# EQUITY AND EFFICIENCY IN AN OVERLAPPING GENERATIONS MODELS

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Abstract. We study intergenerational equity in an overlapping generations framework. We show that, even with only one commodity, no production and constant resources, it is in general not possible to treat all generations in a equitable and efficient way, when equity is defined by variants of the no-envy property.

## 1. Introduction

Many papers have addressed the problem of intergenerational equity and efficiency: from the old and fashionable debate about the discount factor in the social welfare function (started with Ramsey in the early XX century) to the recent concept of sustainability (see among others Chichilnisky (1996) or Dasgupta (1999)), from the attempt to order infinity utility streams (Fleurbaey and Michel (2003) or Bossert and al. (2007)) to the impossibility result contributions (see Basu and Mitra (2003)).

What links together all these contributions to economic research is the assumption about comparability of personal preferences, in some cases through the construction of a social welfare function as discounted sum of the utilities of all agents, in some others with the assumption of the existence of an infinite living representative agent. The approach that will be followed in this paper will depart from these articles in this aspect: in a completely ordinal framework we aim at extending to dynamics the axiomatic approach recently developed in social welfare theory.

Hence, in this paper two different streams of economic theory meet: on one side the macroeconomic framework of overlapping generations models (see Diamond (1965)), introduced to give the necessary dynamic structure to the analysis; on the other side, some well known axioms of equity, largely studied in social welfare theory, are first redefined and then studied in order to evaluate and characterize the allocations. In particular the key property is the no-envy concept introduced by Foley (1967)[1]: it requires that no agent should prefer the consumption bundle of any other agent to his own.

In a static framework where given resources per agent are to be distributed, few assumptions about the preferences are sufficient to guarantee the existence of an allocation rule satisfying Pareto efficiency and no-envy[2]. We will show that in the overlapping generations model, even if some assumptions, like constant resources and a single commodity, are introduced to avoid trivial impossibilities, an allocation satisfying Pareto efficiency and no-envy is in general not possible.

In section 2, we introduce the notation and the model together with the statement of the axioms. The third section contains the possibility and impossibility theorems. A brief conclusion follows.

## 2. Model and the axioms

The model describes a one commodity discrete time economy with two-periods living overlapping agents.

An allocation of the resources is therefore a vector $a = \left(a^{\underline{t}}, a^{\underline{t}+1}, ..., a^{\bar{t}}\right)$, where $a^t$ is the allocation that describes the quantity of good that young and old agents alive at time $t$ get. The lifetime consumption boundle of an agent $i$ born at time $t$[3] is the vector $z_{ti} = (c_{ti}, d_{ti})$, where $c_{ti}$ is consumption when young (so in period $t$) and $d_{ti}$ is consumption when old (so in period $t+1$).

A rational preference relation $\succsim_{ti}$ is defined over the set of consumption vectors $Z$; strict preference relation $\succ_{ti}$ and indifference relation $\sim_{ti}$ are consequent.

The resources of this economy, described by the vector $\phi = \left(\phi^{\underline{t}}, \phi^{\underline{t}+1}, ..., \phi^{\bar{t}}\right)$, are exogenously given and constantly equal to one unit of commodity per period: $\phi^t = \phi^{t'} = 1 \; \forall t \neq t'$.

This assumption is necessary to avoid trivial impossibility results: how could it be possible to be equitable if the resources are distributed in an unfair way through the periods and no time redistribution is allowed? On the other hand, if time redistribution was admitted and costless, the dynamic model would have collapsed in a static problem with a single commodity resource constraint for which more equitable solutions were already proposed.

The time horizon of the model will be assumed to be either finite, in this case we assume without loss of generality $\underline{t} = 0$ and $\bar{t} = T$, or infinite: $t \in [0, \infty)$ or $t \in (-\infty, \infty)$. Notice that if $\underline{t}$ or $\bar{t}$ are finite a special kind of agent will arise: if $\underline{t} = 0$, there is the $-1$ generation for which the model describes only the consumption allocation when old (this generation will be called Adam, by the name of the first

[1]See also Kolm (1972) and Varian (1974).

[2]Notice that this result remains valid even if the number of agents goes to infinity.

