Behavior Based Price Discrimination with Cross Group Externalities

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Abstract

In this paper, we analyze the practice of firms to offer different prices to consumers according to the past purchase behavior (BBPD) in the context of two-sided markets, i.e. markets with cross group externalities. In a two period model, two platforms compete for heterogeneous firms and end-users. Our contribution is that we allow platforms to discriminate prices on the users’ side according to their past purchase behavior. The main findings are two. In the second period game with market shares taken as given, each platform may find it optimal either to offer discounts to rivals’ users or to reward loyalty, depending on the number of users attracted in the past. Moreover, switching towards both platforms occurs if and only if the inherited market partition is symmetric enough. Endogenizing the first period game, BBPD affects both ex-ante and ex-post competition. Ex-post competition is strengthened compared to the regime in which a uniform price is charged in users’ side. Ex-ante competition is relaxed (intensified) if users are the low (high) value group. The overall effect on inter-temporal profits of platforms is negative, confirming the previous results of BBPD literature.

Keywords: Behavior Based Price Discrimination, Two-Sided Markets

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1 Introduction

When a firm knows the identity of its customers, it may decide to charge new customers with a lower price in order to increase its demand. As pointed out by Taylor (2003), price discrimination based on past purchases, called behavior based price discrimination (BBPD), according to which firms offer discounted prices to new customers inducing them to switch, is very common in subscription markets. This is because transactions are never anonymous: once a customer signs the subscription, a firm knows whether she is one of the current customers or not. According to that, it may have an incentive to propose low introductory prices to customers who did not buy its product in the past.

Discounts take different forms such as low introductory prices, trial memberships and free installations. As mentioned in Caillaud and Nijs (2011), a new subscriber for 3 months to the French newspaper "Le Monde", pays 50 euros whereas a previous customer is charged 131.30 euros. A similar strategy is the free trial membership to the program Amazon Prime offered by Amazon\textsuperscript{1}, which offers free streaming contents, magazines and books for Kindle’s owners. Moreover, first subscriptions to credit cards and TVs/internet services are often offered for free.\textsuperscript{2}

Firms selling products as credit cards, magazines and newspapers, satellite TVs, internet access and e-book readers have the common feature that subscribers are not the only customers. Indeed, these firms compete also for another side of the market, e.g. merchants (credit cards), advertisers (media), content providers (e-book readers and internet). These firms are two-sided platforms that serve and allow the interaction between different groups of customers linked to each other by cross-group externalities. Indeed, when a cardholder decides whether to hold a card or not, his utility is increasing in the number of shops in which she can use it. Shops (merchants) represent the other side of the market and in turn

\textsuperscript{1}From Amazon website "Amazon Prime members in the U.S. can enjoy instant videos: unlimited, commercial-free, instant streaming of thousands of movies and TV shows through Amazon Instant Video at no additional cost. Members who own Kindle devices can also choose from thousands of books – including more than 100 current and former New York Times Bestsellers – to borrow and read for free, as frequently as a book a month with no due dates, from the Kindle Owners’ Lending Library. Eligible customers can try out a membership by starting a free trial”

\textsuperscript{2}Taylor (2003) also mentions a 1998 Wall Street Journal’s article by Bailey and Kilman reported that "the 60% of all Visa and MasterCard solicitations include a "teaser" (low introductory rate) on balances transferred from a card issued by another bank".
they are more willing to pay to hold a card reader as the number of card users increases.

The same interaction arises in the other mentioned markets: medias (magazines, newspapers) allow the interaction between readers and advertisers; satellite TVs/internet providers between viewers/surfers and content providers. For what concerns Kindle Owners’ Lending Library and Amazon Instant Video, Amazon is nothing else than a platform that facilitates the interaction between readers/viewers and content providers. Publishers are interested in selling their books to a large number of readers who, in turn, are interested in the variety of contents. Thus, the utility that a reader obtains from subscription increases in the number of publishers, and the utility that a publisher receives to have his book available on Kindle Library is increasing in the number of Amazon Prime subscribers.

