Split-Ticket Voting: An Implicit Incentive Approach

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Abstract

This paper applies an implicit incentive approach in the principal-agent framework to study split-ticket voting, when citizens vote for candidates of different parties in the simultaneous municipal and regional elections (i.e., the elections of a mayor for the city hall and of a governor for the region). The principals (voters), in the presence of moral hazard, reward the agents (mayor and governor) with reelection based on their observed performance but through implicit reward rules. Thus, the voters can influence the politicians’ performance only through the choice of evaluation rules. We first show that, if the voters split tickets and the politicians are committed to their political parties, then the voters prefer a comparative rule conditioned on the incumbents’ performance. Otherwise, the voters adopt a cut-off rule under which an incumbent is reappointed only when her performance exceeds a critical threshold. Second, we show that the stationary probability that the voters split tickets is lower than the stationary probability that they do not split tickets. We find empirical support for the model prediction on the equilibrium transition probabilities between the split-ticket and non-split-ticket states for moderate levels of the politicians’ commitment to their political parties.

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1. Introduction

Split-ticket voting is a common feature of modern political systems, when citizens vote for candidates of different parties in two simultaneous elections (for example, the presidential and congressional elections in the US, or the municipal and regional elections in Spain). The literature has addressed the problem of split-ticket voting in the US elections in the context of strategic voting models analyzed by Alesina and Rosenthal (1995, 1996) and Chari et al. (1997). They show that citizens have incentives to vote split-tickets strategically since the policy outcome is a function of the composition of the executive branch and the legislature. However, Degan and Merlo (2007) have shown empirically that "by and large split-ticket voting is also consistent with sincere voting."\(^1\) Hence, a full microfoundation of split-ticket voting in the US elections is still ahead of researchers. In turn, the problem of split-ticket voting in the simultaneous municipal and regional elections (when citizens elect a mayor for the city hall and a governor for the region this city belongs to), has not been studied yet to our knowledge. This work aims to fill this gap.

In this paper, we apply implicit incentive contracting theory to shed light on split-ticket voting in the local municipal and regional elections that occur simultaneously. We study a principal-agent model of policy implementation in the presence of a moral hazard problem, where the voters are principals and the politicians are office-motivated agents. The politicians want to be reelected for a next term so they are held accountable by the voters at the moment of election and have incentives to satisfy the voters' wishes. Moreover, we assume that the politicians care about overall representation of their party in governing bodies, i.e. a mayor prefers her party co-member to a politician from the rival party for the governor's office (and vice versa for a governor). Each politician performs a single task policy. In turn, the voters care about policy outcomes that are observable but not contractible. The voters evaluate the incumbents' performance and vote accordingly, so they reward (i.e. reelect) the politicians depending on their observed performance, but through implicit reward rules. Obviously, the voters can influence the politicians' performance through the choice of evaluation rules. We restrict the functional space of performance evaluation rules the voters can choose to linear performance evaluation rules. These rules are further required to be sequentially rational.

The society can be found in one of the two states: either a mayor and a governor are the members of the same party (in our framework this is interpreted as non split-ticket voting), or a mayor and a governor belong to different parties (which is interpreted as split-ticket voting). So, split-ticket voting is modeled as the outcome of evaluation rules chosen in the previous

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\(^1\)Degan and Merlo (2007), p. 16.
period. In other words, the voters do decide on evaluation rules to reward the incumbents, and then just vote (or do not vote) split-tickets to follow the chosen rules.

We show that the voters prefer to evaluate the mayor and governor from the same party with a cut-off rule, under which an incumbent is reappointed only when the observed policy outcome she generates exceeds a critical bound given by the equilibrium effort in this office. Under this rule the politician’s effort just increases her own reelection prospects and does not decrease her comember’s chances for reappointment. As for the mayor and governor from different parties, the voters’ choice of an evaluation rule depends on the degree of the politicians’ commitment to their political parties. If the politicians are concerned only about their own reelection prospects rather than their party overall representation in governing bodies, then the voters use a cut-off rule. If the politicians show commitment to their political parties, and thus want their party co-members to be appointed, then the voters prefer a comparative rule to create a competitive environment between the politicians from different parties. This rule is conditioned on the differences between the incumbents’ performance and the equilibrium effort in corresponding offices, and specifies an optimal method to compare and evaluate the politicians’ deeds.

Our results rest on the fundamental assumption about the politicians’ commitment to their political parties, when a mayor (resp. governor) not only cares about her own reelection prospects but also about her party co-member’s chances to be elected for the governor’s (resp. mayor’s) office. The principals choose a comparative rule to create a competitive environment that gives extra implicit incentives to the agents from different parties to perform better. If we relax the assumption about the politicians’ commitment to their parties, then the competition effect weakens and the voters prefer a cut-off rule, under which the reelection chances are more sensitive to effort that increases the politicians’ incentives.

We find the equilibrium transition probabilities between the split-ticket and non split-ticket states, and show that the stationary probability that the voters split tickets is independent of the initial state, and is lower than the stationary probability that they do not split tickets. In other words, in the long run the society is found in split-ticket voting state less frequently than in non split-ticket voting state. We perform a simple testing procedure and find empirical support for the model predictions on the equilibrium transition probabilities between the split-ticket and non split-ticket states for moderate levels of politicians’ commitment to their political parties.

In the paper we ignore the fundamental question of why political constitutions are modeled as incomplete contracts. Firstly, in addition to a sound theoretical framework, this approach has received considerable empirical support (see, for example, Peltzman (1992) and Besley
and Case (1995a, b)). Moreover, we believe that at the municipal and regional levels the politicians’ tasks require mainly managerial skills. This view is supported by the empirical findings of Ferreira and Gyourko (2009) that in US cities the mayors’ party affiliation does not affect the size of the city government and the allocation of spending. In a recent article in the *New York Times*, Glaeser points out that "lack of ideology has become a major feature of big city mayors... They are... managerial mayors, appreciated by voters because they succeed in making the city work."² So, a principal-agent approach may be an appropriate set-up to model local political constitutions. Still, elected politicians can only be offered implicit incentive schemes: public policies are difficult to reward with explicit contracts.


This paper is related to career concerns models pioneered by Holmström (1982). Dewatripont et al. (1999a, b) discuss and extend the literature on agency and career concerns. To our knowledge, this approach was first applied to politics by Persson and Tabellini (2000). Then several authors contrast elected officials ("politicians", "elected regulators") with non-elected ones ("judges", "bureaucrats", "appointed regulators") to study the allocation of decision-making powers in the society (see Alesina and Tabellini (2007, 2008), Besley and Coate (2003), Maskin and Tirole (2004)). In these papers, however, elections address both moral hazard, by holding elected politicians accountable to the voters, and adverse selection, by allowing the voters to select the most competent politicians. In our framework, elections only perform the former function.

