Trading emission permits under upstream-downstream strategic interaction

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In this paper, we study how two strategic firms under environmental regulation based on tradable emission permits interact in both the permits and the output market. We find that *(i)* the outcome in the permits market is determined by the strategic interaction in the output market (and *vice versa*); *(ii)* a price-taker in the permits market can exploit the strategic linkages between the upstream and the downstream market to compensate for the fact that he has no direct influence on the permits price; and *(iii)* the possibility of banking permits reinforces the ability of the price-taker to counterbalance the price-maker's market power in the permits market. Accordingly, we illustrate with a numerical example how the welfare maximizing free allocation of permits depends on output market structure.

Key Words: tradable emission permits; banking behavior; upstream-downstream strategic interaction.
JEL Codes: D43, L13, Q21

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1. INTRODUCTION

The idea of setting up a market for tradable pollution permits is that such market can achieve a given pollution target in a cost-effective manner (Montgomery, 1972). Among other things, whether cost-effectiveness is achieved, depends on (to?) which extent agents are able to influence the equilibrium permits price (Hahn, 1985) and the optimal price path (Liski and Montero, 2005a, 2005b and 2006).

The short history of tradable emission permits[1], has shown how market power in the related output market decreases the efficiency of market-based policies. The introduction of tradable emission permits has contributed to the rise of electricity prices in Europe (Sijm, Neuhoff and Chen, 2006) and in some states of the United States (Kolstad and Wolak, 2003)[2]. In this sense, the theory has addressed the problem of exclusionary manipulation (Misiolek and Elder, 1989) and the impact of free allocation of permits in production efficiency (Eshel, 2005) relying on a dominant-fringe setting, disregarding the possibility of having more than one strategic agent in the permits market or in the product market. Two exceptions are Fehr (1993), which analyzes the possibility of monopolization and the choice of production technology, and Montero (2002), that compares pollution control policies in terms of R&D incentives.

In this paper, we contribute to previous literature by considering two strategic firms that interact both in the permits (upstream) and in the output (downstream) market in two successive periods. Our model differs from the traditional theory on upstream-downstream interaction (Salinger, 1988) in the sense that each firm's position in the upstream market is endogenous to the model. Our purpose is to investigate how firms' interaction in the permits market is determined by the product market structure and by the possibility of banking.

We find that firms' interaction in the permits market depends on how strong is firms' competition in the output market, which in turn depends on price elasticity (i.e. consumers' preferences). Efficiency in the permits market also depends on the way firms compete in the output market. Further, we show that, unlike in a dominant-fringe setting, a price-maker's market power in the permits market can be overruled by a price-taker that is strategic in the output market.

Regarding inter-temporal optimization, we can conclude that the possibility of banking permits reinforces the price-taker's ability to counterbalance the price-maker market power in the permits market, but does not

[1] The first example of a tradable emissions permits market was established by the Clean Air Act Amendment in 1990 to regulate SO2 emissions produced by electricity generators in the United States.

[2] Similarly, the emission trading experiments by Muller and Mestelman (1998) and Godby (2000) show the importance of market power in tradable permits markets.

ensure that firms will set permits prices consistently with the efficient price path.

In Section 2 and 3, we define notation, key assumptions and introduce our game-theoretical model. In section 4 we show the intra-temporal and inter-temporal optimality conditions that determine the equilibrium in both markets. In section 5, we illustrate with a numerical example how the welfare maximizing free allocation of permits depends on output market structure. In section 6 we compare our main results with the existing literature and conclude.

2. ASSUMPTIONS AND NOTATION

Consider two profit maximizing firms (i and j) that produce imperfectly substitute goods and compete in prices p_i and p_j respectively. For simplicity we assume horizontal differentiation à la Hotelling i.e. each good is located at one of the extremes of the interval [0,1].

The aggregate demand for each good is respectively given by $y_i[p_i, p_j]$ and $y_j[p_i, p_j]$, both assumed to be quasi-concave functions of prices.

We assume that goods are produced at a cost $c_k[y_k]$, $k = i, j$, with $c'_k[y_k] > 0$ and $c''_k[y_k] > 0$. The production of these goods generates polluting emissions as a by-product: for a given output level of y_k, firm k generates polluting emissions equal to βy_k with $\beta > 0$.

To encourage the reduction of polluting emissions, this industry is subject to environmental regulation based on tradable emission permits. According to environmental rules, all the polluting emissions of firm k (given by βy_k) must be covered by an equivalent amount of permits (E_k). Uncovered polluting emissions must be abated, i.e.:

$$a_k = \beta y_k - E_k \geq 0, \;\; k = i, j, \tag{1}$$

The abatement cost of uncovered emissions is denoted as $h_k[a_k]$, with $h'_k[a_k] > 0$ and $h''_k[a_k] > 0$.

The environmental regulator determines the amount of tradable permits available in each period (S). The regulator also decides how to allocate total emission permits between firms: firm i receives a flow of free allocation of αS permits, while firm j receives $(1-\alpha)S$. In the first period, in addition to the flow allocation of permits, firms receive an extra bank of permits, which they might use for current production/trading or save (bank) for future utilizations. The total extra bank is equal to R and it is also exogenously determined by the regulator, as well as its allocation between firms: λR for firm i and $(1-\lambda)R$ for firm j. In addition, we assume that $S + R$ is low enough to constraint firms' production decisions, that borrowing is not allowed (i.e. banking cannot be negative) and that agents' stock of permits are publicly observed.

With respect to permits trading, differently from the traditional literature regarding upstream-downstream interaction (Salinger, (1988)), we let firm's position in the upstream market to be endogenous. We consider that firm j has a competitive advantage in the permits market. We model such advantage by considering that firm j sets the price of permits q in the permits market.

At period $t = 1$ (banking period), firms receive an exogenous amount of permits (bank R and flow S allocation) and, conditional on their expectations about subsequent interactions, firms play a three stage game with the following time structure: in the first stage, firm i chooses the price of permits q_t and the bank of permits for period $t + 1$ i.e. $B_{j,t+1}$; in the second stage, firm i observes q_t and decides how many of his current polluting emissions to cover with permits $E_{i,t}$ and how many permits to save for future utilizations $B_{i,t+1}$, while firm j clears the permits market, i.e. he ensures that $x_{j,t} = -x_{i,t}$. Then, given after-trade permits holdings, firms simultaneously set output prices $p_{i,t}$ and $p_{j,t}$ respectively. The reason for considering the previous timing is twofold. First, pollution permit markets are in general less liquid than other commodity or financial markets and therefore we assume that decisions in such markets are harder to reverse than decisions in the output market[3]. Thus, interaction in the permits market must precede strategic interaction in the output market. Second, sequentiality is required to be able to find the price that clears the permits market. This is the case since each agent's position in the permits market as a demander or supplier of permits is decided endogenously depending on the difference between the amount of permits that were allocated to them and their optimal use of permits for production.

At period $t = 2$ (end of compliance period), firms play again the three stage game just described but now initial permits endowments are no longer exogenous: aside to the exogenous flow allocations they will receive from the regulator (αS and $(1 - \alpha)S$ respectively), firms are able to decide their initial permits holdings at $t = 2$ through the saving decisions they took at $t = 1$. Moreover, unlike $t = 1$, firms will not save permits for future utilizations given that $t = 2$ represents the end of the compliance period. In the following section, we describe the two-period dynamic model and, afterwards, we analyze the characteristics of the subgame perfect Nash equilibrium (SPNE) of the game.

3. THE MODEL

[3] Own statistics based on EPA's database at http://camddataandmaps.epa.gov/gdm/ show that the number of private transactions per year in the US-SO2 permits market range from 236 in 1994 to 4950 in 2005. Thus, even in the best case scenario, transactions are less than 15 per day.