Because of the externalities, one of the distinctive features of these markets is the pricing rule, which is different from the general rule that applies in a one-side framework (i.e. market without externalities), both for a monopolistic and a competitive environment. Think for example to Amazon Prime program: because of the cross-group externalities between readers and publishers, the subscription fee charged to the readers affects not only the demand in this group, but also the willingness to pay of publishers to have their books available on Amazon Kindle.

This is the basic reason for which we observe different prices for different sides of the market (cross-group price discrimination): price charged to each group of agents depends on the cross externalities, so that a group whose participation entails a large participation of the other group will be charged less.\textsuperscript{3}

This idea is very clear when we look at medias: since advertisers are only interested in reaching a high number of readers/viewers while the other way around it is not (necessarily) true,\textsuperscript{4} they are charged more and most of medias’ profits are made on ads.

According to this discussion, in many subscription markets two kinds of strategies are used by competing platforms: the mentioned cross-group price discrimination typical of a two-sided market and the within group BBPD in subscribers’ side. These strategies have a common feature: platforms have some informa-

\textsuperscript{3}This result is firstly due to Caillaud and Jullien (2003), which calls this price strategy divide and conquer, and has became the reference point for the succeeding studies on pricing in two-sided markets.

\textsuperscript{4}In particular, the industry specific work of Ferrando et al. (2008) on media market studies provide a two sided model in which a proportion viewers/readers is ad-lovers and the remaining part is composed by ad-averse people.
tion about the characteristics of various groups of customers and exploit this information setting targeted prices to each group.

However, the type of information required to implement these strategies is fundamentally different. On one hand, to engage in cross-group price discrimination, platforms simply sort customers according to their externalities. On the other hand, within group BBPD requires platforms to know the identity and the behavior of customers.

This paper provides a two-sided market analysis to address the following question: “What does it change if platforms are allowed to offer different prices to subscribers according to their past purchase behavior?” Specifically, the aim is to investigate about the effects on prices and platforms’ profits when within group BBPD can be implemented because subscribers are identified.

In order to answer this question, we provide a two period model in which platforms compete in a Hotelling fashion for two different groups of agents, users (subscribers in the examples above) and firms. Two different regimes are provided: the within group uniform price regime and the within group BBPD regime. The first one works as a benchmark to which the second is compared to, to capture the effects of the introduction of BBPD.

In the first period platforms set prices and then each firm and user decide which platform to join. In the second period, platforms come across a new information, the identity of old and new users. The first period competition is common in the two regimes, which differ in the presence (or in the use) of the information above. Under the BBPD regime, platforms are allowed to discriminate prices according to the new information they receive. In the benchmark case platforms are not aware about the identity of the customers or, even if they are, they cannot exploit this information.

**Related literature**

This paper is naturally linked to the two-sided market literature, initially formalized by Rochet and Tirole (2003), Armstrong (2006) and Caillaud and Jullien (2003). The main result around which this literature is built on is the cross-group price discrimination, which follows the concept of Divide & Conquer firstly proposed by Caillaud and Jullien (2003). To develop a business, a platform has to attract a large number of customers on one side, even subsidizing them (divide) and after restore its losses charging a relatively high price to the other side (conquer).
As in Rochet and Tirole (2003), Armstrong (2006) we use an Hotelling model, to capture the idea that customers are not indifferent about joining one platform or another but can be horizontally ordered according to their preferences. The model focuses on the simplest case in which platforms charge only a price independent of the number of interactions with the other side and customers can join at most one platform.

On the other hand our paper is strongly related with BBPD literature, which starts with Villas-Boas (1999) and Fudenberg and Tirole (2000). The main finding of this literature is that BBPD is detrimental for firms, which compete strongerly in prices and face a *prisoners’ dilemma* problem.

In particular, our model is built on Fudenberg and Tirole (2000), which provide a Hotelling model played twice (the world finishes in time 2), allowing firms to know whether a customer in the second period is new (weak market) or he was already buying from the same firm (strong market). They establish that offering discounted prices to new customers is an equilibrium phenomenon that involves a decrease in prices and in profits for firms with consequential increase in the total surplus of consumers.