Our results are also related to the literature on yardstick competition that studies electoral accountability under decentralization, starting with the seminal work of Salmon (1987) and followed by Besley and Case (1995a), Bordignon et al. (2004), Belleflamme and Hindriks (2005), Besley and Smart (2007) and others. The main assumption of this literature is that under decentralization voters use comparative performance evaluation between different local governments to create yardstick competition between jurisdictions. However, this does not really fit with the empirical findings of Lowery and Lyons (1989) and Teske et al. (1993) that voters are poorly informed about fiscal system in jurisdictions other than their own. In

contrast with this literature, we assume that voters can use comparative performance evaluation between local and regional governments rather than between different local governments, since it is reasonable to believe that citizens are well-informed about the mayor’s performance in their own municipality and about the governor’s performance in their own region.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 studies the politicians’ efforts under linear performance evaluation rules. Section 4 analyzes the voters’ choice of optimal evaluation rules. Section 5 presents empirical results. Finally, Section 6 concludes.

2. The Model

Consider a large city that has to elect mayor $M$ for the local municipal government and governor $G$ for the region this city belongs to. The city is inhabited by a large number (formally a continuum) of individuals. The individuals live forever. At the beginning of each period the municipal and regional elections take place simultaneously and a winner is determined by majority rule (mayor at the city level and governor at the region level). Politicians running for the elections belong to one of the two political parties, $L$ or $R$. In particular, we assume that there are exactly two candidates from the opposite parties—the incumbent and an opponent—in each election in each period. The opponent is identical in all respects to the incumbent but is a member of the rival party. The participation constraints of the politicians are always satisfied, and there is no term limit.

While in office, politician $i \in \{M, G\}$ has to implement a corresponding policy which is determined by the politician’s unobservable effort $a_i$. The set of efforts available to each politician is taken to be a non-degenerate interval $[0, \pi] \subset \mathbb{R}$. We assume that the policy outcomes $p_i$ are observed with noise $\varepsilon_i$:

$$p_i = a_i + \varepsilon_i,$$

with $\varepsilon_i \sim N(0, \sigma^2)$, independent and unobservable.

The reward of politician $i$ is labelled by $\Pi_i(a_i)$. Effort is costly, and the cost is labelled by $C_i(a_i)$. Both the mayor and governor choose effort levels $a_i$ to maximize their utility given by

$$\Pi_i(a_i) - C_i(a_i),$$

with $\Pi_i(a_i)$ and $C_i(a_i)$ to be explicitly defined in subsection 2.1.

There is no cost of voting, and we assume that there are no abstainers. The individuals differ in their preferences over political parties. To be more specific, we assume that some
individuals always prefer party $L$ to party $R$, and therefore they vote for candidates from party $L$ in both elections; while other individuals are committed to party $R$, so they always ballot their votes for party $R$ in both elections. Moreover, there is a large group of individuals that share the same preferences, and whose votes are decisive for the outcome of both elections.\footnote{We have an extended version of the model, available upon request, where a region consists of $N$ identical municipalities, and each of them is decisive (i.e. pivotal) for the outcome of the regional election with equal probability. The main insights of the paper do not change with this extension, while exposition and modeling become rather complex.} They are indifferent between political parties and care about the policy outcomes in each period according to a linear utility function 

$$p_M + p_G.$$ 

In what follows we call this decisive voters group simply the voters.

We assume that the voters coordinate on the same retrospective reappointment rules to reelect a mayor and a governor. We follow the literature (e.g. Persson et al. (1997)) and restrict strategy space such that the voters condition the reappointment decision on the policy outcomes in the current period and not in any previous period (which makes sense for a large electorate since it is unrealistic to assume that the voters could coordinate on a same history-dependent reappointment rule). See Persson et al. (1997) for the discussion of the plausibility of this approach. Given this restriction on the strategy space and that the environment is stationary, we drop time subscripts with no risk of confusion.

Thus the timing of events is as follows. At the beginning of each period, the elections take place where, following the reappointment rules chosen in the previous period, the voters reelect the incumbents or not. Next the voters make the choice of reappointment rules to be used in the coming elections. Then the elected politicians exert efforts $a_M$ and $a_G$. Finally, policy outcomes $p_M$ and $p_G$ are observed.

Let us denote by $S$ the state where mayor $M$ and governor $G$ are members of the same party (either $L$ or $R$), and by $D$ the state where mayor $M$ and governor $G$ belong to different parties. In our framework, state $S$ (resp. state $D$) occurs when the voters do not vote (resp. do vote) split-tickets to reward the incumbents for their performance. First, we describe the politicians’ preferences, then we turn to the voters’ problem and define an equilibrium concept.
2.1. Politicians

The politicians’ preferences are as follows. First, once elected, mayor $M$ and governor $G$ want to be reelected themselves in the next period. Moreover, mayor $M$ cares about the chances of her party comembers to be elected for a governor office in the coming elections. If governor $G$ is a member of the same party as mayor $M$, than mayor $M$ prefers her to be reelected. Otherwise, mayor $M$ wants her party comember to be elected for the governor office for a next term. The same intuition works in the case of governor $G$ that wants mayor $M$ to be reelected if they are members of the same party, and wants an opponent to be appointed if mayor $M$ is from the rival party. Let us normalize to 1 the value of holding office, denote by $\lambda_M$ (resp. $\lambda_G$) the value mayor $M$ (resp. governor $G$) associates to her party comember’s holding governor (resp. mayor) office and by $Pr_i(\cdot)$ the probability of being reelected for a corresponding office $i \in \{M, G\}$ in the coming election. Therefore, politician $i$ has the following reward function $\Pi_i : [0, \bar{a}]^2 \to \mathbb{R}$ that depends continuously on both politicians’ efforts:

$$\Pi_i(a_i, a_j) = \begin{cases} Pr_i(a_i, a_j) + \lambda_i Pr_j(a_i, a_j) & \text{if } S \\ Pr_i(a_i, a_j) + \lambda_i (1 - Pr_j(a_i, a_j)) & \text{if } D \end{cases}$$ where $i, j \in \{M, G\}$ and $j \neq i$.\(^4\) With the preferences of this type we try to reflect the politicians’ allegiance to their party with politicians caring about overall representation of their party in governing bodies.\(^5\) Still, the reasonable assumption here is that a mayor (resp. governor) values her own office holding more than her party representation in the governor (resp. mayor) office, so $0 \leq \lambda_i \leq 1$.\(^6\) We call $\lambda_i$ the degree of the politician $i$’s commitment to her party.