At each period $t = 1, 2$, conditional on initial permits endowments, firms i and j play the following three stage game. In the first stage, firm i chooses the price of permits (q_t) and the bank of permits to save for period $t+1$ $(B_{j,t+1})$, with $B_{j,3} = 0$ (since compliance period ends at $t = 2$). More precisely, at the first stage of period t, firm j solves the following optimization problem:

$$\max_{\{q_t, B_{j,t+1}\}} \left\{ \begin{array}{c} p_{j,t} y_{j,t}[p_{i,t}, p_{j,t}] - c_{j,t}[y_{j,t}[p_{i,t}, p_{j,t}]] + q_t x_{j,t} - h_{j,t}\left[a_{j,t}\right] + \\ + \sum_{t=1}^{2} \frac{\pi_{j,t+1}}{(1+r)^t} \end{array} \right\} \tag{2}$$

subject to

$$E_{j,t} + a_{j,t} = \beta y_{j,t} \tag{3}$$

$$B_{j,t+1} = (1-\alpha)S + (1-\lambda)R + B_{j,t} - E_{j,t} - x_{j,t} \tag{4}$$

$$B_{i,t+1} = \lambda R + \alpha S + B_{i,t} - E_{i,t} - x_{i,t} \tag{5}$$

$$x_{j,t} = -x_{i,t} \tag{6}$$

$$\Omega_{j,t} = \{\Omega_{j,t+1}, p_{i,t}[E_{i,t}, E_{j,t}], p_{j,t}[E_{i,t}, E_{j,t}], E_{i,t}[q_t]\} \tag{7}$$

with $R = 0$ if $t = 2$ and where: *(i)* $-x_{i,t} = x_{j,t}$ corresponds to the amount of permits sold (or bought, when $x_{j,t} < 0$) by firm j; *(ii)* $a_{j,t}$ is the amount of polluting emissions abated by firm j and; *(iii)* $\Omega_{j,t}$ represents firm j's information set that includes the rational expectations formulated by this firm with respect to interactions in the subsequent stages.

In the second stage of the game played at each period t, firm i decides how many of its current polluting emissions to cover with permits $(E_{i,t})$ and how many permits to save for future utilizations ($B_{i,t+1}$), with $B_{i,3} = 0$. The optimization problem is:

$$\max_{\{E_{i,t}, B_{i,t+1}\}} \left\{ \begin{array}{c} p_{i,t} y_{i,t}[p_{i,t}, p_{j,t}] - c_{i,t}[y_{i,t}[p_{i,t}, p_{j,t}]] + q_t x_{i,t} - h_{i,t}\left[a_{i,t}\right] + \\ + \sum_{t=1}^{2} \frac{\pi_{i,t+1}}{(1+r)^t} \end{array} \right\} \tag{8}$$

subject to

$$E_{i,t} + a_{i,t} = \beta y_{i,t} \tag{9}$$

$$B_{i,t+1} = \alpha S + \lambda R + B_{i,t} - E_{i,t} - x_{i,t} \tag{10}$$

$$\Omega_{i,t} = \{\Omega_{i,t+1}, p_{i,t+1}[E_{i,t+1}], p_{j,t+1}[E_{i,t+1}]\} \tag{11}$$

Finally, at the third stage of the game, firm i and j interact in the output market simultaneously choosing their output prices. At this stage, firm i and j, solve the following optimization problems, For firm i is

$$\max_{\{p_{i,t}\}} \{p_{i,t} y_{i,t} [p_{i,t}, p_{j,t}] - c_{i,t} [y_{i,t} [p_{i,t}, p_{j,t}]] + q_t x_{i,t} - h_{i,t} [a_{i,t}]\} \tag{12}$$

subject to

$$\beta y_{i,t} = E_{i,t} + a_{i,t} \tag{13}$$

$$B_{i,t+1} = \alpha S + \lambda R + B_{i,t} - E_{i,t} - x_{i,t} \tag{14}$$

$$\Omega_{i,t} = \{\Omega_{i,t+1}\} \tag{15}$$

and for firm j is

$$\max_{\{p_{j,t}\}} \{p_{j,t} y_{j,t} [p_{i,t}, p_{j,t}] - c_{j,t} [y_{j,t} [p_{i,t}, p_{j,t}]] + q_t x_{j,t} - h_{j,t} [a_{j,t}]\} \tag{16}$$

subject to

$$\beta y_{j,t} = E_{j,t} + a_{j,t} \tag{17}$$

$$B_{j,t+1} = (1-\alpha) S + (1-\lambda) R + B_{j,t} - E_{j,t} - x_{j,t} \tag{18}$$

$$B_{i,t+1} = \alpha S + \lambda R + B_{i,t} - E_{i,t} - x_{i,t} \tag{19}$$

$$x_{j,t} = -x_{i,t} \tag{20}$$

$$\Omega_{j,t} = \{\Omega_{j,t+1}\} \tag{21}$$

In the following section, we characterize the SPNE of the game and describe the intra and inter-temporal optimality conditions, which will be used to analyze the role of banking permits under upstream-dowstream strategic interaction.

4. EQUILIBRIUM BEHAVIOR

We use backward induction techniques to characterize firms' behavior at equilibrium. First we analyze strategic interaction in the output market and then we focus on strategic interaction in the permits' market, underlining the results concerning the equilibrium path of permits' price and comparing it with a perfectly competitive price path. Note that we focus on interior solutions where profits are a quasi-concave function of $E_{k,t}$. This is always the case when we consider: *(i)* imperfect substitute goods à la Hotelling; *(ii)* quadratic costs of production. These restrictions on the parameters of the model ensures the reasonable economic properties: $p^*_{k,t} \geq 0$, $E^*_{k,t} \geq 0$ and $a^*_{k,t} \geq 0$ (or equivalently, $\beta y^*_{k,t} - E^*_{k,t} \geq 0$), for $k = i, j$.

4.1. Strategic interaction in the output market

The strategic interaction in the output market is of a static nature. Nevertheless, is important to stress that equilibrium output prices will differ across periods due to firms' banking decisions. In fact, firms' savings for period $t+1$ will affect firms' permit holdings at period t, and consequently affect the output equilibrium prices.

The equilibrium output prices $\left(p^*_{k,t} \geq 0,\ k = i, j\right)$ and the corresponding equilibrium output levels (which determine abatement needs $a^*_{k,t} \geq 0$, $k = i, j$) are determined by the solution to problems (12) and (16). The next lemma shows these optimal output price strategies:

LEMMA 1. *For given "after-trade" permits holdings $(E_{i,t}, E_{j,t})$, optimal price behavior in the output market requires perfect balance between firms' marginal revenue and firms' marginal cost of changing output prices, i.e., the optimal output prices $\left(p^*_{i,t}\left[E_{i,t}\right]; p^*_{j,t}\left[E_{i,t}\right]\right)$ correspond to the solution of the following system:*

$$\left\{ \begin{array}{l} p_{i,t} + \frac{y_{i,t}}{\frac{\partial y_{i,t}}{\partial p_{i,t}}} = c'_{i,t}\left[y_{i,t}\right] + \beta h'_{i,t}\left[\beta y_{i,t} - E_{i,t}\right] \\ p_{j,t} + \frac{y_{j,t}}{\frac{\partial y_{j,t}}{\partial p_{j,t}}} = c'_{j,t}\left[y_{j,t}\right] + \beta h'_{j,t}\left[\beta y_{j,t} - E_{j,t}\right] \end{array} \right. \tag{22}$$