Villas-Boas (1999) makes the same analysis but in infinite time with overlapping generations of consumers while Chen and Zhang (2008) and Esteves (2007) present models with different distributions of consumers types. Except the work of Chen and Zhang, the literature agrees on the result that customers’ recognition and consequent price discrimination hurt firms compared to a situation in which the targeted pricing is not possible. Even if a firm alone would prefer to obtain the information (and so benefit from the surplus extraction), if both get it then a market stealing effect tends to prevail.

Liu and Serfes (2007) is close to our paper in that both of us analyze *within group* price discrimination. In particular, platforms are allowed to engage in perfect price discrimination within each side. Their main finding is that discrimination might be a tool to neutralize cross-group externalities with a positive effect on prices and platforms’ profits. There are two main differences with our work. First of all, they only consider one period, keeping the past behavior of

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5Literature distinguishes between subscription fee and usage fee. In the analysis of the media market of Ferrando et al. (2008) is pointed out how, while readers are charged with the price of the newspaper, advertiser are charged on per readers basis. In this case we can see an access fee in one side and a transaction fee on the other side.

6As a matter of fact, literature points out how often at least one side decides to multi-home, i.e. to join more than one platform. Armstrong (2006) and Armstrong and Wright (2007) provide an analysis on the reasons and on the effects of multi-homing in platforms competition.
consumers and market’s shares as given. Second, they analyze the case of perfect price discrimination, while we focus on a more realistic discrimination based on past purchase behavior.

The rest of the paper is organized as follows. In section 2 we introduce the model. In section 3 we briefly present the benchmark case of the within group uniform price. Afterwards, we analyze the model when we allow for BBPD in section 4. In section 5 we compare the two regimes before providing conclusions (section 6).

2 The model

Two competing platforms \( j = A, B \) aim to sell a service to two different groups of customers. For the sake of expositive clarity, we name end-users the customers on one side and firms on the other side. With end-users’ side we simply refer to the group of final consumers, which can be books/newspapers’ readers, TV’s viewers or cardholders. We call firms’ side, instead, the group of customers composed by advertisers, contents’ providers or merchants.

Both end-users (side or group \( E \)) and firms (side or group \( F \)) are heterogeneous according to their locations: they are assumed to be uniformly distributed along a unit segment. In turn, platforms locations are kept fixed at the end-points of this segment, i.e. platform \( A \)’s location is \( x^A = 0 \), while platform \( B \) is located in \( x^B = 1 \).

Both sides \( i = \{E, F\} \) of the market exhibit linear utilities from joining a platform. An agent located at \( x \in [0,1] \) bears a constant transportation cost \( t \) per unit of distance covered to reach the location of each platform.\(^7\) Moreover, the benefit that an agent receives from joining a given platform depends on the number of agents joining the same platform on the other side.

Specifically, we assume that the utility that an agent receives from joining a platform depends linearly on the number of agents on the other side joining the same platform, where \( \alpha_E \) and \( \alpha_F \) represent the externality parameters respectively of end-users and firms. The parameter \( \alpha_F \) (respectively \( \alpha_E \)) has to be interpreted as the strength of the externality in firms’ (users’) group, i.e. the

\(^7\)Throughout the paper, the transportation cost is assumed to be the same for both sides. This assumption is quite arguable, but since the intuition behind the results provided in the paper remains the same even if we consider two different transportation costs, we use only one transportation cost in order to keep notation as simple as possible.
extra-benefit for a firm (end-user) when an additional user (firm) joins the same platform.

We have two periods $\tau = 1, 2$. In each period platforms set prices. After having observed the prices offered, firms and users decide which platform to join. Defining $p^i_{\tau}$ as the price set by platform $j$ to side $i$ in time $\tau$, the utility function for a user located at $x$ joining platform $j$ in time $\tau$ is simply:

$$U^j_{E\tau}(x) = u + \alpha_E n^j_{E\tau} - p^j_{E\tau} - |x - x^j| t$$  \hspace{1cm} (1)

where $n^j_{E\tau}$ is the total number of firms joining platform $j$. On the other side, the utility function for a firm located at $x$ joining platform $j$ in time $\tau$ is given by what follows:

$$U^j_{F\tau}(x) = u + \alpha_F n^j_{E\tau} - p^j_{F\tau} - |x - x^j| t$$  \hspace{1cm} (2)

where $n^j_{E\tau}$ is the total number of users joining platform $j$. $u$ is a constant representing the standalone utility, i.e. the utility that an agent benefits from joining a platform, regardless which platform he is joining.$^8$