We believe that the incentives of the mayor and governor from the same party differ from the ones of the politicians that belong to the different parties. A simple formulation of this idea models effort costs in terms of politicians’ utility such that the mayor and governor from the same party incur different costs of policy implementation than the ones from the rival

\(^4\)Our model can be generalized for the case where the incumbents maximize their intertemporal utility function (as in Ferejohn (1986)). Still, we want to concentrate on the interactions between politicians and voters rather than dynamic aspects of the model, so we assume, as in Alesina and Tabellini (2007, 2008), that the incumbents are myopic, i.e. they care about reelection only in the next period and not in any other subsequent period.

\(^5\)Alternatively, one can consider the mayor and governor that have to interact with each other while in office. Then each prefers her own party comember to work with rather than a member of the rival party.

\(^6\)In other words, politician $i$ does not mind to reduce her reelection chances by 1% in exchange for an increase in her comember’s reelection probability by $\lambda_i^{-1}\%$. 
parties. So politician $i$ carries a cost

$$C_i(a_i) = \begin{cases} \frac{a_i^2}{2c_i} & \text{if } S \\ \frac{a_i^2}{2} & \text{if } D \end{cases}$$

of exerting effort $a_i$, where $c_i > 0$. Note that this assumption reflects two opposite ways of coordination between municipal and regional governments. First, if $c_i > 1$ then the effort is cheaper for the politicians from the same party. Just as the politicians from the same party tend to give assistance to each other, so may the municipal and regional administrations conduct some joint projects that lead to a decrease in effort costs. As for the politicians from different parties, it is logical to assume more rival behavior and, therefore, more costly efforts. Alternatively, consider the situation where the mayor and governor have to take some joint decision. Then it is easier to come to agreement for the politicians from the same party than for the politicians from the rival parties. One can also consider the synergy interpretation where the mayor and governor from the same party deliver higher joint final outcome—since it is cheaper—than the ones from different parties. Second, if $0 < c_i < 1$ then the opposite scenario is considered, namely, the effort is cheaper for the politicians from different parties. This situation can be the case when the regional government makes considerable investments in the municipality with the mayor from the rival party in office to convince the electorate to vote for an opponent in the coming local municipal election.

### 2.2. Voters

We assume that the final outcomes $p_M$ and $p_G$ (but not their composition between effort and noise) are observed at the end of each period but are not contractible. Public policies are difficult to reward with explicit contracts. It is more natural to use implicit incentive contracting in this situation. We assume that the voters coordinate on the same retrospective voting rule, and that there are no coordination failures among the voters. A coordination problem is serious issue but it is beyond the scope of the paper.

At the end of each period the voters observe the policy outcomes $p_M$ and $p_G$, and in the coming elections they reward the incumbents according to their performance in the current period, i.e. they reappoint the incumbent who has shown "good" results in the current period. A politician thrown out of office is never reappointed. In this case an opponent from the rival party is elected.

Obviously voters can influence the politicians' behavior through the choice of evaluation rules. We restrict the functional space of performance evaluation rules voters can choose to linear performance evaluation rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$ determined by slopes $\beta_M$ and $\beta_G$. 
Figure 2.1: Mayor $M$ and governor $G$ reelection events under rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$ in two-dimensional space of observed policy outcomes $p_M$ and $p_G$.

and intercepts $b_M$ and $b_G$ such that $\beta_M \geq 0$, $\beta_G \geq 0$, $\beta_M \beta_G \leq 1$, $b_M \in \mathbb{R}$, $b_G \in \mathbb{R}$. Under rules $(\beta, b_i)$, $i \in \{M, G\}$, the probability of being reelected for office $i$ in the coming election reads

$$P_{r_i}(a_i, a_j) = P\left(\{p_i(a_i) \geq \beta_i p_j(a_j) + b_i\}\right)$$

with $i, j \in \{M, G\}$ and $j \neq i$. Figure 2.1 depicts possible events of mayor $M$ and governor $G$ reelection under rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$ in two-dimensional space of observed policy outcomes $p_M$ and $p_G$. Note that we restrict $\beta_M \beta_G \leq 1$ to consider reasonable reappointment rules. It is obvious from Figure 2.1 that if $\beta_M \beta_G > 1$ a mayor and a governor with poor performance would be reelected while the politicians with better performance would not.

The restriction to linear rules makes sense for a large number of voters since it is unrealistic to assume that voters could coordinate on the same general reappointment rule as any function of $p_M$ and $p_G$. Moreover, since differentiable functions are linear in first-order approximation, our analysis gives an approximate fit to general performance evaluation rules, and still allows for closed-form solutions. Indeed, linear approximation methods are widely used in macroeconomics to search for time-consistent equilibria (e.g. Krusell et al. (1997)).

The rules are further required to be sequentially rational, that is, no precommitment is allowed. We assume that all the model parameters are common knowledge, so the politicians have rational expectations about the evaluation rules that will be used by the voters for
particular parameters values. We must stress here that the game is played sequentially, so the politicians observe the elections outcome before exerting efforts. Thus, the politicians know whether the voters have used the evaluation rules that have been rational for them or deviated. In the latter case, the politicians conclude that, from this period on, the voters reappoint the incumbents randomly or use some unknown reelection rules that are not based on the politicians’ performance evaluation. As a result, from this period on, the politicians will exert zero effort to minimize their costs. The voters know this, so they have no incentives to deviate and always reward the incumbents according to the rules chosen in the previous period.

As we mentioned above, in our framework split-ticket voting occurs as a result of the evaluation rules chosen in the previous period: that is, the voters decide on performance evaluation rules to reward the incumbents, and then just vote to follow the chosen rules. So, throughout the paper we find it more convenient to refer to $S$ and $D$ as the states characterized by the politicians’ being members of the same party or different parties, keeping in mind that state $S$ (resp. state $D$) occurs when the voters have not split tickets (resp. have split tickets).

2.3. Equilibrium Concept

We search for the subgame perfect equilibrium analyzing the game backwards: first, we solve for the politicians’ efforts $a_M$ and $a_G$ for rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$; second, we turn to the voters’ choice of evaluation rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$. In what follows we introduce a couple of definitions.

Given linear performance evaluation rules $(\beta_M, b_M)$ and $(\beta_G, b_G)$, an equilibrium in effort strategies is a profile of efforts $(a_M, a_G)$ such that

$$\Pi_i (a_i, a_j) - C_i (a_i) \geq \Pi_i (a'_i, a_j) - C_i (a'_i) \quad \text{for each } a'_i \in [0, \bar{a}]$$

where $i, j \in \{M, G\}$, $i \neq j$.