Given optimal price strategies $\left(p^*_{i,t}\left[E_{i,t}\right]; p^*_{j,t}\left[E_{i,t}\right]\right)$, equilibrium output levels are given by

$$\left(y^*_{i,t}\left[p^*_{i,t}\left[E_{i,t}\right], p^*_{j,t}\left[E_{i,t}\right]\right]; y^*_{j,t}\left[\left[p^*_{i,t}\left[E_{i,t}\right], p^*_{j,t}\left[E_{i,t}\right]\right]\right]\right)$$

and, for given $E_{i,t}$, the equilibrium abatement choices are then determined as

$$\begin{array}{rcl} a^*_{i,t}\left[E_{i,t}\right] & = & \beta y^*_{i,t}\left[p^*_{i,t}\left[E_{i,t}\right]\right] - E_{i,t} \\ a^*_{j,t}\left[E_{i,t}\right] & = & \beta y^*_{j,t}\left[p^*_{i,t}\left[E_{i,t}\right]\right] - S + E_{i,t} \end{array}$$

Proof. Firms' profits are quasi-concave functions of output prices: output demands are assumed to be quasi-concave, production and abatement costs are convex functions of prices and permits revenues are linear functions. Quasi-concavity of firms' profit functions is a sufficient condition for the existence of output prices equilibrium that maximize profits. Such prices are the ones that satisfy Kuhn-Tucker conditions for problems (12) and (16) when the non-negativity constraint on prices is not binding (i.e. $p^*_{k,t} > 0$, $k = i, j$) so that we reach interior solutions. Then, the Kuhn-Tucker conditions to reach interior solutions for equilibrium prices can be reduced to the system in (22). ∎

Firms' optimal output prices are set at the margin. Equation (22) can be reformulated as

$$\left\{ \begin{array}{l} L_{i,t} = \frac{1}{\varepsilon^{y_{i,t}, p_{i,t}}} \\ L_{j,t} = \frac{1}{\varepsilon^{y_{j,t}, p_{j,t}}} \end{array} \right. \tag{23}$$

where $\varepsilon^{y_{k,t},p_{k,t}}$ is the own-price elasticity of demand[4] for good $k = i, j$ and $L_{k,t}$ is the augmented Lerner index in the sense that now it includes the effect of marginal abatement costs (with $L_{k,t} = \frac{p_{k,t} - c'_{k,t} - \beta \frac{h'_{k,t}}{p_{k,t}}}{p_{k,t}}$).

We observe that the firm with the less efficient abatement technology will be less competitive in the output market and, all the rest being equal, will set a higher output price to compensate its higher cost of environmental compliance. From equation (23) we can draw the following conclusion:

PROPOSITION 1. *Strategic firms are able to pass-through the cost of environmental regulation to consumers and will get windfall profits by increasing their mark-ups.*

When strategic interaction in the output market is relevant, firms will take into account the effect on the output market of their decisions in the permits market (i.e. $\frac{\partial p_{k,t}}{\partial E_{k,t}}, \frac{\partial p_{-k,t}}{\partial E_{k,t}}$) and, consequently, differently from what is established in the literature (both under perfectly competitive and dominant-fringe setting), firm i will no longer be passive. This is the case since i will acknowledge the influence he exerts on his rival's payoff trough changes in his permits demand ($E_{i,t}$).

Accordingly, when firms exert some degree of market power in the output market, any analysis of market power in the permits market that neglects the technological link with the output market is misleading because it does not account for the whole motivation behind firms decisions.

4.2. Strategic interaction in the permits market

According to our time structure, at period $t = 1$, firm face a static decision, in relation to the amount of emissions to be covered with permits (or the price of permits respectively) and a dynamic decision in relation to the amount of permits to save. For this reason, firms' initial endowment of permits at period $t = 2$ are endogenously determined by firms' saving decisions in period 1, and, consequently, firms' dynamic decisions at period 1 will influence market outcomes at period 2. Instead, since it must be the case that $B_{i,3} = 0$, at period $t = 2$ firms take only static decisions.

4.2.1. End of the compliance period

At $t = 2$, firms' interaction in the permits market is modeled in the context of a sequential two-stage game. In the second stage, firm i observes the price of permits and decides the amount of emissions ($E_{i,2}$) to be covered by permits (which residually determines its demand/supply of

[4] This means that $\varepsilon^{y_{i,t},p_{i,t}} = -\frac{\partial y_{i,t}}{\partial p_{i,t}} \frac{p_{i,t}}{y_{i,t}} > 0$ and, since goods are assumed to be Hotelling substitutes, it is also the case that $\varepsilon^{y_{i,t},p_{j,t}} = \frac{\partial y_{i,t}}{\partial p_{j,t}} \frac{p_{j,t}}{y_{i,t}} > 0$.

permits). Since it must be the case that $B^*_{i,3} = 0,$ the problem faced by firm i in period $t = 2$ is of a static nature. Considering again that the domain of values of the parameters is restricted to the one that entails an interior solution, firm i's choices in the permits market will be optimal if and only if the condition in the following lemma holds.

LEMMA 2. *Given permits price (q_2), firm i chooses $E^*_{i,2}[q_2]$ such that the following condition holds:*

$$h'_{i,2}\left[a_{i,2}\right] = q_2 - \left(\frac{\partial p_{i,2}}{\partial E_{i,2}} y_{i,2} + \frac{dy_{i,2}}{dE_{i,2}} \mu_{i,2}\right) \tag{24}$$

where $\mu_{i,2} = p_{i,2} - c'_{i,2}[y_{i,2}] - \beta h'_{i,2}[y_{i,2}]$ is the total mark-up in the output market. When condition (24) holds, and given $B^*_{i,3} = 0$, the conditional demand/supply of permits is then determined by

$$x^*_{i,2}[q_2] = \alpha S + B_{i,2} - E^*_{i,2}[q_2], \tag{25}$$

where $E^*_{i,2}[q_2]$ is implicitly given by (24).

Proof. We restrict our analysis to parameters for which firm i's profit function is a quasi-concave functions of $E_{i,2}$ to ensure the existence of a maximum. Such maximum satisfies Kuhn-Tucker conditions for problem (8) when the condition on the non-negativity of emissions is not binding (i.e. $E^*_{i,2} > 0$) so that we reach an interior solution. Then, the Kuhn-Tucker conditions to reach an interior solution for emissions can be reduced to (24). ∎

Lemma 2 shows that firm i's choice concerning emissions trading are optimal if and only if the gain from buying an additional emission permit (corresponding to the reduction in marginal abatement cost, $h'_{i,2}\left[a_{i,2}\right]$) is perfectly offset by the net marginal cost of buying such permit. This cost includes the direct cost of buying permits (the price of permits, q_2) and an indirect cost, which steams from the technological link between permits and output market. Indeed, equation (24) shows that, even if firm i behaves as a price-taker in the permits' market, as long as he is strategic in the output market, when considering whether to buy an additional permit, he anticipates the effect in $p_{i,2}$ due to the lower abatement effort he will undertake when buying permits, and the effect on the demand for good i due to the change both in $p_{i,2}$ and $p_{j,2}$. These effects are respectively the first and second term of the following expression:

$$\frac{\partial p_{i,2}}{\partial E_{i,2}} y_{i,2} + \frac{dy_{i,2}}{dE_{i,2}} \mu_{i,2}. \tag{26}$$

The sign of (26) is positive if $i's$ output price decreases when he increases his use of permits for production $\left(\frac{\partial p_{i,2}}{\partial E_{i,2}} < 0\right)$ and negative otherwise (the proof is provided in Appendix B). The sign of this last derivative ultimately depends on which firm owns the most efficient abatement technology and on the efficiency gap between one firm and the other (the proof is provided in Appendix A).

Previous literature based on a dominant-fringe setting found that price-takers optimize when $q = h_i'[a_i]$ and static inefficiency is due to the gap between the dominant firm's marginal abatement cost and the price of permits q.