Platforms seek to maximize inter-temporal profits, bearing unitary cost normalized to 0 for both sides. The profit of a firm in time $\tau$ is simply given by the sum of the products between the price charged to each group (or sub-group, as we will see afterwards) and the number of joiners belonging to the same group. Thus, the profit of platform $j$ in time $\tau$ when charging prices $p^j_{E\tau}$ and $p^j_{F\tau}$ to each side is indicated in equation by the following:

$$\pi^j_{\tau} = \sum_{i=E,F} p^j_{i\tau} n_{i\tau}$$  \hspace{1cm} (3)

Platforms set prices in each time period in order to maximize the sum of inter-temporal profits, given by:

$$\Pi^j = \pi^j_1 + \delta \pi^j_2$$  where $\delta \equiv$ discount factor  \hspace{1cm} (4)

Three main assumptions are used throughout the paper: demand is fully served, transportation cost is big enough so to have single-homing in both sides and time profit functions are concave. Using a formal jargon, we write down the assumptions $A1$, $A2$ and $A3$.

$^8$Here it is assumed to be the same for both sides of the market just to keep notation as simple as possible but it is not crucial in the analysis of the model.
ASSUMPTION A1 (Market fully served): \( u \) big enough.

If the standalone utility \( u \) is big enough, every agent prefers to join at least one platform instead of joining none. In this way we insure that the market is fully covered. For simplicity, an agent who does not join any platform is assumed to receive no utility.

ASSUMPTION A2 (Single-homing): \( t > \alpha_E \) and \( t > \alpha_F \).

As shown in Armstrong and Wright (2007), this assumption means that the transportation cost is high enough to have that each agent joins at most one platform. In words, agents are interested in reaching the other side, but not so much to decide to join both platforms and bear price and transportation cost twice. Following the usual phrasing of two-sided markets literature, they opt for single-homing instead of multi-homing.

ASSUMPTION A3 (Concavity): \( t^2 > 2(\alpha_E + \alpha_F)^2 \).

This is simply the assumption we need for the profit functions to be concave. Proof is provided in the appendix.

In the analysis of the model we consider two cases. In the first one platforms set uniform prices within each side, i.e both platforms charge the same amount to all agents belonging to the same group. In this case within group price discrimination is not possible, either because the identity of end-users is not known or because it is banned. Throughout the paper, we refer to this case as the within group uniform price regime and use it as a benchmark.

The second case refers to the regime of price discrimination in the users’ side according to past purchase behavior: in the second period, we indeed allow platforms to offer discounted prices to consumers who did not subscribe to them in the first period.

Namely, both platforms are allowed choose different prices for old and new users’ subscriptions in stage 2, while in the firms’ side prices \( p_{F2}^A \) and \( p_{F2}^B \) are chosen following the same reasoning of the first period. Since our objective is to analyze subscription markets in which BBPD is used, we focus only on price discrimination in the users’ side, the only one for which we have evidence about BBPD.\(^9\)

\(^9\)As a matter of fact, some price discrimination may be used also in firms’ side, but we keep this possibility out of our analysis.
Formally, platform $j$ in period 2 offers this pair of prices for users

$$p_{E1}^{jj} \equiv \text{price chosen by platform } j \text{ for users who have already bought from it in period 1 (agents who are loyal to } j)$$

$$p_{E2}^{jj} \equiv \text{price chosen by platform } j \text{ for users who have bought from platform } j' \neq j \text{ in period 1 (i.e. agents who are supposed to switch from } j' \text{ to } j)$$

Some insights should be highlighted about this model. First of all, prices chosen in time 1 have an effect on the inter-temporal profit twice: on the $\pi_1$ directly and on $\pi_2$ indirectly (prices in time 1 have an effect on the market share of platforms and then on the effective possibility to steal customers from the other platform).

Potentially, given this pair of prices, some agents remain loyal to one platform while some others may decide to switch to the rival, because of a lower introductory price offered by the latter. If switching occurs, the Hotelling segment would be partitioned in four sub-segments: segment of old consumers who remain in the network $A$, consumers who switch from $A$ to $B$, consumers who switch from $B$ to $A$ and old consumers who remain in the network $A$.