[3]In case every generation is composed by only one agent, the subscript $i$ will be omitted and the agent will be identified by the generation he belongs to.

man on earth that was, in some sence, “born” old); if $\bar{t} = T$, there is the $T$ generation for which the reverse occurs and only consumption when young is described (this generation will be called Lionel[4]). In accordance to these special generations, in the hypothesis of a finite time horizon we will speak of the Lionel model; if $t \in [0, \infty)$, it will be the Adam model and if $t \in (-\infty, \infty)$ it will be the infinite time model.

Before stating the possibility/impossibility theorems, we need to introduce the axioms of efficiency and of intertemporal equity that the allocation rules will be required to satisfy.

The first trivial requirement pertains the feasibility of an allocation:

**Condition 2.1.** *(Feasibility)* An allocation $a$ is feasible if for every period $t$ the total commodities distributed is not greater than the resources available: $\sum_i c_{ti} + \sum_j d_{t-1j} \leq 1 \; \forall t$.

The classic concept of Pareto efficiency property directly transfers from the static context:

**Axiom 2.2.** (Pareto efficiency) *A feasible allocation $a$ is Pareto efficient if there is no other feasible allocation $a'$ such that no agent is worse off and at least one agent is strictly better off; that is if $\neg\exists \; a'$ s.t. $z'_{ti} \succsim_{ti} z_{ti} \; \forall ti$ and $a'_{t'j} \succ_{t'j} a_{t'j}$ for some $t'j$.*

For intergenerational equity, this paper readapts the well known no-envy axiom[5] to the dymamic context requiring that interpersonal comparison is allowed not only among agents of the same generation, but also among members of different generations.

**Axiom 2.3.** (No-envy) *An allocation $a$ is envy free if no agent strictly preferes the consumption bundle of some other agent to the own; that is if $\forall ti$ it is true that $z_{ti} \succsim_{ti} z_{t'j} \forall i \neq j$.*

The allocation of Adam and Lionel should be made clear for No-envy as for the next equity axioms: since the allocation $a$ does not define completely the lifetime consumption bundle, it will be assumed that the missing allocation puts these agents in the worse position in the comparison. So if other agents are comparing their allocation to that of these agents, it will be assumed that $c_{-1i} = d_{Tj} = 0$, while if Adam and Lionel generation agents compare their allocation to the one of

[4]By the name of the last man on earth in the apocalyptic science fiction novel by Mary Shelley “The last man”

[5]see Varian (1974).

the other agents (included other agents of the same generation) it will be assumed that $c_{-1i} = d_{Tj} = \frac{1}{N}$, where $N$ is the number of agents per generation.

It should be noticed that this transfert of no-envy concept into dynamics is not for free. In the static concept of no-envy it was included the possibility of moving resources from an agent to another that made Pareto efficiency and no-envy partially substitutes: if for example a cycle of envy realizes (agent $i$ envies agent $j$ and reverse) the allocation can not be Pareto efficient since a switch of allocations would be Pareto improving. In the dynamic context this exchange of resources is not possible.

Even if other alternative dynamic adaptations of no-envy where introduced (see Shinotsuka and al. (2007)) like no-envy in current consumption (so independently from the generation of the agent), we believe that the most intergenerationally equitable allocation follows no-envy in life time consumption bundles: it is not socially accepted that a young agent complains about how much more the contemporaneous old agent is consuming; much more interesting is a young that complains about the fact that he will not be given the same lifetime opportunities of his father.

Mantaining the idea of interpersonal comparisons among lifetime consumptions bundles, some weaker properties can be defined:

**Axiom 2.4.** (Equal treatment of equals) *An allocation a satisfies equal treatment of equals if for any two agents that share the same preferences the lifetime allocations they are assigned are for them indifferent; that is if for some* $ti, t'j$ $\succsim_{ti} \equiv \succsim_{t'j} \equiv \succsim^*$ *then* $z_{ti} \sim^* z_{t'j}$.

This is directly implied by no-envy: it requires that the no-envy axiom applies uniquely to “equal” agents; where agents are considered equal if they share the same preferences independently of the generation they belong to.

**Axiom 2.5.** (No domination) *An allocation a satisfies no domination if no agent is assigned a consumption bundle which is striclty less then the consumption bundle of some other agent; that is if* $\forall ti$ *it is never verified that* $z_{t'j} \gg z_{ti}$ $\forall t'j \neq ti$.