Let us define an equilibrium in rule strategies as a tuple $(\beta_M^*, b_M^*, \beta_G^*, b_G^*)$ of performance evaluation rules $(\beta_M^*, b_M^*)$ and $(\beta_G^*, b_G^*)$ such that

$$a_M (\beta_M^*, b_M^*, \beta_G^*, b_G^*) + a_G (\beta_M^*, b_M^*, \beta_G^*, b_G^*) = \max_{\beta_M, \beta_G \geq 0, b_M, b_G \in \mathbb{R}, \beta_M, \beta_G \leq 1} a_M (\beta_M, b_M, \beta_G, b_G) + a_G (\beta_M, b_M, \beta_G, b_G)$$

where $(a_M (\cdot), a_G (\cdot))$ is an equilibrium in effort strategies. Finally, we denote by $a_i^* \equiv a_i \left( \beta_i^*, b_i^*, \beta_j^*, b_j^* \right)$, $i, j \in \{M, G\}$, $i \neq j$, the politicians’ equilibrium efforts.
3. Politicians’ Efforts under Linear Performance Evaluation Rules

Let the voters use evaluation rules \((\beta_M, b_M)\) and \((\beta_G, b_G)\) to reappoint the incumbents. Note that the voters compare their utilities from the policies implemented by mayor \(M\) and by governor \(G\). In reality, public policies often pursue many different goals that could be hard to measure and compare. Still, we assume that the voters can compare their utility levels from implemented policies if not the policies themselves. To be more specific, the voters use the following performance evaluation rules: to reappoint the incumbent for office \(i\) if the voters’ utility from policy \(p_i\) exceeds the linear transformation of their utility from policy \(p_j\) with slope \(\beta_i\) and intercept \(b_i\). Under these rules the politician \(i\)’s reward reads

\[
\Pi_i(a_i, a_j) = \begin{cases} 
P \left( \{p_i(a_i) \geq \beta_i p_j(a_j) + b_i\} \right) + \lambda_i P \left( \{p_j(a_j) \geq \beta_j p_i(a_i) + b_j\} \right) & \text{if } S \\
\lambda_i \left( 1 - P \left( \{p_j(a_j) \geq \beta_j p_i(a_i) + b_j\} \right) \right) & \text{if } D
\end{cases}
\]

Our first result establishes the existence of an equilibrium in effort strategies. The proofs of this and the following propositions are given in the Appendix.

Proposition 1. Under linear performance evaluation rules \((\beta_M, b_M)\) and \((\beta_G, b_G)\) such that \(\beta_M \geq 0, \beta_G \geq 0, \beta_M \beta_G \leq 1, b_M \in \mathbb{R}, b_G \in \mathbb{R}\), there exists an equilibrium in effort strategies \((a_M, a_G)\). Furthermore, this equilibrium is defined implicitly by

\[
f_{\epsilon_i - \beta_i \epsilon_j} (\beta_i a_j - a_i + b_i) = \begin{cases} 
\frac{a_i \epsilon_j + a_j \epsilon_i \beta_j}{c_i c_j (1 - \lambda_i \lambda_j \beta_i \beta_j)} & \text{if } S \\
\frac{a_i - a_j \lambda_i \beta_j}{1 - \lambda_i \lambda_j \beta_i \beta_j} & \text{if } D
\end{cases}
\]

where \(i, j \in \{M, G\}, i \neq j\), and \(f_{\epsilon_i - \beta_x \epsilon_j}(\cdot)\) is the probability density function of \(\epsilon_i - \beta_i \epsilon_j \sim N(0, (1 + \beta_i^2) \sigma^2)\).

Note that nowhere does the proof of the proposition use the quasi-concavity of \(\Pi_i(\cdot, a_j) - C_i(\cdot)\) on \([0, \bar{a}]\). Continuity properties of the politicians’ best response functions and Brouwer’s Fixed Point Theorem are used to provide the result.

4. Optimal Performance Evaluation Rules

What evaluation rules do the voters prefer to reappoint the incumbents? The voters have rational expectations about the politicians’ effort in each state, \(S\) or \(D\). Thus, to decide on an evaluation rule they just maximize their expected utility. In expectations the noise is equal to 0, so the voters prefer evaluation rules under which the politicians are expected to exert higher effort. Our next result characterizes an equilibrium in rule strategies.
Proposition 2. There exists a unique equilibrium in rule strategies \((\beta^*_i, b^*_i; \beta^*_j, b^*_j)\) given by

\[
(\beta^*_i, b^*_i) = \begin{cases} 
(0, a^*_i) & \text{if } S \\
(\lambda_j, a^*_i - \lambda_j a^*_j) & \text{if } D
\end{cases}
\]

\(i, j \in \{M, G\}, i \neq j,\) where politicians’ equilibrium efforts \(a^*_i\) are equal to

\[
a^*_i = \begin{cases} 
\frac{c_i}{\sqrt{2}\pi\sigma^2} & \text{if } S \\
\frac{1}{\sqrt{2}\pi\sigma^2} \left( \frac{1}{1 + \lambda_j^2} + \frac{\lambda_j^2}{1 + \lambda_i^2} \right) & \text{if } D
\end{cases}
\]  

\[(4.1)\]

According to Proposition 2, in the case where the politicians are members of the same party (state \(S\)), the voters adopt a "cut-off" rule. Under the cut-off rule, an incumbent is reappointed only when the observed policy outcome that she generates exceeds a critical bound given by the equilibrium effort in this office. So, the probability of being reelected for office \(i\) – that depends only on \(a_i\) here – reads

\[P r_i (a_i) = P \{p_i (a_i) \geq a^*_i\}\]

Thus, if the offices are held by the members of the same party they exert higher effort under a cut-off rule. The result is due to the fact that the mayor wants to be reelected herself and she wants the governor to be reappointed as well. She knows that under a cut-off rule, her effort only affects her own reelection prospects: the higher the effort is, the greater the chances to be reappointed are. Under a linear performance evaluation rule with a positive slope her effort affects both her own and the governor reelection prospects: the higher the effort is, the greater the mayor’s chances and the lower the governor’s chances to be reappointed are. The same intuition works in the case of a governor. This trade-off explains why the politicians from the same party exert higher effort under the cut-off rule. As for the choice of a critical bound, rational voters realize that the alternative to reelecting the incumbent is to appoint an opponent who will exert the equilibrium level of effort \(a^*_i\).

In the case of the politicians from the different parties (state \(D\)), the voters choose a "comparative" rule. Under the comparative rule, a mayor is reelected only when the difference between her observed performance and the equilibrium effort in the mayor’s office exceeds the weighted difference between the governor’s observed performance and the equilibrium effort in the governor’s office where the weight is equal to the degree of the governor’s commitment to her party, \(\lambda_G\); and vice versa for the governor reappointment. Thus, the probability of being reelected for office \(i\) under the comparative rule is equal to

\[P r_i (a_i, a_j) = P \{p_i (a_i) - a^*_i \geq \lambda_j (p_j (a_j) - a^*_j)\}\]
Under the comparative rule, politician $i$’s performance affects the reelection prospects of politician $j$, so the reappointment chances are less sensitive to effort that decreases the politicians’ incentives. However, rewarding based on performance comparison gives additional incentives to the politicians from the different parties to exert higher effort (when each politician has to perform better for the incumbent from the rival party not to be reappointed). If politician $j$ cares only about her own reelection chances and not about her party overall representation in governing bodies (so $\lambda_j = 0$), then the former effect dominates and the optimal rule to reappoint politician $i$ simplifies to a cut-off rule. In the case politician $j$ does care about her party comembers’ chances to be appointed for office $i$ (so $\lambda_j \neq 0$), the latter competition effect dominates and a comparative rule is adopted by the voters to reappoint politician $i$.