According to Figure 1, we define $n_{E1}$ as the number of users who joined platform $A$ in time 1 and $x_2^A$ (respectively $x_2^B$) represents the user who subscribes to platform $A$ ($B$) in time 1 and in time 2 is indifferent between switching to $B$ ($A$) and staying with $B$ ($A$).
In Figure 1 the situation in which the two-directions switching (from $A$ to $B$ and the other way around) arises is represented. To be precise, a priori this may be not the case, that is to say that $x_A^2$ may be higher as well as $x_B^2$ lower than $n_{E_1}$. Thus, the number of switchers from platform $A$ to platform $B$ will be $n_{E_2}^{BA} = \max\{n_{E_1} - x_A^2, 0\}$ and the switchers going in the opposite direction are $n_{E_2}^{AB} = \max\{x_B^2 - n_{E_1}, 0\}$. Alike, the number of users loyal to $A$ is $n_{E_2}^{AA} = \min\{x_A^2, n_{E_1}\}$ while the loyal to $B$ are $n_{E_2}^{BB} = \min\{1 - x_B^2, 1 - n_{E_1}\}$.

According to that, the profit function in time $\tau = 2$ turns out to be slightly different from the one defined in (3). Namely, it takes the following form:

$$
\pi_j^{\tau=2} = \begin{cases} 
  p_{E_2}^{AA} n_{E_2}^{AA} + p_{E_2}^{AB} n_{E_2}^{AB} + p_{E_2}^A n_{E_2}^A & \text{if } j=A \\
  p_{E_2}^{BB} n_{E_2}^{BB} + p_{E_2}^{BA} n_{E_2}^{BA} + p_{E_2}^B n_{E_2}^B & \text{if } j=B 
\end{cases}
$$

The model is solved by backward induction. First we analyze the decisions of customers in time 2, then we solve the maximization problem of platforms and we find the equilibrium prices and profits in time 2 for given equilibrium in the first period game. Assuming that all agents anticipate the second period result, we solve the game in time 1. In the next section, we briefly discuss the benchmark case of within group uniform price. In section 4, the analysis of the within group price discrimination regime is provided.

### 3 Within Group Uniform Price Regime.

First of all, we consider the case in which price discrimination among users is not possible, either because their identity is not known or because price discrimination is banned. Since platforms do not have any information about past purchase behavior, they cannot distinguish between old and new users. Thus, they can only set different prices between the two sides, but not within them. It means that $p_{E_2}^{AA} = p_{E_2}^{AB} = p_{E_2}^A$ and $p_{E_2}^{BB} = p_{E_2}^{BA} = p_{E_2}^B$.

The final result is that we have exactly the same Hotelling competition game played twice, so that prices chosen in time 1 are kept in time 2. Since platforms have no more information in time 2, prices set in time 1 are optimal also in time 2, so that nothing changes at equilibrium. Moreover, equilibrium prices are the same for both platforms because of the symmetry of the model. Solving the maximization problem of the function defined in (3) choosing prices, equilibrium prices are indeed $\bar{p}_E = t - \alpha_F$ for end-users and $\bar{p}_F = t - \alpha_E$ for firms, with a
consequent level of profits:

\[ \bar{\Pi}^A = \bar{\Pi}^B = \bar{\Pi} = \left[ 2t - (\alpha_E + \alpha_F) \right] \frac{(1 + \delta)}{2} \] (6)

Equilibrium prices simply reflect the presence of externalities. As an example, when a user subscribes for the service offered by the platform, he is not only "bringing" himself to the platform but also a number of firms, which join the platform because of his presence. In particular, each user who joins the platform carries \( \alpha_F \) firms so that he is rewarded for that. Thus, a profit maximizer platform may well set a negative price on users’ side and recoup the loss made in this side by charging a high price to firms, which willingness to pay is high when a lot of users subscribe. Hereafter, this case is used as a benchmark because it allows to see which are the effects of the the low introductory subscription fees charged by the platforms on equilibrium prices and profits.