Let us now suppose that firm i (the price taking firm in the market for permits) is in fact on the demand side. In such case, demand for permits is given by (25) so that (24) is satisfied. If i is the owner of the less efficient abatement technology $\left(\text{i.e. } \frac{\partial p_{i,2}}{\partial E_{i,2}} < 0\right)$, permits demand is higher with respect to the dominant-fringe setting, increasing permits price inefficiency. If instead he is the owner of the most efficient abatement technology and the relative efficiency of such technology is very high, so that (26) is negative, demand is lower than in the dominant-fringe setting, decreasing permits price inefficiency. Finally, if i is on the supply side, the sign of the effect in (26) stills depend on who owns the most efficient abatement technology but it would be the supply curve the one moving upwards or downwards as just described. The following proposition summarizes this result:

PROPOSITION 2. *A price-taker is able to counterbalance the market power of a price-maker in the permits market through his actions in the output market. Whether his demand (or supply) is higher or lower than in a dominant-fringe setting depends on wether an increase in the use of permits for production increases his own output price (or vice versa). This effect on output price is influenced by the relative efficiency of his abatement technology.*

The key for the price-taker abatement choices is the effect that the use of permits has on output profits and not on which side of the permits market he is. Optimal use of permits $E_{i,2}^*[q_2]$, and therefore $h_{i,2}^*\left[a_{i,2}^*\right]$, is such that:

$$h_{i,2}'\left[a_{i,2}^*\right] \quad < \quad q_2 \text{ if } \frac{\partial p_{k,2}}{\partial E_{k,2}} < 0 \tag{27}$$

$$h_{i,2}'\left[a_{i,2}^*\right] \quad > \quad q_2 \text{ if } \frac{\partial p_{k,2}}{\partial E_{k,2}} > 0 \tag{28}$$

Since $E_{i,2}$ is a function of q_2 which is set by firm j we derive the following lemma.

In stage 1, when firm j decides the optimal price of permits (q_2), he anticipates that the conditional demand/supply of permits is given by (25) and, despite the static nature of the problem ($B_{j,3} = 0$), the optimal value of q_2 will be influenced by firm i's past savings ($B_{i,2}$) as follows:

LEMMA 3. *At $t = 2$, the equilibrium price of permits is given by*

$$q_2 - \frac{x_{i,2}[q_2]}{\frac{\partial E_{i,2}[q_2]}{\partial q_2}} = h'_{j,2}\left[a_{j,2}\right] - \left(\frac{\partial p_{j,2}}{\partial E_{i,2}} y_{j,2} + \frac{dy_{j,2}}{dE_{i,2}} \mu_{j,2}\right) \tag{29}$$

Proof. Given our assumption regarding quasi-concavity of profits, the equilibrium value q_2^* is given by the FOCs of problem (2), with Ω_j including the information in system (22) and condition (24), i.e. $\Omega_i = \left\{p_{i2}^*\left[E_{i,2}^*[q_2]\right]; p_{j,2}^*\left[E_{i,2}^*[q_2]\right]; E_{i,2}^*[q_2]\right\}$. ∎

Lemma 3 shows that, in equilibrium, firm j chooses the price of permits for which the marginal revenue from price changes is exactly offset by the marginal cost generated by such changes, balancing the marginal profitability of permits price changes in the output and in the permits market.

Notice that the marginal revenue in (29) is the standard monopolist's marginal revenue that we note $M_2^* = \left(q_2^* - \frac{x_{i,2}[q_2]}{\frac{\partial E_{i,2}[q_2]}{\partial q_2}}\right)$. The RHS of the intra-temporal condition depends on the choice that his rival i made in the previous period (in (25)). This result restated in the following proposition, as we will show, may drastically change market outcomes.

PROPOSITION 3. *A price-taker in the permits market can influence the price of permits at $t = 2$ trough his saving decisions in $t = 1$.*

The LHS of (29) represents the marginal cost of price changes, that can be explained as the sum of a direct and an indirect effect. The direct marginal cost corresponds to the marginal cost of abatement $h'_{j,2}\left[a_{j,2}\right]$: changes in permits prices, affect permits trading and, therefore, firm j's choices in terms of abatement. The indirect cost is given by the last term in the RHS of (29) and it captures the effect of permits' price on output's price (and quantity) trough emissions trading: a change in the price of permits q_2 impacts on the optimal use of permits by i, changing "*after-trade*" permits holdings and, subsequently, affecting output prices. The discussion after lemma 2 regarding the sign of (26) also applies to the last term in the RHS of (29) since:

$$-\left(\frac{\partial p_{j,2}}{\partial E_{j,2}} y_{j,2} + \frac{dy_{j,2}}{dE_{j,2}} \mu_{j,2}\right) = \left(\frac{\partial p_{j,2}}{\partial E_{i,2}} y_{j,2} + \frac{dy_{j,2}}{dE_{i,2}} \mu_{j,2}\right) \tag{30}$$

Condition (30) implies that, if (26) is positive and firm j is the seller of permits, supply of permits is higher than supply in a dominant-fringe setting. Moreover, when (26) is positive, it is also the case that the demand for permits is higher than the demand faced by the dominant firm in a dominant-fringe setting. Instead, if (26) is negative, these two curves move in opposite directions as well as if j is the buyer and i is the seller. In this last case, permits price efficiency may be higher when there is more than one strategic agent, interplaying in the permits and in the output market, than when there is only one strategic firm and many competitive ones.

PROPOSITION 4. *Firms' competition in the output market influences the price-maker's permits price choice. Whether the resulting permits price is the perfect competition one or a price even higher than the one resulting from a dominant-fringe interaction depends on the way both firms interact in the output market taking into account each firm's relative abatement technology.*

Given that both demand and supply in our framework are different from the ones in a dominant-fringe setting or in a perfectly competitive setting, we find:

$$h'_{j,2}\left[a^*_{j,2}\right] \quad < \quad M^*_2 \text{ if } \frac{\partial p_{k,2}}{\partial E_{k,2}} < 0, \tag{31}$$

$$h'_{j,2}\left[a^*_{j,2}\right] \quad > \quad M^*_2 \text{ if } \frac{\partial p_{k,2}}{\partial E_{k,2}} > 0 \ . \tag{32}$$

Propositions 2 to 4 may be summrized as follows: when j is the owner of the most efficient abatement technology, i.e. $\frac{\partial p_{k,2}}{\partial E_{k,2}} < 0$, it is the case that

$$h'_{j,2}\left[a^*_{j,2}\right] < h'_{i,2}\left[a^*_{i,2}\right] < q^*_2$$

if either j is the seller of permits, or; $\left|\frac{\partial p_{j,2}}{\partial E_{i,2}} y_{j,2} + \frac{dy_{j,2}}{dE_{i,2}}\mu_{j,2}\right| > \left|-\frac{x_{i,2}[q_2]}{\frac{\partial E_{i,2}[q_2]}{\partial q_2}}\right|$ when j is the buyer. Similarly, it is the case that

$$q^*_2 > h'_{i,2}\left[a^*_{i,2}\right] > h'_{j,2}\left[a^*_{j,2}\right]$$

if either j is the buyer of permits, or; $\left|\frac{\partial p_{j,2}}{\partial E_{i,2}} y_{j,2} + \frac{dy_{j,2}}{dE_{i,2}}\mu_{j,2}\right| > \left|-\frac{x_{i,2}[q_2]}{\frac{\partial E_{i,2}[q_2]}{\partial q_2}}\right|$ if j is the seller.

Signs reverse when j is not the owner of the most efficient technology, i.e. $\frac{\partial p_{k,2}}{\partial E_{k,2}} > 0$.