How do the politicians’ equilibrium efforts $a_i^*$ given by (4.1) depend on parameters’ values? First, a larger variance of noise $\sigma^2$ decreases the politicians’ efforts. Intuitively, the higher the variance of noise $\sigma^2$ is, the more random the observed policy outcomes $p_i$ are, so reelection chances are less sensitive to effort that reduces the politicians’ incentives. Next, in state $S$ where the politicians are from the same party, a higher value of cost parameter $c_i$ increases the effort of politician $i$, $a_i^*$: that is, the cheaper it is to exert efforts, the better the politicians’ performance is. Finally, when the politicians belong to rival parties, a higher value of the degree of politician $i$’s commitment to her party, $\lambda_i$, increases the equilibrium effort of politician $i$, $a_i^*$, and decreases the equilibrium effort of politician $j$, $a_j^*$. The result comes from the nature of rewarding according to the comparative rule. Politician $i$ does not want the incumbent from the rival party to be reappointed for office $j$, which gives her incentives to perform better to reduce the reelection chances of incumbent $j$ (that are decreasing in $a_i$ under the comparative rule). Then, the more politician $i$ cares about her party comembers’ chances to be appointed for office $j$ (the higher $\lambda_i$ is), the more incentives she has to perform better, so the higher her equilibrium effort, $a_i^*$, is. As for politician $j$’s equilibrium effort, $a_j^*$, note that under the comparative rule politician $j$’s reappointment chances decrease in the degree of politician $i$’s commitment to her party, $\lambda_i$; that is, the more politician $i$ cares about her party comembers’ chances to be appointed for office $j$ (the higher $\lambda_i$ is), the better she performs, so the lower incumbent $j$’s reelection chances, which weakens incumbent $j$’s incentives to exert an effort.

Next, we calculate the equilibrium probabilities of transition between state $S$ (the politicians are members of the same party) and state $D$ (the politicians belong to the different parties). Let us denote by $P_{kl}$ a probability that the city in state $k$ will next be in state $l$ with $k, l \in \{S, D\}$. So, we establish the following result.
Proposition 3. The matrix of the equilibrium one-step transition probabilities, $P$, is equal to

$$P \equiv \begin{bmatrix} P_{SS} & P_{SD} \\ P_{DS} & P_{DD} \end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan \left( \lambda_M \right) \right) & \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan \left( \lambda_M \right) \right) \\ 1 - \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan \left( \lambda_M \right) \right) & \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan \left( \lambda_M \right) \right) \end{bmatrix}$$

where $\arctan \left( \cdot \right)$ is an arctangent function.

If the politicians care only about their reelection and not about their party overall representation in governing bodies (so $\lambda_i = 0$), then the voters always adopt a cut-off rule and in each state the equilibrium transition probability is equal to $\frac{1}{2}$. In the case where the politicians care about their party comembers’ reelection chances as much as about their own (so $\lambda_i = 1$), in state $D$ the voters use a "strict" comparative rule that reads: to reelect incumbent $i$ only if the difference between the policy outcome she generates and the equilibrium effort in office $i$ exceeds the difference between the policy outcome of incumbent $j$ and the equilibrium effort in office $j$. Under this rule, there is always only one incumbent reappointed, so the equilibrium probability that the city in state $D$ will stay in state $D$ is equal to zero, $P_{DD} = 0$. In other words, if the politicians are fully committed to their political parties, then the city in state $D$ will always next be in state $S$. Note that equilibrium probability $P_{DD}$ is decreasing in $\lambda_i$, so the more committed to their parties the politicians are, the lower the probability is that the city in state $D$ will stay in state $D$.

It is straightforward to find a limiting (or stationary) probability that the municipality will be in each state after a large number of periods. Let $P_{kl}^n$ be a probability that the city in state $k$ will be in state $l$ after $n$ additional periods, $k, l \in \{S, D\}$. The matrix of the equilibrium $n$-step transition probabilities, $P^{(n)}$, is obtained by multiplying matrix $P$ by itself $n$ times:

$$P^{(n)} \equiv \begin{bmatrix} P_{SS}^n & P_{SD}^n \\ P_{DS}^n & P_{DD}^n \end{bmatrix} = P^n$$

Note that

$$\lim_{n \to \infty} P_{SS}^n = \lim_{n \to \infty} P_{DS}^n = \frac{P_{DS}}{1 - P_{SS} + P_{DS}} = 1 - \frac{\pi}{3\pi - 2 \arctan \left( \frac{1}{\lambda_G} \right) + 2 \arctan \left( \lambda_M \right)}$$

$$\lim_{n \to \infty} P_{SD}^n = \lim_{n \to \infty} P_{DD}^n = \frac{1 - P_{SS}}{1 - P_{SS} + P_{DS}} = \frac{\pi}{3\pi - 2 \arctan \left( \frac{1}{\lambda_G} \right) + 2 \arctan \left( \lambda_M \right)}$$

Therefore, a stationary probability of state $S$ (resp. $D$) is independent of the initial state and is equal to $1 - \frac{\pi}{3\pi - 2 \arctan \left( \frac{1}{\lambda_G} \right) + 2 \arctan \left( \lambda_M \right)} \in \left[ \frac{1}{2}, \frac{2}{3} \right]$ (resp. $\frac{\pi}{3\pi - 2 \arctan \left( \frac{1}{\lambda_G} \right) + 2 \arctan \left( \lambda_M \right)} \in \left[ \frac{1}{3}, \frac{1}{2} \right]$).
This probability can be interpreted as the long-run proportion of time that the city is in the corresponding state. So we conclude that in the long run the mayor and governor offices are held more often by the members of the same party than by the politicians from the rival parties. In our framework, state $S$ (resp. state $D$) occurs when the voters do not vote (resp. do vote) split-tickets to reward the incumbents for their performance. The following proposition summarizes the discussion.