4 Within Group Price Discrimination regime.

4.1 Time 2 competition game.

Now, we analyze the decisions of players in the second period, treating the initial subscription to each platform as exogenous. Thus, we have a game with three types of players, which equilibria depend on what occurred in the first period.

Platforms set prices in each side. In firms’ side, the price is uniform, i.e. each firm joining the same platform is going to pay the same price. For what concerns users, we allow platforms to set different prices according to their past (observed) purchase behavior. Before setting their prices, platforms form expectations about the participation of both sides of the market, since the utility functions are common knowledge.

After having observed the price offers of platforms, users decide whether to confirm or not the decision they have taken in the past. If they join the same platform as in the first period, they are loyalists, while if they change, they are switchers. Their decision is also taken looking at prices for the firms’ side. According to the prices they observe, users have expectations about participation on the other side. The firms’ participation as well as the prices they have to pay will determine the number of loyalists and switchers.

Firms’ decisions follow exactly the same reasoning. They observe prices they have to pay in each case and then form expectations about the participation of
At equilibrium, each customer joins the platform which gives him the highest utility. Anticipating the participation and the choices of the rival, each platform maximizes profits choosing prices.

**Users** Who is going to switch and who is going to stay? Users simply compare utility that they get joining each platform and decide upon which platform to join given offered prices and given the expected number of firms’ contents available in both platforms. As explained in section 2 for given prices offered by platforms, we can find where the indifferent users between switching and staying are located.

Consider an end-user who has bought from platform A in period 1. According to (1), he prefers buying again from platform A (paying price charged by A to its old customers, \( p^A_E \)) rather than switching to B (and paying price chosen by B for new customers, \( p^B_E \)) if the following inequality holds:

\[
\alpha E n^A_{F2} - p^A_{E2} - xt > \alpha E n^B_{F2} - p^B_{E2} - (1 - x) t
\]  (7)

An agent for who this inequality is reversed is a switcher from A to B, while the agent for who LHS is exactly equal to RHS is indifferent between switching and keeping the same decision of time 1. As defined in section 2 the location of this agent is \( x^A_2 \), which turns out to be what follows, simply by equalizing the two sides of (7) and solving for \( x^A_2 \).

\[
x^A_2 = \frac{1}{2} + \frac{\alpha E n^A_{F2} - \alpha E n^B_{F2} + p^B_{E2} - p^A_{E2}}{2t}
\]  (8)

The same reasoning is followed to define \( x^B_2 \), threshold representing the agent who has decided to join platform B in time 1 and in 2 is indifferent between choose to buy again from B (and pay \( p^B_{E2} \)) and switch to platform A (paying price \( p^A_{E2} \)), i.e:

\[
x^B_2 = \frac{1}{2} + \frac{\alpha E n^A_{F2} - \alpha E n^B_{F2} + p^B_{E2} - p^A_{E2}}{2t}
\]  (9)
According to the fact that the number of users loyal to $A$ is given by $n_{E2}^{AA}$ while the number of switchers to $A$ is $n_{E2}^{AB}$, the total number of users joining platform $A$ in time 2 will be:

$$n_{E2}^A = n_{E2}^{AA} + n_{E2}^{AB} = \min \{x_2^A, n_{E1}\} + \max \{x_2^B - n_{E1}, 0\} \quad (10)$$

Different cases may arise. If $n_{E1} \in (x_2^A, x_2^B)$, then switching to both directions occurs and (10) becomes $n_{E2}^A = x_2^A + x_2^B - n_{E1}$. Depending on $n_{E1}$, switching to both directions may be not the case. If $x_2^A > n_{E1}$, then there are no switchers from $A$ to $B$ and $n_{E2}^A = x_2^B$; if $x_2^B < n_{E1}$, then there are no switchers from $B$ to $A$ and then $n_{E2}^A = x_2^A$.\(^{10}\) Moreover, according to assumption $A1$, $n_{E2}^B = 1 - n_{E2}^A$.

**Firms**  
Firms take their decision following basically the same reasoning as users. They observe prices offered by both platforms and according to how many users they expect to subscribe to each platform, they decide which platform to join.