Another possibility of weakening the previous concepts is by adding the prefix “one directional” (OD) to the corresponding axiom: this will restrict intergenerational lifetime bundle comparisons only in one direction, in this case the equity concept will schrink to a kind of sustainability concept since only future generations’ bundles will be compared to the ones of the previous generations and not the reverse. For example we state one directional no-envy, that requires that no agent envies the bundles given to agents of previous (or contemporany) generations

**Axiom 2.6.** (One directional no-envy) *An allocation $a$ is one directional envy free if no agent strictly preferes the consumption bundle of some previous living agent to the own; that is if $\forall ti$ it is true that $z_{ti} \succsim_{ti} z_{t'j} \forall t'j \neq ti$ with $t' \leq t$.*

For the sake of simplifying the notation of the theorems, it will be introduced the helpful concept of constant allocation:

**Definition 2.7.** (Constance) An allocation $a$ is constant if every agent obtains the same lifetime allocation; that is if $\forall ti, t'j \;\; z_{ti} = z_{t'j}$.

## 3. Theorems

In an economy with at least four generations a first impossibility result can be given:

**Theorem 3.1.** *In a one commodity economy with 2 period living agents, one agent per generation, and given constant resources $\phi^t = 1 \, \forall t$, if the domain of the preference relations is restricted to satisfy monotonicity, it is not possible to define an allocation rule that satisfies both Pareto efficiency and no domination (or equal treatment of equals).*

*Proof.* Take agent $t-1$ and $t$'s preferences to care only about consumption when old, i.e. $(c_t, d_t) \succsim_t \left(\bar{c}_t, \bar{d}_t\right)$ if and only if $d_t \geq \bar{d}_t$, and agent $t+1$ and $t+2$'s preferences to care only about consumption when young, i.e. $(c_{t+1}, d_{t+1}) \succsim_{t+1} \left(\bar{c}_{t+1}, \bar{d}_{t+1}\right)$ if and only if $c_{t+1} \geq \bar{c}_{t+1}$; by efficiency $c_t = d_{t+1} = 0$ and $d_{t-1} = c_{t+2} = 1$. By no dominance (or by equal treatment of equals) agent $t$ should get $d_t = 1$ (since $d_{t-1} = 1$); accordingly agent $t+1$ should get $c_{t+1} = 1$ (since $c_{t+2} = 1$). By the resource constraint this is not possible, leading to the impossibility result. $\square$

Since the example that makes the proof needs agents to be interested in only one argument of the preferences, it will be assumed, from now on, that the preference relations satisfy strong monotonicity:

**Assumption 3.2.** (Strong monotonicity) *The preferences of the agents are strongly monotonic; that is $(x_{ti}) \succ_{ti} (x'_{ti})$ whenever $x_{ti} \geq x'_{ti} \wedge x_{ti} \neq x'_{ti}$.*

This assumption will now turn the previous impossibility result to possibility: in the Lionel model all constant allocations satisfy Pareto efficiency and no-envy. The no-envy property is a direct consequence of giving to all agents the same lifetime consumption bundle; the Pareto efficiency follows from the presence of the Adam and Lionel generations: given a constant allocation $a^*$, any different allocation $a'$ will affect strictly negatively Adam or Lionel if the other agents are weakly better off[6].

Also in the Adam model some allocations that satisfy Pareto efficiency and no-envy for every possible preferences of the agents emerge:

**Theorem 3.3.** *Assuming $t \in [0, \infty)$, in a one commodity economy with 2 period living agents, one agent per generation, given constant resources $\phi^t = 1 \, \forall t$, if the domain of the preference relations is restricted to satisfy strong monotonicity and convexity, there exists a constant allocation $a = (1 - c_{Lia}, c_{Lia}, ..., 1 - c_{Lia}, c_{Lia}, ...)$ such that all the constant allocations $(1 - \lambda c_{Lia}, \lambda c_{Lia}, ..., 1 - \lambda c_{Lia}, \lambda c_{Lia}, ...)$ with $\lambda \in [0, 1]$ satisfy both Pareto efficiency and no-envy.*

*Proof.* Let's first construct $c_{Lia}$[7]: it will be defined as the lowest preferred consumption level when young, under the constraint that consumption when old is given by $d_t = 1 - c_t$ among all the agents $t \in [0, \infty)$; so $c_{Lia} = inf \left[ \{c_t\}_{t \in [0,\infty)} \right]$ with $(c_t, 1 - c_t) \succsim_t (c, 1 - c) \; \forall c \in [0, 1]$.