**Proposition 4.** The stationary probability that the voters split tickets is independent of the initial state, and is equal to

$$\frac{\pi}{3\pi - 2\arctan\left(\frac{1}{\lambda_c}\right) + 2\arctan(\lambda_M)} \in \left[\frac{1}{3}, \frac{1}{2}\right].$$

The more committed the politicians are to their political parties (the higher $\lambda_M$ and (or) $\lambda_G$ are), the lower the stationary probability of split-ticket voting state is. When the politicians care about their party comembers’ reelection chances as much as their own (i.e. $\lambda_i = 1$), the stationary probability of split-ticket voting takes its minimal value $\frac{1}{3}$. If the politicians care only about their reelection chances (i.e. $\lambda_i = 0$), the stationary probability of split-ticket voting state takes its maximal value $\frac{1}{2}$. In this case the voters always use a cut-off rule, and a symmetric distribution of noise implies the equal stationary probabilities of the split-ticket voting and non split-ticket voting states.

5. Empirics

In Spain, local municipal and regional elections occur simultaneously every four years in 15 out of 19 regions.\(^7\) The two leading parties are Partido Popular (PP) and Partido Socialista Obrero Español (PSOE).\(^8\) We use the available data on these elections to see if the theoretical model fits the data significantly. Due to the scarcity of data, our empirical analysis is quite limited.\(^9\) Still, we can perform a simple testing exercise to find empirical support for the model. According to our results, the city dynamics is described by a Markov chain with matrix $P$ of one-step transition probabilities given in (4.2). Therefore, a testable prediction reads:

\(^7\)The mayor and governor elections take place simultaneously in Aragón, Canarias, Cantabria, Castilla-La Mancha, Castilla y León, Ceuta, Comunidad Foral de Navarra, Comunidad de Madrid, Comunitat Valenciana, Extremadura, Islas Baleares, La Rioja, Melilla, Principado de Asturias and Región de Murcia. In Andalucía, Cataluña, Galicia and País Vasco local and regional elections are held on different dates.

\(^8\)There are a lot of minor parties (e.g. Izquierda Unida that has considerable support in some regions).

\(^9\)To conduct a proper empirical analysis in our framework one would need data on individuals’ voting techniques (ideological versus retrospective voting) and on politicians’ commitment to their political parties. To the best of our knowledge, available polls do not provide such data.
Null hypothesis. In large municipalities with decisive voters caring about policy outcomes rather than parties’ ideology, the transitions between the split-ticket and non split-ticket states follow matrix $\mathbf{P}$ given in (4.2).

5.1. Data Description

We use the panel data on the simultaneous local and regional elections for the years 1995, 1999, 2003 and 2007 in six Spanish regions–Castilla-La Mancha, Comunidad de Madrid, Comunitat Valenciana, Islas Baleares, Principado de Asturias and Región de Murcia–partially available online at the official websites of the regional governments and the Spanish Ministry of Interior (see Appendix D for details).

Initially, we consider all municipalities with more than 15,000 inhabitants (2007 census). Each observation consists of census, number of abstainers, votes to PP, votes to PSOE, votes to other parties for both municipal and regional elections in the considered municipality. From the initial sample of 115 cities we discard 7 cities where the number of voters in local elections differs significantly from the number of voters in regional elections at least once during the analyzed period (we allow for maximal difference of 5%) to ensure that almost the same electorate participates in the local and regional elections. Next, we exclude 14 more cities where either PP or PSOE is not a leading party at least twice in 1995-2007 elections (i.e. the other parties win more votes than PP or more votes than PSOE) to consider the municipalities with the same two leading parties. Then we can interpret an observation in the following way: the voters do not split tickets if the same party (either PP or PSOE) wins in both local and regional elections; the voters do split tickets if different parties win in local and regional elections. In our theoretical analysis we assume that the voters are indifferent between political parties and care about policy outcomes (i.e. no ideological voting). To meet this requirement we discard 51 more municipalities where the voters have never split tickets during the analyzed period (which can be considered as ideological voting). So our final sample covers 43 cities, which is around 40 percent of the initial sample.

We have 172 observations for 43 municipalities for the years 1995, 1999, 2003 and 2007, with 67 observations of split-ticket voting and 105 observations of non split-ticket voting (see Appendix E for the list of the municipalities included in the final sample). During the analyzed period split-ticket voting has taken place once in 23 municipalities, twice in 16 municipalities and thrice in 4 municipalities.

\footnote{Some data for Castilla-La Mancha and Región de Murcia have been kindly provided by the statistical institutes of the corresponding regions, and are available upon request.}
5.2. Analysis

In the sample there are 44 (resp. 34) transitions from non split-ticket state, $S$, to non split-ticket state, $S$ (resp. split-ticket state, $D$), and 33 (resp. 18) transitions from split-ticket state, $D$, to non split-ticket state, $S$ (resp. split-ticket state, $D$). We assume that in 1995– the first period of the sample– the split-ticket and non split-ticket states occur with the corresponding stationary probabilities. Then, taking into account that in 1995 elections 28 (resp. 15) cities do not split (resp. split) tickets, we can find a maximum likelihood estimate of the matrix of one-step transition probabilities, $\hat{P}$. To be more specific, the likelihood function of the sample data based on Markov chain distribution model– denoted by $L(\cdot, \cdot)$– reads

$$L(P_{SS}, P_{DS}) = \left(\frac{P_{DS}}{1 - P_{SS} + P_{DS}}\right)^{28} \left(\frac{1 - P_{SS}}{1 - P_{SS} + P_{DS}}\right)^{15} (P_{SS})^{44} (1 - P_{SS})^{34} (P_{DS})^{33} (1 - P_{DS})^{18}$$

and is maximized by $\hat{P}_{SS} = 0.5774$ and $\hat{P}_{DS} = 0.6591$. So a maximum likelihood estimate of the matrix of one-step transition probabilities is equal to

$$\hat{P} = \begin{bmatrix} \hat{P}_{SS} & 1 - \hat{P}_{SS} \\ \hat{P}_{DS} & 1 - \hat{P}_{DS} \end{bmatrix} = \begin{bmatrix} 0.5774 & 0.4226 \\ 0.6591 & 0.3409 \end{bmatrix}$$

Note that in the sample the transitions to non split-ticket state $S$ are more frequent than the transitions to split-ticket state $D$ (independently of the state in the previous period). In other words, in the municipalities included in the sample, voters do not split tickets more often than they split tickets.