Consider a firm located at $x$. According to (2), it prefers to join platform $A$ instead of $B$ if the following inequality holds:

$$\alpha_F n_{E2}^A - p_{F2}^A - xt > \alpha_F n_{E2}^B - p_{F2}^B - (1 - x) t$$

The equation above is also telling us that a firm for which this inequality is reversed will join platform $B$, while the the firm located at the $x$ such that LHS is exactly equal to RHS is indifferent between $A$ and $B$. In order to define this indifferent firm, we directly consider that the total number of users joining each platform is the sum of loyalists and switchers as described in equation (10). We define this location as $n_{F2}$, which turns out to be what follows, simply by equalizing the two sides of (11) and solving for $n_{F2}$.

$$n_{F2} = \frac{1}{2} + \frac{\alpha_F}{t} \left( n_{E2}^{AA} + n_{E2}^{AB} - \frac{1}{2} \right) + \frac{1}{2t} \left( p_{F2}^B - p_{F2}^A \right) \quad (12)$$

According to this threshold and assumption $A1$, the number of firms joining platform $A$ (respectively $B$) is simply $n_{F2}^A = n_{F2}$ (resp. $n_{F2}^B = 1 - n_{F2}$).

\(^{10}\)As it will be explained afterwards, it cannot exist any situation in which switching does not occur at all, i.e. $x_2^B < n_{E1} < x_2^A$. In the equilibrium, at least in one direction, some end-users are going to change platform.
Platforms Platforms act to maximize profits, knowing what is the respectively market share inherited from period 1. As already said, there is a priori uncertainty about the fact that the strategy of setting introductory prices can be useful for platforms. Namely, if the inherited number of subscribers is very high, it may be too costly for a platform to attract the small residual number of users (and consequently of firms, because of externalities) who have subscribed to the rival in period 1.

Consider as an example the case in which \( n_{E1} \) is very close to 1, meaning that almost all users have subscribed to platform A in time 1: competition for the residual number of subscribers is very unbalanced in favor of platform B, since these users are very close to its location. It means that even if platform A is very aggressive in pricing B’s previous subscribers, it is unlikely to have switching from B to A. These considerations will be clarified soon looking to the various segments of the demands, that we obtain simply putting together (8), (9) and (12).

\[
x_A^2 = \frac{2^2 - \alpha E \alpha F}{k t} (p_{E1}^{BA} - p_{E2}^{BA}) + \frac{\alpha E \alpha F}{k} (p_{E1}^{BB} - p_{E2}^{BB}) + \frac{\alpha E}{k} (p_{E1}^{BB} - p_{E2}^{BB}) + \frac{\alpha F}{k} (p_{E1}^{BB} - p_{E2}^{BB}) + \frac{2^2 - \alpha E \alpha F (1 + 2n_{E1})}{k}
\]

\[
x_B^2 = \frac{2^2 - \alpha E \alpha F}{k t} (p_{E1}^{BA} - p_{E2}^{BA}) + \frac{\alpha E \alpha F}{k} (p_{E1}^{AB} - p_{E2}^{AB}) + \frac{\alpha E}{k} (p_{E1}^{AB} - p_{E2}^{AB}) + \frac{\alpha F}{k} (p_{E1}^{AB} - p_{E2}^{AB}) + \frac{2^2 - \alpha E \alpha F (1 + 2n_{E1})}{k}
\]

\[
n_{F2} = \frac{1}{2} + \frac{\alpha E}{k} [1 - 2n_{E1}] + \frac{\alpha F}{k} (p_{E1}^{BB} - p_{E2}^{BB}) + \frac{\alpha E}{k} (p_{E1}^{BA} - p_{E2}^{BA})
\]

where \( k \equiv 2t^2 - 4\alpha_E \alpha_F > 0 \) by the concavity conditions stated in assumption A3. It is very easy to see how having a high inherited number of subscribers makes it more difficult for a platform to retain old customers. Indeed, platform A retains less users in the second period as long as the difference between the inherited market share \( n_{E1} \) and the indifferent users between staying and switching to B, \( x_A^2 \) is higher. The higher this difference, the more users will switch to B. Computing the derivative of this difference with respect to \( n_{E1} \), gives:

\[
\frac{\partial (n_{E1} - x_A^2)}{\partial n_{E1}} = 1 + \frac{2\alpha_E \alpha_F}{k} = \frac{t^2 - \alpha_E \alpha_F}{t^2 - 2\alpha_E \alpha_F} > 0
\]