4.2.2. Banking period

At period $t = 1$ strategic interaction in the permits market is again modelled in the context of a two-stage game. As before, in the second stage, firm i observes the contemporaneous price of permits (q_1) and decides the amount of emissions to be cover with permits ($E_{i,1}$). But now, differently from the end of the compliance period, given expectations on the path of permits' price, firm i also decides how many permits he wants to save for next period ($B_{i,2}$). These decisions will be optimal if and only if conditions in the following proposition are satisfied.

LEMMA 4. *Given permits price (q_1) and firm i's expectations on future interactions, firm i chooses optimally the amount of polluting emissions to* *cover with permits ($E_{i,1}$) and the amount of permits to save for future use ($B_{i,2}$) if and only if the following conditions hold:*

$$h'_{i,1}\left[a_{i,1}\right] = q_1 - \left(\frac{dy_{i,1}}{dE_{i,1}}\mu_{i,1} + \frac{\partial p_{i,1}}{\partial E_{i,1}}y_{i,1}\right) \quad (33)$$

$$q_1 = \frac{1}{1+r}\left(q_2 + x_{i,2}\frac{\partial q_2}{\partial B_{i,2}}\right) \quad (34)$$

When conditions (33) and (34) hold, the conditional demand/supply of permits is

$$x^*_{i,1}[q_1] = \alpha S + \lambda R - B^*_{i,2} - E^*_{i,1}[q_1] \quad (35)$$

where $E^*_{i,1}[q_1]$ is implicitly given by (33) and the optimal saving rule B^*_{i2} conditional on current prices and expectations on future interactions is implicitly given by (34).

Proof. Given quasi-concavity of profit functions, the equilibrium values $E^*_{i.1}[q_1]$ and $B^*_{i,1}$ are given by the FOCs of problem (8), with Ω_i including the information in system (22) and subsequent interactions at $t = 2$, i.e. $\Omega_{i,1} = \left\{p^*_{i,1}\left[E_{i,1}, E_{j,1}\right]; p^*_{j2}\left[E_{i,1}, E_{j,1}\right], \Omega_2\right\}$. ∎

Lemma 4 defines the intra and the inter-temporal optimality conditions for firm i. First, concerning the use of permits ($E_{i,1}$), condition (33) is analogous to condition (24) in lemma 2. This is the case because, given the saving decisions, the abatement decisions are, indeed, of a static nature (intra-temporal optimality). Second, condition (34) states the inter-temporal optimality condition for firm i during the banking period. Such condition requires that, in equilibrium, it is not profitable to reallocate permits intertemporally, and consequently, abatement effort across periods. In the case of firm i, the inter-temporal optimization is guaranteed when saving the amount of permits that makes equal the current cost of permits (either the direct cost from buying or the opportunity cost of not selling) and the (discounted) expected future profits due to the additional stock of permits $\left(\frac{d\pi_{i,2}}{dB_{i,2}}\right)$.

In a dominant-fringe setting, optimal fringe's savings equalize current opportunity cost of saving permits (q_1) and the discounted expect profits of saving permits:

$$\left(B^*_{i,2}\right)^{competitive} : q_1 = \frac{q_2}{1+r}.$$

Instead, when the price-taker in the permits market is strategic in the output market, the discounted expected profits of saving permits is not limited to the direct effect given by q_2. As a matter of fact, in the presence of banking under upstream-downstream strategic interaction, the discounted profits of saving permits include: *(i)* the direct effect of savings (firm i will not need to spend q_2 to buy the permit that saved today $x_{i,2}$); and *(ii)* the indirect effect of savings on future permits' prices (since as shown in proposition 4, $B_{i,2}$ influences firm $j's$ decision with respect to q_2). Let us restate (34) as follows:

$$q_1 = \frac{1}{1+r}\left(q_2 + H\right) \tag{36}$$

As a reaction to q_1 fixed by j, if firm i is the buyer next period ($x_{i,2} = \alpha S + B_{i,2} - E_{i,2} < 0$), $B_{i,2}$ is such that the previous equality is satisfied with $H > 0$. Then, given that $B_{i,2} \geqslant 0$ (no borrowing possible), when $B_{i,2}$ tends to $\alpha S - E_{i,2}$ the inter-temporal relationship between prices is

$$\lim_{B_{i,2}\to\alpha S - E_{i,2}} H = 0 \Rightarrow q_1 = \frac{q_2}{1+r} \tag{37}$$

Instead, when $B_{i,2} \to 0$ the relationship satisfies

$$\lim_{B_{i,2}\to 0} H = \frac{\partial q_2}{\partial B_{i2}}\left(\alpha S - E_{i,2}\right) \Rightarrow q_1 >> \frac{q_2}{1+r} \tag{38}$$

If firm i is the seller next period the relationship is inverse.

To our knowledge, this effect has not been considered in previous literature based on a competitive or dominant-fringe setting (Liski and Montero, 2005a). In these two settings, the price path of permits is the competitive one as long as the price-takers in the permits market hold positive stocks.

Proposition 5. *During the banking period, the price-taker's inter- temporal optimality condition is not the competitive one and, for that reason, the price-maker in the permits market cannot manipulate the price path alone.*

In the rest of this subsection, we solve the first stage of the game and characterize the equilibrium behavior of firm j. That is, at the first stage of the game played at $t = 1$, firm j decides on the permits price (q_1) and the amount of permits he is willing to save for future use ($B_{j,2}$). The next proposition points out the conditions that must be satisfied for $j's$ behavior to be optimal at $t = 1$.

Lemma 5. *In equilibrium, the price of permits at $t = 1$ is implicitly given by*

$$\begin{aligned} & \frac{dp_{j,1}}{dq_1}y_{j,1} + \frac{dy_{j,1}}{dq_1}\mu_{j,1} - x_{i,1}[q_1, B_{j,2}] - \frac{dx_{i,1}}{dq_1}\left(q_1 - h'_{j,1}\right) \\ = & \frac{1}{1+r}\left(\frac{\partial B_{i,2}}{\partial q_1}\left(q_2 - h'_{j,2}\right)\right) \end{aligned} \tag{39}$$

where $x_{i,1}$ includes λR.

Analogously, firm j's optimal bank of permits ($B_{j,2}$) is implicitly given by the following optimal saving rule:

$$\begin{aligned} & \frac{dp_{j,1}}{dB_{j,2}}y_{j,1}+\frac{dy_{j,1}}{dB_{j,2}}\mu_{j,1}+\left(q_1-h'_{j,1}\right)C-h'_{j,1} \\ = & -\left(\frac{dp_{j,2}}{dB_{j,2}}y_{j,2}+\frac{dy_{j,2}}{dB_{j,2}}\mu_{j,2}-\frac{dq_2}{dB_{j,2}}x_{i,2}+\left(q_2-h'_{j,2}\right)C-h'_{j,2}\right) \end{aligned} \tag{40}$$

where $C=\frac{\partial B_{i,2}}{\partial B_{j,2}}+\frac{dE_{i,1}}{dB_{j,2}}$.

The optimal amount of polluting emissions to be covered by permits is then determined by the following transition law:

$$E^*_{j,1}=(1-\alpha)S+(1-\lambda)R-B^*_{j,2}-x^*_{j,1}. \tag{41}$$

where $B^*_{j,2}$ is determined by (40) and $x^*_{j,1}$ is equal to $-x^*_{i,1}[q^*_1]$, obtained from replacing condition (39) into the equilibrium $x^*_{i,1}[q_1]$ in Proposition 5.