Since no-envy is a byproduct of all stationary allocations, it remains to proof efficiency in Pareto's sense. Assume by absurd that the allocation $a = ((d_{-1}, c_0), (d_0, c_1), ..., (d_{t-1}, c_t), ...) = ((1 - \lambda c_{Lia}, \lambda c_{Lia}), (1 - \lambda c_{Lia}, \lambda c_{Lia}), ..., (1 - \lambda c_{Lia}, \lambda c_{Lia}), ...)$ is not Pareto efficient; there will exist another allocation $a' = \left( \left( d'_{-1}, c'_0 \right), \left( d'_0, c'_1 \right), ..., \left( d'_t, c'_{t+1} \right), ... \right)$ that will differ for at least one element from the previous one. Suppose it differs for a greater consumption when young $c_t$ for any agent $t \in [0, \infty)$; since a higher $c_t$ implies a lower $d_{t-1}$, $a'$ should be characterized by a higher consumption when young $c_{t-1}$; recursively the necessary compensations for making the previous agents reach (at least) the same utility level of allocation $a$ will lead to a reduction in consumption when old of the first old, Adam, and is therefore not a Pareto improvement.

Assume, instead, $a'$ is different from $a$ by a higher consumption when old $d'_t$ for one agent $t \in [-1, \infty)$. The allocation for agent $t + 1$, $z'_{t+1}$, in order to be weakly preferred to the previous, will be characterized by $c'_{t+1} + d'_{t+1} > 1$: by the resource constraint $d'_t > d_t$ implies $c'_{t+1} < c_{t+1}$, but since $c'_{t+1} < c_{Lia}$ it must be that the allocation that makes agent $t + 1$ indifferent with the previous one is above the $c + d = 1$ line (otherwise contradicting the definition of $c_{Lia}$). This will imply that $c'_{t+1} < c'_t$ and $d'_{t+1} > d'_t$ and, by recursivity, $\forall \tau \geq t \; c'_{\tau+1} < c'_\tau$ and $d'_{\tau+1} > d'_\tau$. Convexity of the preferences provides the additional requirement that the series of consumption when young (decreasing) and consumption when old (increasing) are not converging and make the allocation $a'$ unfeasible. □

[6]The argument proving this statement will become clear from the following proof.
[7]Lia is the acronym for least impatient agent.

This theorem shows that a Pareto efficient and no-envy allocation follows from the presence of the first old generation, Adam, that having only consumption when old in the model, behaves as an agent that doesn't care about consumption when young. This is the origin of his dictatorial power: since there is no possible compensation in terms of consumption when young, the allocation that gives all the resources to the old generations at every period of time is straightforward Pareto efficient and satisfies no-envy.

This result should not surprise too much since it confirms the Ferejohn and Page (1978) extension of the Arrow's theorem in a dynamic setting: Arrow's axioms, together with stationarity, lead to the dictatorialship of the first generation.

Before extending this weak possibility result to more agents, it is necessary to introduce the concept of more patient agent and of diversity:

**Definition 3.4.** *(More patient relation)* An agent $i$ is defined more patient than agent $j$ if the marginal rate of substitution of consumption when young for consumption when old is lower at any lifetime consumption bundle $x$: that is, $i\, mp\, j \iff MRS_i\left(x^*\right) \leq MRS_j\left(x^*\right)\ \forall x^*$. An agent is strictly more patient if strict inequality holds.

**Definition 3.5.** *(Diversity)* Agent $i$ and $j$ of generation $t$ and agent $k$ and $l$ of generation $t'$ are diverse if the contract curve between $i$ and $j$ has no interior intersection with the contract curve between $k$ and $l$; that is, if $z_{ti}, z_{tj} \in \mathbb{R}^2 \setminus \{0\}$ is a Pareto efficient allocation for agents $i$ and $j$ such that $z_{ti} + z_{tj} \in Int\left\{\mathbb{R}^2\right\}$, it is not Pareto efficient for agents $k$ and $l$.