Our null hypothesis reads

$$H_0 : P = \begin{bmatrix} 1 - \frac{1}{\pi} \left(\arctan \left(\frac{1}{\lambda_i}\right) - \arctan (\lambda_M)\right) \\ \frac{1}{\pi} \left(\arctan \left(\frac{1}{\lambda_i}\right) - \arctan (\lambda_M)\right) \end{bmatrix}$$

with $0 \leq \lambda_i \leq 1$, $i \in \{M, G\}$. We run the likelihood ratio test taking into account that the two restrictions are tested and that the Chi-Square 0.05 percentile with 2 degrees of freedom is equal to 5.99. After simple calculations we conclude that the null hypothesis is not rejected at 5% significance level for the following range of the degrees of politicians’ commitment to their political parties ($\lambda_M, \lambda_G$):

$$H_0 \text{ is not rejected for } (\lambda_M, \lambda_G) \text{ such that } \frac{1}{\pi} \left(\arctan \left(\frac{1}{\lambda_G}\right) - \arctan (\lambda_M)\right) \leq 0.4546$$

Figure 5.1 depicts the range of $(\lambda_M, \lambda_G)$ for which the null hypothesis is not rejected.
The model prediction on the transition probabilities fits the data only for moderate levels of the politicians’ commitment to their political parties, $\lambda_M$ and $\lambda_G$ (where by "moderate" we mean that both $\lambda_M$ and $\lambda_G$ are neither too high nor too low). The null hypothesis is rejected when both $\lambda_M$ and $\lambda_G$ are too high or too low. Still, $H_0$ is not rejected when one of the politicians is highly committed to her political party ($\lambda_i$ is high) while the other cares mainly about her own reappointment ($\lambda_j$ is low). To sum up, we do not reject the hypothesis that the transition probabilities between split-ticket and non split-ticket states in Spanish municipalities included in the sample follow the pattern (4.2) for moderate levels of the politicians’ commitment to their political parties.

6. Conclusion

This paper applies an implicit incentive approach to study split-ticket voting phenomenon in the simultaneous municipal and regional elections. In our analysis, the principals (voters) reward the agents (politicians) through implicit reward rules. In turn, the politicians can be committed to their political parties, caring about the overall representation of their party in governing bodies.

We show that, if the voters do vote split-tickets and if the politicians from the rival parties are committed to their political parties, then the voters adopt a comparative rule.
under which the differences between the incumbents’ performance and the equilibrium effort in corresponding offices are compared. In case the voters do not split tickets or the politicians only care about their reappointment prospects, the voters prefer a cut-off rule under which an incumbent is reappointed only when her performance exceeds a critical bound. We find the equilibrium transition probabilities between the split-ticket and non split-ticket states, and show that in the long run the voters split tickets less frequently than do not split tickets. Finally, we find empirical support for the model prediction on the equilibrium transition probabilities in panel data analysis for moderate levels of politicians’ commitment to their political parties.

We have focused on single task policies in the presence of a moral hazard problem. However, in reality public policies can pursue many goals, so it is of interest to study the split-ticket voting problem under a more realistic assumption of multiple tasks policies when the problem of effort allocation among tasks can create policy trade-offs. Alternatively, one can add an adverse selection problem by assuming that policy outcomes are determined by effort and ability. We leave these extensions of the model for future research.

Appendix

Throughout the Appendix we use $F$ to denote the normal distribution function and $f$ for the corresponding density.

A. Proof of Proposition 1

Under linear performance evaluation rules $(\beta_i, b_i)$ the probability of being reelected for office $i$ reads

$$Pr_i(a_i, a_j) = P\left(\{\varepsilon_i - \beta_j \varepsilon_j \geq \beta_i a_j - a_i + b_i\}\right) = 1 - F_{\varepsilon_i-\beta_j \varepsilon_j} (\beta_i a_j - a_i + b_i),$$

where noises $\varepsilon_i$ and $\varepsilon_j$ ($i, j \in \{M, G\}, i \neq j$) are independent normally distributed random variables, so by the convolution formula $\varepsilon_i - \beta_j \varepsilon_j \sim N \left(0, (1 + \beta_i^2) \sigma^2\right)$.

Politicians’ equilibrium efforts, $a_i$, are defined implicitly by the first-order conditions

$$f_{\varepsilon_i-\beta_i \varepsilon_j} (\beta_i a_j - a_i + b_i) - \lambda_i \beta_j f_{\varepsilon_j-\beta_i \varepsilon_i} (\beta_j a_i - a_j + b_j) = \frac{a_i}{\varepsilon_i} \quad \text{if } S$$

$$f_{\varepsilon_i-\beta_i \varepsilon_j} (\beta_i a_j - a_i + b_i) + \lambda_i \beta_j f_{\varepsilon_j-\beta_i \varepsilon_i} (\beta_j a_i - a_j + b_j) = a_i \quad \text{if } D$$

or

$$f_{\varepsilon_i-\beta_i \varepsilon_j} (\beta_i a_j - a_i + b_i) = \begin{cases} \frac{a_i \varepsilon_j + a_j \varepsilon_i \lambda_i \beta_j}{\varepsilon_i \varepsilon_j (1 - \lambda_i \lambda_j \beta_i \beta_j)} & \text{if } S \\ \frac{a_i - a_j \lambda_i \beta_j}{1 - \lambda_i \lambda_j \beta_i \beta_j} & \text{if } D \end{cases} \quad \text{(A.1)}$$
Define the best response functions by \( R_i : [0, \bar{a}] \to [0, \bar{a}] \) such that
\[
R_i(a_j) = \arg \max_{a'_i \in [0, \bar{a}]} \Pi_i(a'_i, a_j) - C_i(a'_i)
\]
Then the best response functions are determined implicitly by
\[
f_{\varepsilon_i, \varepsilon_j}(\beta_i; a_j - R_i(a_j) + b_i) = \begin{cases} 
\frac{R_i(a_j)c_j + a_j\lambda_i\beta_j}{c_ic_j(1-\lambda_i\lambda_j\beta_i\beta_j)} & \text{if } S \\
\frac{R_i(a_j) - a_i\lambda_i\beta_j}{1-\lambda_i\lambda_j\beta_i\beta_j} & \text{if } D
\end{cases}
\]
Since \( \Pi_i(\cdot, \cdot) - C_i(\cdot) \) is continuous, \( R_i(\cdot) \) is continuous. Therefore, a composite function \( R_i \circ R_j : [0, \bar{a}] \to [0, \bar{a}] \) (defined as \( (R_i \circ R_j)(a_i) = R_i(R_j(a_i)) \)) is a continuous function from \([0, \bar{a}]\) into itself, where \([0, \bar{a}]\) is a nonempty, compact, convex set. Then by Brouwer’s Fixed Point Theorem, \( R_i \circ R_j \) has a fixed point; that is, there exists \( a_i \in [0, \bar{a}] \) such that \( a_i = (R_i \circ R_j)(a_i) \). So there exists a profile \((a_M, a_G)\) such that \( a_M = R_M(a_G) \) and \( a_G = R_G(a_M) \). Thus, there exists \((a_M, a_G)\) such that
\[
\Pi_i(a_i, a_j) - C_i(a_i) = \max_{a'_i \in [0, \bar{a}]} \Pi_i(a'_i, a_j) - C_i(a'_i)
\]
where \( i, j \in \{M, G\}, i \neq j \), so an equilibrium exists.