Proof. Given quasi-concavity of profit functions, the equilibrium q^*_1 is given by the FOCs of problem (2), with Ω_j including the information contained in system (22) and condition (24), i.e.
$\Omega_{j,1}=\left\{p^*_{i,1}\left[E^*_{i,1}[q_1]\right];p^*_{j,1}\left[E^*_{i,1}[q_1]\right];E^*_{i,1}[q_1];B^*_{i,2},\Omega_2\right\}$. ∎

Lemma 5 illustrates two points. First, with respect to the optimal saving rule, it shows that in the presence of upstream-downstream strategic interaction, the optimal q_2 chosen by agent j is affected by his rival's decision in relation to $B_{i,2}$. This is the case since such saving decisions will affect the way firms interact strategically in the output market (trough the influence that the use of permits has both on output prices and quantities). The dominant-fringe setting takes into account how changes in firm j's decisions affect saving decisions of firm i, and consequently, the choice of $E_{i,2}$ (either directly, or trough $B_{i,2}$). Instead, in our approach, both strategic agents' decisions interplay.

Second, in relation to the optimal pricing behavior in the permits market, lemma 5 shows that it is not a static decision. Firm j knows that the price at period $t=1$ will influence the saving decisions of firm i in that *same* period and, for that reason, it will influence the market outcomes (of both firms) in *next* period. Therefore, firm j' s optimal choice in period 1 must be such that the current marginal profitability of a change in permits' price (considering both the permits and the output markets) is perfectly balanced by the future marginal profitability of that change in permits price, which is induced by the effect of q_1 on $B_{i,2}$ and the subsequent effects *(i)* of $B_{i,2}$ on q_2; *(ii) of* q_2 and $B_{i,2}$ on E_{i2} and E_{j2}; *(iii)* and of E_{i2} and E_{j2} on the output market (trough upstream-downstream strategic linkages). When strategic interaction in the output market is relevant, our model shows that the dominant-fringe settings cannot be used to predict the equilibrium path of permits price.

4.2.3. Optimal path of permits price

Under perfect competition both in the output and in the permits market, the equilibrium path for permits prices is:

$$q_1 = \frac{q_2}{1+r}, \tag{42}$$

which, given intra-temporal static optimization, is also the path that maximizes total welfare (or minimizes total abatement cost):

$$h_1' = q_1 = \frac{q_2}{1+r} = \frac{h_2'}{1+r}. \tag{43}$$

If we consider a dominant-fringe framework for the permits market (without considering the output market[5]) as in Liski and Montero (2006), the path of prices may be different from the perfectly competitive one depending on the parameters α, λ, S and R. For instance, Liski and Montero (2006) focus on parameter values such that, when the dominant firm is the only one holding banks, it is the case that

$$q_1 > \frac{q_2}{1+r}. \tag{44}$$

This means that the firm exerting market power manipulates permits prices to accelerate the depletion of the fringe's bank of permits.

Our model adds to this literature by pointing out that, in the presence of strategic interaction in the output market, upstream-downstream technological linkages create a new determinant of firm $j's$ choice with respect to the optimal price of permits (which reduced form is implicitly given by the system of equations formed by both conditions in the previous lemma). Then, we can conclude that:

PROPOSITION 6. *The optimal price in the first period is chosen taking into account intra and inter-temporal strategic linkages between upstream and downstream markets in the second period. The price-maker takes into account the fact that this period's price influences directly or trough his rival's reaction, outcomes in present and future output prices, and therefore, markups and profits.*

5. WELFARE IMPLICATIONS

Using horizontal differentiation *à la* Hotelling, we represent the spectrum of possible versions of the good produced in this economy by the unit line $[0, 1]$. Moreover, we assume that firms are located at opposite extremes

[5] This market structure is equivalent to consider a perfectly competitive output market together with a dominant-fringe interaction in the permits market.

of the unit line: firm i is located at the extreme 0 and firm j at the extreme 1. There is a unit mass of atomic consumers uniformly distributed on the unit line according to their ideal specification of the good. The position of each consumer in the unit line is $x \in [0,1]$ and we consider that consumers face quadratic transport costs *à* la d'Aspremont et al. (1979).

Thus, the utility of consuming good i is given by:

$$U(V, x, p_i) = V - \tau x^2 - p_i, \tag{45}$$

where: V is the utility of consuming the ideal specification of the good, $x \in [0,1]$ denotes the position of consumer x in the spectrum of possible variants, τ is the unit transportation cost and p_i is the price charged by firm offering good i. Similarly, the utility of consuming good j is given by:

$$U(V, x, p_j) = V - \tau(1-x)^2 - p_j. \tag{46}$$

Define the indifferent consumer located at $\tilde{x}$ as the consumer for which the following relations are satisfied:

$$U(V, \tilde{x}, p_i) = U(V, \tilde{x}, p_j) \tag{47}$$

$$V - \tau\tilde{x}^2 - p_i = V - \tau(1-\tilde{x})^2 - p_j$$

$$\tilde{x} = \frac{1}{2} - \frac{p_i}{2\tau} + \frac{p_j}{2\tau}. \tag{48}$$

Consumers located to the left of $\tilde{x}$ buy good i while those located to the right of $\tilde{x}$ buy good j instead. Accordingly, demand functions are given by:

$$y_i[p_i, p_j] = \frac{1}{2} - \frac{p_i}{2\tau} + \frac{p_j}{2\tau}, \tag{49}$$

$$y_j[p_i, p_j] = \frac{1}{2} + \frac{p_i}{2\tau} - \frac{p_j}{2\tau}. \tag{50}$$

Let us leave the possibility of banking aside for a moment and consider the case of $t = 2$ where firms maximize static profits. Let us also assume that $R = 0$. We can substitute the general demand functions in the previous model by the ones specified in (49) and (50) to obtain output prices as a function of the parameters, i.e. $p_i^*(\alpha, S)$, and $p_j^*(\alpha, S)$. Accordingly, it is possible to measure social welfare conditional on the regulator's choice in terms of S and α.

Social welfare in a period t can be obtained by aggregating consumers' individual utility and adding total profits, i.e.:

$$W^*(\alpha, S) = \Pi^*(\alpha, S) + C^*(\alpha, S) \tag{51}$$

where $W^*(\alpha, S)$ is the social welfare in equilibrium[6], $\Pi^*(\alpha, S)$ is the firms' joint-profits in equilibrium, and $C^*(\alpha, S)$ is the consumers' welfare in

[6] We consider social welfare in equilibrium in the sense that, conditional on regulator's decisions, both firms and consumers are maximizing their respective payoffs.

equilibrium. Formally:

$$\Pi^* (\alpha, S) = \pi_i^* (\alpha, S) + \pi_j^* (\alpha, S), \tag{52}$$

where $\pi_k^* (\alpha, S)$, $k = i, j$, is the profits of firm k evaluated at the equilibrium values of q^*, E_k^* and p_k^* that solve our three-stage game for period t.

Similarly, consumers' welfare is given by:

$$\begin{aligned} C^* (\alpha, S) &= \int_0^{x\left(p_i^*(\alpha,S),p_j^*(\alpha,S)\right)} \left(V - \tau x^2 - p_i^* (\alpha, S)\right) dx \\ &+ \int_{x\left(p_i^*(\alpha,S),p_j^*(\alpha,S)\right)}^1 \left(V - \tau (1-x)^2 - p_j^* (\alpha, S)\right) dx, \end{aligned} \tag{53}$$

where the first integral provides an aggregate measure of the welfare of consumers that buy good i and the second integral provides an aggregate measure of the welfare of consumers that buy good j.

Thus, total welfare in equilibrium, as a function of regulator's choices, is given by:

$$\begin{gathered} W^* (\alpha, S) = \pi_i^* (\alpha, S) + \pi_j^* (\alpha, S) \\ + \int_0^{x\left(p_i^*(\alpha,S),p_j^*(\alpha,S)\right)} \left(V - \tau x^2 - p_i^* (\alpha, S)\right) dx \\ + \int_{x\left(p_i^*(\alpha,S),p_j^*(\alpha,S)\right)}^1 \left(V - \tau (1-x)^2 - p_j^* (\alpha, S)\right) dx \end{gathered} \tag{54}$$

For a given polluting target ($\underline{S}$), it is possible to identify which allocation rule maximizes social welfare (considering that both consumers and firms behave rationally). For a given $\underline{S}$, the optimal allocation rule (α^*) is the solution to the following optimization problem:

$$\begin{aligned} &\max_{\alpha} \{W^* (\alpha, \underline{S})\} \\ &s.t.\ 0 \leq \alpha \leq 1 \end{aligned} \tag{55}$$

From (55) we see that, in the presence of strategic interaction in the output market, the optimal allocation rule α may coincide with the one that ensures

$$h_i' = h_j' \tag{56}$$

only by chance.

