developer showing a lower minimum price, once this price has been adjusted for the different cumulative externalities enjoyed by h_3 in each neighborhood. The winner of the competition for h_3 will set an equilibrium price equal to the minimum price of the other developer. In this particular case,

$$\text{Iff } \frac{WP_{\mathbf{RD}}^{l_{4(l,h|h,l)}}}{(1+r)} < \frac{WP_{PD}^{l_{4(l,l|h,h)}}}{(1+r)} + \left(WP_{\mathbf{RD}}^{h_{3(l,l|h,h)}} - WP_{PD}^{h_{3(l,h|h,l)}}\right), \text{then the } \mathbf{RD} \text{ will attract } h_3, \text{ provided that the } PD \text{ attracted } l_2. \quad (3)$$

The previous expression shows the condition under which h_3 is attracted by the **RD** in scenario (l_2-, h_1-). If we express the previous condition in terms of the underlying parameters, we can obtain that

$$\text{Iff } (1+r) > \frac{x_{hl}-x_{ll}}{x_{hh}-x_{lh}}, \text{then the } \mathbf{RD} \text{ will attract } h_3, \text{ provided that the } PD \text{ attracted } l_2 \quad (4)$$

Here we can see that if the discount factor $(\frac{1}{1+r})$ is low enough, and if x_{hh} and x_{ll} are high enough relative to x_{hl} and x_{lh}, the previous inequality is easier to be satisfied. This means that a low discount factor is likely to induce the **RD** to compete toughly for h_3, since otherwise he would have to wait one period for l_4. By the same token, and given that the **RD** already hosts a high-type (h_1), the **RD** will be more willing to attract h_3 the larger are the externalities generated by this buyer (i.e. the higher is x_{hh}) and the softer is the competition coming from the PD (i.e. the lower are x_{hl} and x_{lh}, and the higher is x_{ll}).

If $(1+r) < \frac{x_{hl}-x_{ll}}{x_{hh}-x_{lh}}$ then we would always have integrated neighborhoods regardless of the speed of the arrival rate, since the **RD** would never want to capture h_3. Therefore, we are specially interested in the case where $(1+r) > \frac{x_{hl}-x_{ll}}{x_{hh}-x_{lh}}$, and henceforth we will assume that this condition is satisfied. Which are the equilibrium prices set on h_3 by both developers?

As a consequence of the competition, the equilibrium prices set on h_3 by both competitors in this situation will be the following:

$$p_{3(l|h)}^{PD} = \max\left(\underbrace{\frac{WP_{\mathbf{RD}}^{l_{4(l,h|h,l)}}}{(1+r)} - \left(WP_{\mathbf{RD}}^{h_{3(l,l|h,h)}} - WP_{PD}^{h_{3(l,h|h,l)}}\right)}_{If\ PD\ wins\ h_3}, \underbrace{\frac{WP_{PD}^{l_{4(l,l|-)}}}{(1+r)}}_{If\ \mathbf{RD}\ wins\ h_3}\right); \quad (5)$$

$$p_{3(l|h)}^{\mathbf{RD}} = \max\left(\underbrace{\frac{WP_{PD}^{l_{4(l,l|h,h)}}}{(1+r)} + \left(WP_{\mathbf{RD}}^{h_{3(l,l|h,h)}} - WP_{PD}^{h_{3(l,h|h,l)}}\right)}_{If\ \mathbf{RD}\ wins\ h_3}, \underbrace{\frac{WP_{\mathbf{RD}}^{l_{4(l,h|h,l)}}}{(1+r)}}_{If\ PD\ wins\ h_3}\right) \quad (6)$$

3.2 The arrival of l_2 in period 2

Let us analyze now the arrival of l_2; i.e. we are in the following scenario:

There are two possible cases here:

1) Assume that PD is the *loser* in 2, in which case he has monopoly power over h_3 and l_4. Then the profits enjoyed by the PD in case he loses l_2 are:

$$\pi^{PD}_{2(-|h_1)loser} = \frac{p^{PD}_{3(h|h,l)}}{(1+r)} + \frac{p^{PD}_{4(h,l|h,l)}}{(1+r)^2} = \frac{WP^{h_{3(h|h,l)}}_{PD}}{(1+r)} + \frac{WP^{l_{4(h,l|h,l)}}_{PD}}{(1+r)^2} \tag{7}$$

2) Assume that PD is the winner in 2, in which case we have:

PD's expected profit from attracting l_2, given that our condition (3) holds and **RD** attracts h_3, will be the following:

$$\pi^{PD}_{2(l_2,l_4|h_1,h_3)winner} = p^{PD}_{2(l_2,l_4|h_1,h_3)} + \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)^2} \tag{8}$$

By comparing (8) and (7) we get that charging $p^{PD\min}_{2(l_2,l_4|h_1,h_3)}$ for l_2 leaves the PD indifferent between setting a monopoly price on h_3 and l_4 or attracting l_2 and l_4 instead:

$$\pi^{PD}_{2(-|h_1)loser} - \pi^{PD}_{2(l_2,l_4|h_1,h_3)winner} = 0 = \frac{p^{PD}_{3(h|h,l)}}{(1+r)} + \frac{p^{PD}_{4(h,l|h,l)}}{(1+r)^2} - \left(p^{PD\min}_{2(l_2,l_4|h_1,h_3)} + \frac{p^{PD}_{4(l,l|h,h)}}{(1+r)^2} \right)$$

$$p^{PD\min}_{2(l_2,l_4|[h_1,h_3])} = \frac{p^{PD}_{3(h|h,l)}}{(1+r)} + \frac{p^{PD}_{4(h,l|h,l)}}{(1+r)^2} - \frac{p^{PD}_{4(l,l|h,h)}}{(1+r)^2} = \frac{WP^{h_{3(h|h,l)}}_{PD}}{(1+r)} + \frac{WP^{l_{4(h,l|h,l)}}_{PD}}{(1+r)^2} - \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)^2} \tag{9}$$

In period 2 the **RD** will be also able to quote a minimum price which will leave him indifferent between capturing l_2 or not. This minimum price can be derived by comparing the **RD**'s payoff in the case of winning and losing l_2:

$$p^{\mathbf{RD}\min}_{2(h_3,l_4|h_1,l_2)} = \frac{p^{\mathbf{RD}}_{3(l|h)}}{(1+r)} = \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)^2} + \frac{(WP^{h_{3(l,l|h,h)}}_{\mathbf{RD}} - WP^{h_{3(l,h|h,l)}}_{PD})}{(1+r)} \tag{10}$$

The integrated equilibrium holds when the **RD** attracts l_2 and the segregated equilibrium takes place when **RD** loses l_2. Again, the developer that captures l_2 will be the one with a lowest minimum price, once that price has been adjusted for the difference in cumulative externalities. In this particular case and if

$$p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} \leq \min\left(p_{2(l_2,l_4|[h_1,h_3])}^{PD\min} + \left(WP_{\mathbf{RD}}^{l_{2(h,l|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right), WP_{\mathbf{RD}}^{l_{2(h,l|h,l)}}\right),$$

the **RD** will attract l_2.

In principle, it is conceivable that neither of the developers wants to accept l_2 in period 2. Particularly, the PD may want to reject l_2 in order to compete more efficiently for h_3 in period 3, when he would have two free lots and the **RD** only one, or to enjoy from substantial monopoly profits over our low types. We will prove that under certain conditions that is exactly what happens: it will be profitable for PD to reject l_2 in period 2 and wait for monopoly profits in period 3, once the neighborhood of his competitor is already filled. The intuition is the following: the competition for h_3 would be so fierce that the PD would be unable to get a high-type later, because of the **RD**'s advantage in terms of offered cumulative externalities, but

Under these circumstances, the profitability from waiting until period 3 dominates the gain from extracting *partially* the surplus from l_2 before (specially if the instantaneous residential utility that can be extracted in period 3 is high enough). Therefore, since the PD will always accept l_2 in period 3, the integrated or segregated nature of the equilibrium configuration depends only on the **RD**'s willingness to capture l_2 or not.