The following assumption avoids the possibility that all the agents of a generation strictly prefer the same boundary allocation (all resources to young or all resources to old):

**Assumption 3.6.** (Interior first best allocation) *We assume that for at least one agent per generation the preferred allocation over the constant feasible allocations is interior.*

**Theorem 3.7.** *In a one commodity Adam economy with 2 periods living agents, two agents per generation and given constant resources $\phi^t = 1\, \forall t$, preferences of the agents are represented by continuous, differentiable, quasi-concave and strictly increasing utility functions and satisfy assumption (3.6). Whenever there exist a sequence $[t, t+2]$ in which agent $A, B$ of generation $t$, $C, D$ of generation $t+1$, $E, F$ of generation $t+2$ are such that:*

- *there are two generations among them that are diverse and*
- *one of the following group of relations is verified:*
  1. $C\, smp\, A\, mp\, B$, $C\, mp\, D\, smp\, B$, $C\, smp\, E\, mp\, F$, $C\, mp\, D\, smp\, F$, *or alternatively*
  2. $A\, smp\, C\, mp\, D$, $A\, mp\, B\, smp\, D$, $E\, smp\, C\, mp\, D$, $E\, mp\, F\, smp\, D$,

*there exists a unique allocation* $a = \left(\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ..., \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ...\right)$ *that satisfies both Pareto efficiency and no-envy.*

*Proof.* Suppose there exist a sequence $t, t+1, t+2$ for which case 1 applies. If $c_C + c_D + d_C + d_D > 1$ it implies that for the next generation there is less available consumption when young: $c_E + c_F < c_C + c_D$; but since $C\, smp\, E$ and $D\, smp\, F$, an allocation that satisfyes no-envy must show $c_E \leq c_C$ and $c_F \leq c_D$ leading to a contradiction. If instead $c_C + c_D + d_C + d_D < 1$, the previous generation will have a higher available consumption when old: $d_A + d_B < d_C + d_D$; but, again, since $C\, smp\, A$ and $D\, smp\, B$, an allocation that satisfies no-envy must show $d_A \geq d_C$ and $d_B \geq d_D$ leading to a contradiction. If case 2 applies, the reverse order of argumentation applies.

The remaining case is with $c_C + c_D + d_C + d_D = 1$: to satisfy no envy, the only possible allocation should satisfy $z_A = z_C = z_E$ and $z_B = z_D = z_F$; but if $z_C + z_D \in Int\left\{\mathbb{R}^2\right\}$, by the assumption of diversity of two of these generations, this allocation can not be Pareto efficient. The allocation $a = \left(\left(0, 0, \frac{1}{2}, \frac{1}{2}\right), ..., \left(0, 0, \frac{1}{2}, \frac{1}{2}\right), ...\right)$ is not Pareto efficient by the assumption of interior first best allocation: it is evident that the allocation $a' = \left(\left(0, \alpha, \frac{1}{2}, \frac{1}{2} - \alpha\right), ..., \left(0, \alpha, \frac{1}{2}, \frac{1}{2} - \alpha\right), ...\right)$ with $\alpha > 0$ and small enough is strictly preferred by all generations to $a$: in every generation there is an agent that strictly preferes the new allocation $\left(\frac{1}{2} - \alpha, \alpha\right)$ and all the other agents receive the same allocation. The other possible no-envy allocation on the border of resources is $a = \left(\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ..., \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ...\right)$; in this case Pareto efficiency is the consequence of the presence of the Adam generation: if for some agents $i, j$ at time $t$ consumption when old is such that $d_{ti} + d_{tj} < 1$, they have to be compensated with a $c_{ti} + c_{tj} > 0$ in order not to be worse off; but recursively this will end up with Adam agents to have strictly less consumption when old and be strictly worse off. □

It is interesting that it is no longer possible to construct a Lia allocation: whenever the constant allocation is interior the exchange possibilities among the agents and the assumption of diversity impede to have an allocation that satisfyes Pareto efficiency and no-envy. The only allocations that avoid any exchange possibility (that would require equality of the MRS) are on the boundary of the possibility set since the agents are not any more given two commodities (when young and when old) but only one of them.

Notice that $a = \left(\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ..., \left(\frac{1}{2}, \frac{1}{2}, 0, 0\right), ...\right)$ results in being the only possible allocation satisfying Pareto efficiency and no-envy even if, by assumption (3.6) on the preferences of the agents, all the other generations would be strictly better off by moving to an interior solution. This shows that the role of Adam is properly dictatorial in this framework.

Of course, in a model in which two agents per generation have to share two commodities per period, the fragmentation of the markets due to the dynamic overlapping generations framework and the impossibility to avoid exchange possibilities among agents on border solutions, the impossibility result is the only possible outcome.