The allocation that ensures the relationship in (56) is the allocation that entails maximization of social welfare in the absence of strategic behavior. This is the case given the duality between the welfare maximization problem and the abatement cost minimization problem. When we observe market power in the output market, such duality is no longer present: firms decisions in the permits market (permits trading and abatement) will be reflected in the output equilibrium prices (and therefore consumer's welfare) and vice versa.

Accordingly, when maximizing social welfare, under certain circumstances, the regulator could be willing to give up on abatement efficiency in order to lower prices of output and increase consumers' welfare (which should more than compensate welfare losses due to inefficiency in abatement).

In what follows we maximize welfare for a certain period t, under strategic interaction in the output market using specific functional forms. On the side of consumers, we consider utility functions (45) and (46), normalizing transport costs to one, i.e. $\tau = 1$. In this case, demand faced by firm $k = i, j$ is given by:

$$y_k\left(p_i, p_j\right) = \frac{1}{2} - \frac{p_k}{2} + \frac{p_{-k}}{2}. \tag{57}$$

Moreover, we assume production and abatement costs satisfy:

$$c_k(y_k) = \frac{1}{2}y_k^2, \; k = i, j. \tag{58}$$

$$h_i(a_i) = \frac{1}{2}a_i^2 \tag{59}$$

$$h_j(a_j) = \frac{\varphi}{2}a_j^2, \; \varphi < 1 \tag{60}$$

where φ implies that firm j owns a more efficient abatement technology.

Considering these specific functions we solve the three-stage game proposed in Section 2 for $t = 2$ with $R = 0$ (see Appendix D). Considering reasonable values for the parameters, we get the allocation of permits that maximizes joint profits:

$$\alpha_\Pi = 0.42. \tag{61}$$

The allocation of permits that maximizes joint profits for the chosen value of the parameters is lower than a half, i.e. $\alpha_\Pi < \frac{1}{2}$, implying that the less efficient firm in terms of abatement should receive less permits than the most efficient one. This result, contrary to previous literature's result, is due to strategic interaction in the output market. As underlined before, in this setting the regulator considers both the profits in the permits and in the output market. Even if abatement costs are not minimized, the higher profit in the output market compensates this inefficiency. This is the reason why $\alpha_\Pi < \frac{1}{2}$.

After calculating consumer's surplus as shown in Appendix D, the allocation that maximizes total welfare is:

$$\alpha^* = 0.75. \tag{62}$$

For comparison purposes we compute, for same value of the parameters, the allocation of permits that would lead to abatement efficiency in a perfectly competitive framework, i.e., the allocation that ensures $h_i' = h_j'$ and that, in such a framework, leads to no trade $(x_i = 0)$. Then, considering the relation between prices and permits allocations we get:

$$\alpha_a = 0.53. \tag{63}$$

Comparing our optimal allocation rule (62) with (61) and (63) we observe that:

$$\alpha^* = 0.75 > \alpha_a = 0.53 > \alpha_\Pi = 0.42 \tag{64}$$

From (64) we learn that, under strategic interaction both in the output and the permits market, in order to choose the optimal allocation of permits, the regulator should not just focus on abatement efficiency. For the chosen values of the parameters, we find that the allocation that maximizes joint profits, α_Π, is lower than the allocation that only considers abatement efficiency, α_a since, while in the former case the regulator must balance profits both in permits and in output market[7], in the latter he only looks at the permits market.

When the regulator does not only consider firms' joint profits but also consumers' surplus, the optimal allocation of permits changes to α^*. For the chosen value of the parameters, the latter allocation is the highest. This is because consumers would prefer the regulator to give *all* permits to the less efficient firm to decrease output's price. Then, maximization of joint profits both in the permits and in the output market counterbalance the extreme position of consumers resulting in the value $\alpha^* = 0.74515$.

We now look at optimal allocation of permits considering the maximization of welfare across time in our two-period model $(R > 0)$. Doing this implies finding the optimal initial extra allocation of permits λ that maximizes:

$$\max_\lambda \left\{ C_1(\alpha^*, \lambda) + \Pi_1(\alpha^*, \lambda) + \frac{C_2(\alpha^*, \lambda) + \Pi_2(\alpha^*, \lambda)}{1+r} \right\} \tag{65}$$
$$s.t.\ 0 \leq \lambda \leq 1$$

In period 3, after bank is exhausted, the static game repeats itself. Therefore, optimal λ in the previous problem is conditional on the value of α that maximizes welfare for a given t with $R = 0$. This is the reason why in (65) we have already included the optimal allocation rule found in the static case $\alpha^* = 0.74515$. For shortness we will not derive the value of λ for the chosen value of the parameters but we summarize the procedure in the

[7] The relation $\alpha_a > \alpha_\Pi$ is due to the fact that, for the chosen values of the parameters, profits in the output market from the firm that recieves $\alpha_\Pi S$ is positive and therefore higher than in the case where agents just interact in the permits market. In this last case optimal allocation is α_a.

following three steps: (i) substitute expressions (57)-(60) in the FOCs of our dynamic model and solve the system; (ii) given optimal behavior find λ that maximizes inter-temporal profits; (iii) add consumer's inter-temporal surplus that, since price decisions are of a static nature, will simply be the discounted sum of the consumer surplus as derived for the static case, i.e. $C_1(\alpha^*) + \frac{C_2(\alpha^*)}{1+r}$.

6. CONCLUSION

In this paper, we use a game-theoretical model to account for two strategic firms interacting both in the permits and in the output market. In terms of intra-temporal strategic interaction, first, we show that firms pass-through the costs of pollution abatement to consumers and are able to realize windfall profits. Second, differently from previous literature, we find that a price-taker in the permits market can counterbalance the price-maker's market power trough his actions in the output market. In this context, we show that the price inefficiency in the permits market, which typically characterizes a dominant-fringe setting, disappears under some output market conditions.

Regarding inter-temporal optimization, our main result is that the possibility of banking permits reinforces the price-taker's possibility of counterbalance the price-maker's market power in the permits market. Nonetheless, the fact that the price-taking firm holds positive stocks does not ensure that the price path of permits coincides with the competitive one.

Our results are of big relevance for market design. Since permits market outcomes and efficiency depend on output market characteristics, authorities should not ignore such interdependence when choosing optimal regulatory policy. Further proof is provided in a numerical example where we derive the optimal flow allocation of permits and the rule for allocating the initial bank of permits optimally.

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Appendix A

Let us consider that firms compete à la Hotelling in the output market which implies

$$y_{k,t}[p_{i,t}, p_{j,t}] = \frac{1}{2}\left(1 - p_{k,t} + p_{-k,t}\right), \ k = i, j \tag{66}$$

Moreover, let us assume production and abatement costs are of the form:

$$\begin{aligned} c_{k,t}\left[y_{k,t}\right] &= \frac{1}{2}y_{k,t}^2, & (67)\\ h_{i,t}[a_{i,t}] &= \frac{1}{2}a_{i,t}^2, & (68)\\ h_{j,t}\left[a_{j,t}\right] &= \frac{\varphi}{2}a_{j,t}^2, & (69) \end{aligned}$$

where, if $\varphi < 1$ implies that firm j is the owner of the most efficient abatement technology.