This means that if the **RD** does not want to attract l_2 in period 2, then the PD will do it sooner or later. We will elaborate below on this characteristic of the equilibrium. By now let us summarize this result in the following lemma:

Lemma 1 *Provided that* $(1+r) > \frac{x_{hl}-x_{ll}}{x_{hh}-x_{lh}}$*, and for a sufficiently high level of momentary residential utility (u), we can prove the following facts:*

Even if the PD *postpones the attraction of* l_2 *in period 2, he will not be able to capture* h_3 *in period 3.*

The PD *decides to postpone the attraction of* l_2 *till period 3.*

The **RD** *would also attract* h_3 *if* PD *decided to accept* l_2 *in period 2.*

In the next paragraphs we are going to describe and explain the proof to Lemma 1. First of all, from (9) and (10), it is easy to show that a compact way of expressing these equilibrium prices is the following:

$$p_{2(-|h_1)}^{\mathbf{RD}} = \max\left(\min\left(p_{2(l_2,l_4|h_1,h_3)}^{PD\min} + \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right), WP_{\mathbf{RD}}^{l_{2(-|h,l)}}\right), p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min}\right)$$
$$p_{2(-|h_1)}^{PD} = \max\left(\min\left(p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} - \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right), WP_{PD}^{l_{2(l,l|-)}}\right), p_{2(l_2,l_4|[h_1,h_3])}^{PD\min}\right) \tag{11}$$

Is there any way for us to know which term will be lower inside the maximum operator in first and second row of expression (11)? Here the level of instantaneous residential utility (u) will play a crucial role: when the residential utility level is high enough, the maximum (instantaneous) sum of the cumulative externality and the residential utility level extractable from l_2 in period 2 is very large. Competition does not allow the duopolists to extract all that surplus, and therefore the relevant prices they quote are always inferior to the whole sum of discounted residential utility plus cumulative externalities. Thus, for a suitably chosen value of u [5] we know that

$$\min\left(p_{2(l_2,l_4|h_1,h_3)}^{PD\min} + \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right), WP_{\mathbf{RD}}^{l_{2(-|h,l)}}\right) = p_{2(l_2,l_4|h_1,h_3)}^{PD\min} + \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right)$$

and

$$\min\left(p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} - \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right), WP_{PD}^{l_{2(l,l|-)}}\right) = p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} - \left(WP_{\mathbf{RD}}^{l_{2(-|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right).$$

In other words, we choose a certain value of u that guarantees a non-binding participation constraint for the buyer l_2.

In order to prove the statement a) in the previous lemma, we must find the equilibrium prices for $l_{2[3]}$ and h_3 in case the PD decided to postpone and reject l_2 in period 2. To that purpose, we must explore the possibility that the PD aims to capture buyer h_3. It is important to realize here that, if the PD attracted h_3, he would be able to compete *evenly* with the **RD** for $l_{2[3]}$. In fact, he would capture $l_{2[3]}$ with probability 0.5. Thus, the PD has the option to attract $l_{2[3]}$ with probability one or to try capturing h_3(and then either $l_{2[3]}$ or l_4 with probability 0.5).

Again, the winner of the competition for h_3 will be the duopolist showing the lowest minimum price, once that price has been adjusted for the difference in cumulative externalities. Let us proceed now to compute the minimum prices for both developers: $p_{3h_3\left(h_3,0.5l_{2(3)}|h_1\right)}^{PD\min}$ and $p_{3h_3(-|h_1,h_3)}^{\mathbf{RD}\min}$. We know from (1) that $p_{3h_3(-|h_1,h_3)}^{\mathbf{RD}\min} = \frac{WP_{\mathbf{RD}}^{l_{4(h,l|h,l)}}}{(1+r)}$. Furthermore, the PD needs to compare $\pi_{rechazar\ l_2 \text{ in period 2 y } h_3,0.5l_{2(3)}}^{PD(\text{in prices of per 3})}$ with $\pi_{3\left(l_{2[3]},l_4|h_1,h_3\right)}^{PD}$ to solve for $p_{3h_3\left(h_3,0.5l_{2(3)}|h_1\right)}^{PD\min}$.