The last theorem will test the robustness of the model by analyzing the infinite time horizon model. As $t \in (-\infty, +\infty)$, even in the economy with one commodity and one agent per generation, the impossibility result is restored: there is no allocation rule satisfying both Pareto efficiency and no-envy. Morover, even weakening the axiom into equal treatment of equals (or one directional version of no-envy) the impossibility persists.

**Theorem 3.8.** *In the infinite time horizon model, if the domain of the preference relations is restricted to satisfy strict monotonicity and convexity, it is not possible to define an allocation rule that satisfies both Pareto efficiency and equal treatment of equals (one directional no-envy) .*

*Proof.* Take agent $t$'s preferences described by the following functions:

$$U_t\left(c_t, d_t\right) = \begin{cases} U_\theta\left(c_t, d_t\right) & if \quad t \leq 0 \\ U_\tau\left(c_t, d_t\right) & if \quad t > 0 \end{cases}$$

such that the preferred stationary allocation for agent $\theta$ (so under the constraint that $c_t + d_t = 1$) is $(c_\theta, d_\theta) = (1, 0)$ while the equivalent allocation for agent $\tau$ is $(c_\tau, d_\tau) = (0, 1)$.

Suppose the allocation at time $t = 1$, when the two kind of agents coexist, is described by $a^1 = (d_0, c_1)$. Given $c_1$, also $d_1$ is determined: a $d_1'$ above the indifference curve (of preferences $\tau$) that passes through the preferred stationary allocation $(0, 1)$ would not satisfy equal treatment of equals (one directional no-envy), since that level of utility is not sustainable for all the next agents[8]; while a $d_1'$ below that indifference curve will not be Pareto efficient since, by the previous result it's not possible to make some generations reach a higher indifference curve while

[8]There is no stationary allocation on that indifference curve, so moving on that curve, the feasibility constraint can not hold.

that level is available for all agents of kind $\tau$. If $a^1 = (d_0, c_1)$ is such that $c_1 > 0$, the allocation that makes all agents on the indifference curve passing through $(0, 1)$ is not Pareto efficient since dominate by a different one that assignes $(c_1, 1)$ to agent 1 and $(0, 1)$ to all the following agents. So it should be that $a' = (0, 1)$. By equal treatment of equals all the agents of kind $\theta$ can not get more that the indifference curve passing through the preferrend allocation $(1, 0)$, and this can be made in an efficient way only if $a' = (1, 0)$ which is in contradiction with the previous result. (In order for $a^1 = (0, 1)$ the previous generation should get $z_0 = (c_0, 1)$; if $c_0 = 0$ agent 0 will envy some previous generations that by Pareto efficiency will reach at least the indifference curve passing through the preferred allocation; if $c_0 > 0$ he will be envied by the future generations).

Therefore there is no allocation rule that satisfies both Pareto efficiency and equal treatment of equals (one directional no-envy). □

## 4. Conclusions

We introduce the no-envy axiom, and some other weaker concepts deriving from it, in a two periods living overlapping generations model with heterogeneous agents.

The dynamic structure, that comes together with the overlapping generations model, brings a fragmentation and a kaleidoscopic view of the commodities: even if there is a unique kind of commodities in the economy (ex. corn), the good is characterized by the period in which it is available, so in economic sense the commodities become one per period; moreover the agents consider only two economic goods, forming consumption when young and consumption when old, but, for the sake of interpersonal comparison, own consumption when old is equivalent to the consumption when old of another generation.

Even assuming one commodity only, constant resources and no production in the economy, we show that existence of an allocation rule satifying both Pareto efficiency and no-envy is crucially depending on two conditions: the presence in the economy of Adam, the special first generation for which the model assigns only consumption when old, and the possibility of finding an allocation that avoids exchange possibilities among agents, the allocation on the border in the multi agent generation serve this scope. Whenever these conditions are fulfilled, the allocation is characterized by dictatorialship of the first old, restoring the Arrow's possibility result, unless this dictatorial role is contended by Lionel.

To overcome this impossibility result two possible solution can be proposed: on one side the comparability approach through construction of a social welfare function or by assuming the representative agent, on the other side, an approach that we would like to undertake in future research consists in weakening the proposed axioms by introducing lotteries of allocations in order to offset the dependence of advantages/disadvantages of possible exchanges on the preferences of the other agents.

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