To determine the impact of $E_{i,t}$ in the equilibrium prices $p_{i,t}^*$ and $p_{j,t}^*$ we substitute (66)-(68) in the general problem, solve the system of equations in (22), and then make the derivative of equilibrium values $p_{i,t}^*$ and $p_{j,t}^*$ w.r.t. $E_{i,t}$. We obtain

$$\frac{dp_{i,t}^*}{dE_{i,t}} = \beta\frac{3\varphi - 5}{\beta^2 + \beta^2\varphi + 8} \tag{70}$$

which is negative as long as $\varphi < \frac{5}{3}$. This is always the case if the price-maker is also the owner of the most efficient abatement technology ($\varphi < 1$).

Similarly, it is the case that

$$\frac{dp_{j,t}^*}{dE_{i,t}} = \beta\frac{5\varphi - 3}{\beta^2 + \beta^2\varphi + 8} \tag{71}$$

which is positive if and only if $\varphi > \frac{3}{5}$, i.e. asymmetry in firms' abatement cost functions is not too strong.

Appendix B

The inequality

$$\frac{\partial p_{i,t}}{\partial E_{i,t}}y_{i,t} + \frac{dy_{i,t}}{dE_{i,t}}\mu_{i,t} > 0 \tag{72}$$

can be re-expressed as

$$\frac{\partial p_{i,t}}{\partial E_{i,t}}\frac{y_{i,t}}{p_{i,t}}\left(\left(\varepsilon^{y_{i,t},p_{j,t}}\frac{\partial p_{j,t}}{\partial p_{i,t}}\frac{p_{i,t}}{p_{j,t}}\frac{\partial y_{i,t}}{y_{i,t}}\frac{y_{i,t}}{\partial y_{i,t}} - \varepsilon^{y_{i,t},p_{i,t}}\right)\mu_{i,t} + y_{i,t}\frac{p_{i,t}}{y_{i,t}}\right) > 0 \tag{73}$$

where $\varepsilon^{y_{i,t},p_{i,t}} = -\frac{\partial y_{i,t}}{\partial p_{i,t}}\frac{p_{i,t}}{y_{i,t}} > 0$ and $\varepsilon^{y_{i,t},p_{j,t}} = \frac{\partial y_{i,t}}{\partial p_{j,t}}\frac{p_{j,t}}{y_{i,t}} > 0$ and therefore, after rearranging we find

$$\frac{\partial p_{i,t}}{\partial E_{i,t}} y_{i,t} \left(1 - 2\varepsilon^{y_{i,t},p_{i,t}} \frac{\mu_{i,t}}{p_{i,t}}\right) > 0. \tag{74}$$

The previous sign is respected if either:
a) it is the case that $\frac{\partial p_{i,t}}{\partial E_{i,t}} < 0$ and $1 < 2\varepsilon^{y_{i,t},p_{i,t}}\frac{\mu_{i,t}}{p_{i,t}}$, or:
b) instead $\frac{\partial p_{i,t}}{\partial E_{i,t}} > 0$ and $1 > 2\varepsilon^{y_{i,t},p_{i,t}}\frac{\mu_{i,t}}{p_{i,t}}$.
On the other hand, from (23) we know that

$$\frac{\mu_{i,t}}{p_{i,t}} = \frac{1}{\varepsilon^{y_{i,t},p_{i,t}}}. \tag{75}$$

Then, the sign is respected only when $\frac{\partial p_{i,t}}{\partial E_{i,t}} < 0$ and is reversed otherwise. Then, when the permit's price effect is positive $\left(\frac{\partial p_{i,t}}{\partial E_{i,t}} > 0\right)$, permit's quantity effect is negative $\left(\frac{dy_{i,t}}{dE_{i,t}} < 0\right)$.

Appendix D
Output equilibrium prices for the mentioned functions are:

$$\begin{aligned} p_i^* &= \frac{\left(4\beta^2 - 5\beta E_i + 3\beta^2\varphi + \beta^4\varphi - 3S\beta\varphi + 3\beta\varphi E_i - S\beta^3\varphi + 12\right)}{\beta^2 + \beta^2\varphi + 8} \\ p_j^* &= \frac{\left(3\beta^2 - 3\beta E_i + 4\beta^2\varphi + \beta^4\varphi - 5S\beta\varphi + 5\beta\varphi E_i - S\beta^3\varphi + 12\right)}{\beta^2 + \beta^2\varphi + 8} \end{aligned} \tag{76}$$

Given equilibrium prices as a function of demand/supply of permits, we now solve the second-stage of the game finding:

$$E_i^* = E_i^*\left[q, \beta, S, \varphi\right] \tag{77}$$

that satisfies

$$\frac{dE_i}{dq} = -\frac{\left(\beta^2 + \beta^2\varphi + 8\right)^2}{-5\beta^2\varphi^2 + 6\beta^2\varphi + 11\beta^2 + 64}. \tag{78}$$

The previous expression is negative if the denominator is positive. This is the case if and only if $\varphi \in \left]\frac{1}{5\beta}\left(3\beta - 8\sqrt{\beta^2+5}\right), \frac{1}{5\beta}\left(3\beta + 8\sqrt{\beta^2+5}\right)\right[$ that, for feasible values of β includes[8] all possible values of $0 < \varphi < 1$.

Finally, we solve the first stage of the game taking into account (76) to (78) finding:

$$q^* = q^*\left[\beta, S, \varphi, \alpha\right] \tag{79}$$

Considering reasonable values of the parameters, say: $\beta = 0.8$, $S = 0.5$ and $\varphi = 0.75$ the previous expression can be simplified to:

$$q^* = 0.28 - 0.16\alpha \tag{80}$$

[8] For example for say $\beta = 0.8$, the interval is: $\varphi \in \left]-4.1497, 5.3497\right[$.

The conditional use of permits is then given by:

$$E_i^* = 0.18\alpha + 0.13 \tag{81}$$

and equilibrium prices in the output market are:

$$p_i^* = 1.64 - 4.44 \times 10^{-2}\alpha \tag{82}$$
$$p_j^* = 1.21 \times 10^{-2}\alpha + 1.59 \tag{83}$$

Thus, equilibrium profits conditional on regulator decision about α are respectively given by:

$$\pi_i = 0.16\alpha - 6.51 \times 10^{-2}\alpha^2 + 0.60 \tag{84}$$
$$\pi_j = 3.99 \times 10^{-2}\alpha^2 - 0.14\alpha + 0.73 \tag{85}$$

with

$$\frac{d\pi_i}{d\alpha} = 0.16 - 0.13\alpha, \tag{86}$$
$$\frac{d\pi_j}{d\alpha} = 0.08\alpha - 0.14. \tag{87}$$

The first expression is positive while the second one is negative for any feasible value of α.

Then, the allocation of permits between firms that maximizes joint profits is:

$$\alpha_\Pi = 0.42. \tag{88}$$

Concerning consumers' welfare, given equilibrium output prices in (82), the position of the indifferent consumer in the Hotelling interval is:

$$\tilde{x} = 2.83 \times 10^{-2}\alpha + 0.48 \tag{89}$$

which is always inside the interval $[0, 1]$ for relevant values of α.

Thus, consumers' total welfare is given by:

$$C(\alpha) = 7.99 \times 10^{-4}\alpha^2 + 1.50 \times 10^{-2}\alpha + V - 1.70 \tag{90}$$

and the economy's total welfare $W(\alpha)$ is then

$$C(\alpha) + \Pi(\alpha) = -2.44 \times 10^{-2}\alpha^2 + 3.64 \times 10^{-2}\alpha + V - 0.37. \tag{91}$$

Then, the value of α that maximizes total welfare in (91) is:

$$\alpha^* = 0.75. \tag{92}$$