[5] It is easy to prove that all we require from u is that $u > \max\{\bar{u}_1, \bar{u}_2\}$, where

$$\bar{u}_1 = \frac{x_{lh} + x_{hl} - x_{ll}((1+r)^2+1)}{(1+r)\,r}$$

and

$$\bar{u}_2 = \frac{(1+r)\,(x_{hh} - x_{lh}) + x_{ll} - x_{hl}(1+r)^2}{r\,(2+r)}$$

Since the PD has monopoly power over both $l_{2[3]}$ and l_4 once he has renounced to h_3, it is straightforward to derive that

$$\pi^{PD}_{3\left(l_{2[3]},l_4|h_1,h_3\right)} = WP_{PD}^{l_{2(3)}\left(l_{2(3)},l_4|-\right)} + \frac{WP_{PD}^{l_4\left(l_{2(3)},l_4|-\right)}}{(1+r)}$$

. Then the PD's expected profit in period 3 conditional on attracting $h_3, 0.5l_{2(3)}$ is:

$$\pi^{PD(\text{in prices of per 3})}_{rechazar\ l_2\ \text{in period 2 y}\ h_3,0.5l_{2(3)}} = p^{PD}_{3h_3\left(h_3,0.5l_{2(3)}|h_1\right)} + 0.5\frac{WP_{PD}^{l_{4(h_3,l_4|-)}}}{(1+r)} + 0.5\frac{WP_{PD}^{l_{4(h_3,l_4|-)}}}{(1+r)} \tag{12}$$

By equalizing the two previous expressions and solving for $p^{PD}_{3h_3\left(h_3,0.5l_{2(3)}|h_1\right)}$, it is possible to obtain that

$$p^{PD\min}_{3h_3\left(h_3,0.5l_{2(3)}|h_1\right)} = WP_{PD}^{l_{2(3)}\left(l_{2(3)},l_4|-\right)} + \frac{WP_{PD}^{l_4\left(l_{2(3)},l_4|-\right)}}{(1+r)} - \frac{WP_{PD}^{l_{4(h_3,l_4|-)}}}{(1+r)} \tag{13}$$

We can compare expressions (1) and (13) to conclude that a sufficient condition for the **RD** to win the competition for h_3 is that

$$(1+r) > \frac{2WP_{\mathbf{RD}}^{l_{4(h,l|h,l)}} - WP_{PD}^{l_4\left(l_{2(3)},l_4|-\right)}}{\left(WP_{PD}^{l_{2(3)}\left(l_{2(3)},l_4|-\right)} + WP_{\mathbf{RD}}^{h_{3(-|h_1,h_3)}} - WP_{PD}^{h_3\left(h_3,0.5l_{2(3)(3+\epsilon)}+0.5l_4|-\right)}\right)}$$

And, after replacing variables by the underlying parameters and rearranging, we come up with our sufficient condition

$$u > \frac{2\left(x_{hl} - x_{ll}\right) - (1+r)\left(x_{hh} - x_{lh}\right)}{r} = \bar{u}_3 \tag{14}$$

We can see how the competitiveness of the **RD** will be enhanced by higher levels of x_{hh} and x_{ll}, and by lower levels of x_{hl} and x_{lh}. Finally, high values of u favor the competitive position of the **RD**. Why? If the PD did not try to attract h_3, he would capture a high level of residential utility and cumulative externalities from $l_{2[3]}$. This raises the minimum price he is willing to set on h_3 and weakens the PD's competitive position.