### B. Proof of Proposition 2

From (A.1) we find \( b_i \) as a function of \( a_i \) \((i, j \in \{M, G\}, i \neq j)\)
\[
b_i = \begin{cases} 
\frac{a_i - \beta_i a_j \pm \sqrt{(1 + \beta_i^2) \sigma^2 \left( \log(2\pi) + 2 \log \left( \frac{\sqrt{1 + \beta_i^2} \sigma^2 (a_i c_j + a_j c_i \lambda_i \beta_j)}{c_ic_j (1-\lambda_i \lambda_j \beta_i \beta_j)} \right) \right)}}{1 + \beta_i^2 \sigma^2 \left( \log(2\pi) + 2 \log \left( \frac{\sqrt{1 + \beta_i^2} \sigma^2 (a_i - a_i \lambda_i \beta_j)}{1-\lambda_i \lambda_j \beta_i \beta_j} \right) \right)} & \text{if } S \\
\frac{a_i - \beta_i a_j \pm \sqrt{(1 + \beta_i^2) \sigma^2 \left( \log(2\pi) + 2 \log \left( \frac{\sqrt{1 + \beta_i^2} \sigma^2 (a_i - a_i \lambda_i \beta_j)}{1-\lambda_i \lambda_j \beta_i \beta_j} \right) \right)}}{1 + \beta_i^2 \sigma^2 \left( \log(2\pi) + 2 \log \left( \frac{\sqrt{1 + \beta_i^2} \sigma^2 (a_i - a_i \lambda_i \beta_j)}{1-\lambda_i \lambda_j \beta_i \beta_j} \right) \right)} & \text{if } D
\end{cases}
\]
One can show that \( a_i \) is maximized when the expression under the square root is equal to zero, which yields
\[
a_i = \begin{cases} 
\frac{c_i}{\sqrt{2\pi\sigma^2}} \left( \frac{1}{\sqrt{1 + \beta_i^2}} - \frac{\lambda_i \beta_j}{\sqrt{1 + \beta_i^2}} \right) & \text{if } S \\
\frac{1}{\sqrt{2\pi\sigma^2}} \left( \frac{1}{\sqrt{1 + \beta_i^2}} + \frac{\lambda_i \beta_j}{\sqrt{1 + \beta_i^2}} \right) & \text{if } D
\end{cases}
\]
Maximizing \( a_M + a_G \) with respect to \( \beta_i \) \((i, j \in \{M, G\}, i \neq j)\) yields equilibrium slopes \( \beta_i^* \) of linear performance evaluation rules:
\[
\beta_i^* = \begin{cases} 
0 & \text{if } S \\
\lambda_j & \text{if } D
\end{cases}
\]
Politicians’ efforts in equilibrium read

\[ a_i^* = \begin{cases} \frac{\epsilon_i}{\sqrt{2\pi}\sigma^2} & \text{if } S \\ \frac{1}{\sqrt{2\pi}\sigma^2} \left( \frac{1}{\sqrt{1+\lambda_j^2}} + \frac{\lambda_j^2}{\sqrt{1+\lambda_j^2}} \right) & \text{if } D \end{cases} \]

Finally, equilibrium intercepts \( b_i^* \) are given by

\[ b_i^* = \begin{cases} a_i^* & \text{if } S \\ a_i^* - \lambda_j a_j^* & \text{if } D \end{cases} \]

which completes the proof.

C. Proof of Proposition 3

The transition from state \( k \) back to state \( k, k \in \{S, D\} \), occurs either when both incumbents are reappointed or when none of them is reappointed (so, opponents are elected). The equilibrium transition probabilities read

\[
\begin{align*}
P_{SS} &= P \left( \{p_M(a_M^* \geq a_M^*) \cap \{p_G(a_G^* \geq a_G^*)\} \right) + P \left( \{p_M(a_M^* < a_M^*) \cap \{p_G(a_G^* < a_G^*)\} \right) = \frac{1}{2} \\
P_{DD} &= P \left( \{p_M(a_M^* - a_M^* \geq \lambda_G (p_G(a_G^*) - a_G^*)) \cap \{p_G(a_G^* - a_G^* \geq \lambda_M (p_M(a_M^*) - a_M^*)) \right) + \\
& P \left( \{p_M(a_M^* - a_M^* < \lambda_G (p_G(a_G^*) - a_G^*)) \cap \{p_G(a_G^* - a_G^* < \lambda_M (p_M(a_M^*) - a_M^*)) \right) = \\
& \frac{1}{\lambda_M} \int_{\frac{1}{\lambda_G}}^{+\infty} f_{\varepsilon_M}(x) \int_{\frac{1}{\lambda_G}} f_{\varepsilon_G}(y) \, dy \, dx = \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan (\lambda_M) \right) \\
P_{SD} &= 1 - P_{SS} = \frac{1}{2} \\
P_{DS} &= 1 - P_{DD} = 1 - \frac{1}{\pi} \left( \arctan \left( \frac{1}{\lambda_G} \right) - \arctan (\lambda_M) \right)
\end{align*}
\]

where \( \arctan (\cdot) \) is an arctangent function.

D. Data Sources

**Ministry of Interior**: http://elecciones.mir.es/MIR/jsp/resultados/index.htm;


**Com. de Madrid**: http://www.madrid.org/iestadis/fijas/estructu/general/otros/eleccionescm.htm;

**Com. Valenciana**: http://www.gva.es/jsp/portalgv.jsp?deliberate=true;
E. Municipalities Included in the Final Sample

Castilla-La Mancha: Azuqueca de Henares (2), Ciudad Real (1), Guadalajara (1), Hellín (2), Toledo (2), Tomelloso (2), Valdepeñas (1);

Com. de Madrid: Alcobendas (3), Alcorcón (1), Aranjuez (1), Collado Villalba (2), Coslada (1), Leganés (2), Móstoles (1), Pinto (1), San Sebastián de los Reyes (1), Torrejón de Ardoz (1), Tres Cantos (1);

Com. Valenciana: Alaquàs (1), Alcalá (1), Algemesí (3), Burjassot (1), Catarroja (1), Elche (2), Elda (2), Gandia (3), Ibi (2), Manises (1), Mislata (2), Oliva (3), Ondara (1), Paterna (2), Quart de Poblet (1), San Vicente del Raspeig (2), Torrent (2), Xirivella (2);

Islas Baleares: Eivissa (1), Mahón (1);

Princ. de Asturias: Gijón (1), Siero (2);

Región de Murcia: Caravaca de la Cruz (1), Jumilla (2), Totana (1);

where the number in brackets denotes how many times voters split tickets in the considered municipality during the analyzed period.

References