Therefore, we have just shown that under assumption (14) the **RD** will attract h_3 for sure. We could wonder now whether still the PD has any incentive to reject l_2 in period 2 and try to postpone. By postponing the attraction of l_2, the PD would get (in prices of period 2):

$$\pi^{PD\ (\text{prices of period 2})}_{3\left(l_{2[3]},l_4|h_1,h_3\right)} = \frac{WP_{PD}^{l_{2(3)}\left(l_{2(3)},l_4|-\right)}}{(1+r)} + \frac{WP_{PD}^{l_4\left(l_{2(3)},l_4|-\right)}}{(1+r)^2} \tag{15}$$

On the other hand, his payoff from accepting l_2 in period 2 is, from (10) and (11),

$$\pi^{PD\ (\text{prices of period 2})}_{2(l_2,l_4|h_1,h_3)} = 2\frac{WP_{PD}^{l_{4(l,l|h,h)}}}{(1+r)^2} + \frac{\left(WP_{\mathbf{RD}}^{h_{3(l,l|h,h)}} - WP_{PD}^{h_{3(l,h|h,l)}}\right)}{(1+r)} - \left(WP_{\mathbf{RD}}^{l_{2(h,l|h,l)}} - WP_{PD}^{l_{2(l,l|-)}}\right) \tag{16}$$

Now we can compare (15) and (16) to see that the following condition needs to be satisfied for the PD to prefer rejecting l_2 in period 2:

$$u > \frac{(x_{hh} - x_{lh})(1+r) - x_{hl}(1+r)^2 + x_{ll}}{r} = \bar{u}_4 \tag{17}$$

The intuition behind this last result could be spelled out as follows: the PD will enjoy a monopoly position over the poor residents if he decides to postpone; that position will be specially interesting if the value of u is high, and if the competition for l_2 from the **RD** is specially tough. The last condition (17) guarantees the statement b) of lemma 1[6]. The statement c) was ensured above by our initial restriction (4) on the values of the parameters. This completes the proof of lemma 1.

It may seem odd that - in equilibrium - the PD decides to postpone one period the attraction of l_2 (for sufficiently high values of u). The full monopoly power he enjoys over the poor residents (l_2 and l_4) depends on very specific features of our model: we just have two lots per developer (instead of many more), and there are no outside opportunities for the rejected residents to leave these two neighborhoods and find accommodation elsewhere. These conditions are apparently unrealistic in the residential case, and therefore we should probably focus on our main result, as we do in the next section. We conjecture that this result should be robust to the inclusion of more realistic assumptions in the model.

3.3 The influence of the arrival rate on the nature of the equilibrium

According to the conditions established in lemma 1, we know that *at least one* of the developers is interested in attracting l_2, either in period 2 (the **RD**) or in period 3 (the PD). This means that the integrated or segregated nature of the equilibrium only depends on which of the two developers attracts buyer l_2. Then, the winner of the competition for l_2 will be the developer whose minimum price (once again, adjusted for the differences in cumulative externalities) is lower. As a result of this, we are facing the following scenario. If

$$p_{2(l_2,l_4|[h_1,h_3])}^{PD\min} > p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} - (WP_{\mathbf{RD}}^{l_{2(h,l|h,l)}} - WP_{PD}^{l_{2(l,l|-)}})$$

the **RD** captures l_2 (integrated equilibrium) and if

$$p_{2(l_2,l_4|[h_1,h_3])}^{PD\min} < p_{2(h_3,l_4|h_1,l_2)}^{\mathbf{RD}\min} - (WP_{\mathbf{RD}}^{l_{2(h,l|h,l)}} - WP_{PD}^{l_{2(l,l|-)}})$$

the PD captures l_2 in period 3 (segregated equilibrium).

[6] In fact the restrictions implied by the previous lemma require that $u > \max\{\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{u}_4\}$,where $\bar{u}_1$ and $\bar{u}_2$ were defined in the previous footnote, $\bar{u}_3$ is defined in (14) and $\bar{u}_4$ in (17).

We are ready now to introduce a proposition that clarifies how the equilibrium configuration of neighborhoods depends on the speed of the arrival rate.

Proposition 2 *Under the conditions established in lemma 1, the slower is the arrival rate (represented by a higher value of r) the more likely will be the equilibrium configuration to be integrated. Identically, the faster is the arrival rate (represented by a lower value of r) the more likely will be the configuration to be segregated.*

Let us now proceed to sketch a proof of the statement in the proposition.

If $p^{PD\min}_{2(l_2,l_4|[h_1,h_3])} < p^{\mathbf{RD}\min}_{2(h_3,l_4|h_1,l_2)} - (WP^{l_{2(h,l|h,l)}}_{\mathbf{RD}} - WP^{l_{2(l,l|-)}}_{PD})$, from (9) and (10) this implies that

$$\begin{aligned} & \frac{WP^{h_{3(h|h,l)}}_{PD}}{(1+r)} + \frac{WP^{l_{4(h,l|h,l)}}_{PD}}{(1+r)^2} - \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)^2} + \left(WP^{l_{2(h,l|h,l)}}_{\mathbf{RD}} - WP^{l_{2(l,l|-)}}_{PD}\right) \\ < \quad & \frac{WP^{l_{4(l,l|h,h)}}_{PD}}{(1+r)^2} + \frac{(WP^{h_{3(l,l|h,h)}}_{\mathbf{RD}} - WP^{h_{3(l,h|h,l)}}_{PD})}{(1+r)} \end{aligned}$$

It is straightforward to show that, after expressing every cumulative total utility in terms of the parameters and rearranging, the previous inequality can be rewritten as:

$$3\frac{x_{ll}+u}{r(1+r)} + \frac{x_{hh}+u}{r} - 2\frac{x_{lh}+u}{r} > \frac{x_{hl}+u+(x_{hl}+u)(1+r)^2}{r(1+r)} - u\frac{2+r}{1+r} \tag{18}$$

Multiplying both sides of the inequality by r and simplifying, we can find that (18) is equivalent to

$$\frac{3x_{ll}}{(1+r)} + u\left(\frac{2-(2+r)(1+r)}{1+r}\right) + x_{hh} - 2x_{lh} - x_{hl}\left(\frac{1+(1+r)^2}{(1+r)}\right) > 0 \tag{19}$$

Once we check that $\left(\frac{1+(1+r)^2}{(1+r)}\right)$ is a monotone-increasing function of r, it is clear that the previous inequality will hold more easily the lower r is. That is intuitive: the faster the arrival rate is - which is equivalent to a low r - the easier it is for the equilibrium configuration to be segregated, since the **RD** will prefer to wait just a little and capture a high type instead. As a result, the **RD** will quote a high price in period 2 and l_2 will be attracted by the PD. If the arrival rate were very slow, both developers would compete very toughly for l_2 and, since **RD** offers a more attractive location in terms of externalities, he would be the winner and the configuration would be integrated.

Notice also that, no matter how large the level of u is, segregation is always possible for a small enough r provided that the different externalities keep the right proportions.

Moreover, notice that in expression (19) the parameters that facilitate a segregated configuration are x_{hh} and x_{ll}, since for a high x_{hh} the **RD** becomes more prone to wait for h_3 instead of competing for l_2, and the higher x_{ll} is the more likely the PD becomes to compete fiercely for l_2. On the contrary, high values of x_{lh}, x_{hl} and u tend to favor integration: a high momentary residential utility u encourages the **RD** to obtain agglomeration in the short run (as opposed to good matches in the long run) and compete for l_2; similarly, high values of x_{lh} and x_{hl} raise the appreciation of l_2 by the **RD**.

4 Conclusions and possible extensions

In this paper we have studied the conditions under which a low enough discount rate and / or a high arrival rate of new residents can result in higher levels of segregation by income in new neighborhoods. One natural extension seems to be exploring the constrained-optimal allocation of residents by a monopolist that can not differentiate the product. Then, we could compare this constrained-efficient outcome with that resulting from our duopoly model in order to obtain some normative implications: are the conditions for segregation more stringent under monopoly or under duopoly? ; are the poor people worse-off under monopoly or under duopoly? And the rich people?

Another possible extension may consist of an empirical study about the connections between low interest rates, urban demographic growth (or income distribution) and segregation by income. The impact of interest-rate variations could be identified with time-series data, whereas the effect of urban demographic growth could be captured in the cross-section. Therefore, panel-data is apparently the right empirical strategy.

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