Since the difference is clearly increasing in \( n_{E1} \) for given prices, equation (16) means that the probability of platform A to retain old customers is strictly decreasing in the inherited market share. The same discussion can be done for platform B, which probability to retain users is decreasing in the number of users joining this platform in time 1, i.e. \( 1 - n_{E1} \).
Moreover, few comments should be done about prices. The first consideration is that the higher is the difference between the price charged by platform \( j \) to its old users (\( p^j_{E2} \)) and the price charged to the rival to induce switching (\( p^{j'}_{E2} \) with \( j' \neq j \)), the higher will be the likelihood to have switching from \( j \) to \( j' \). It is clear that \( x^A_2 \) is lower (more switching to \( B \)) when \( p^{AA}_{E2} - p^{BA}_{E2} \) is higher and that \( x^B_2 \) is higher (more switching to \( A \)) when \( p^{BB}_{E2} - p^{AB}_{E2} \) is higher.

A distinctive feature of this two-sided model is that the indifferent users (respectively firms) depend not only on the prices of their side, but also on the prices charged to the firms (respectively users). The dependence on other side prices is nothing surprising: since there are cross group externalities, when decisions about subscription are taken, what matters is not only the price and the location but also the number of agents joining in the other side.

Slightly more puzzling is the result that the user indifferent between switching or staying with platform \( j \), not only depends on prices charged to the interested locations (i.e. platform \( j' \)'s inherited turf) but also on the prices charged in the other territory (platform \( j' \)'s turf with \( j' \neq j \)). This is a feedback effect of externalities: since the competition in \( j' \)'s turf affects the participation of firms on the other side, it indirectly affects the utility of agents in \( j \)'s turf when they take their subscription decisions.

The platform’s total number of users in time 2 is given by old customers and new customers who it tries to poach from the rival. Hereafter, we assume that \( n_{E1} \) is close enough to \( \frac{1}{2} \), so that bi-directional switching may occur at equilibrium.

Accordingly, the first part of the demand is represented by the segment going from 0 to \( x^A_2 \) and the price charged is \( p^{AA}_{E2} \). The second part is the segment \( x^B_2 - n_{E1} \) and the price charged is \( p^{AB}_{E2} \). The last part of the demand is represented by the number of firms joining the platform, which in turn depends on prices faced by firms as well as on the number of loyalists and switchers. Thus, platform \( A \) solve the following maximization problem:

\[
\max_{p^{AA}_{E2}, p^{AB}_{E2}, p^{AF}_{E2}} p^{AA}_{E2} x^A_2 + p^{AB}_{E2} (x^B_2 - n_{E1}) + p^{AF}_{E2} n_{F2}
\]

Using the first order conditions of the maximization problem, we obtain the best response function of platform \( A \), represented by prices \( p^{AA}_{E2}, p^{AB}_{E2}, p^{AF}_{E2} \) in function of prices charged by the rival platform, which are relegated to the appendix.
Solving the system of the best responses, we obtain the following equilibrium prices:

\[
\begin{align*}
    p^A_{F2} &= t - \alpha_E + \frac{t^2(2n_{E1}-1)(\alpha_E - \alpha_F)}{4\Omega} \\
    p^B_{F2} &= t - \alpha_E - \frac{t^2(2n_{E1}-1)(\alpha_E - \alpha_F)}{4\Omega} \\
    p^{AA}_{E2} &= \frac{5}{12} t - \alpha_F + \frac{1}{2} t n_{E1} + \frac{3t(2n_{E1}-1)}{4\Omega} \\
    p^{AB}_{E2} &= \frac{3}{12} t - \alpha_F - \frac{1}{2} t n_{E1} - \frac{3t(2n_{E1}-1)}{4\Omega} \\
    p^{BA}_{E2} &= -\frac{5}{12} t - \alpha_F + \frac{3}{2} t n_{E1} - \frac{3t(2n_{E1}-1)}{4\Omega} \\
    p^{BB}_{E2} &= -\frac{1}{12} t - \alpha_F - \frac{3}{2} t n_{E1} - \frac{3t(2n_{E1}-1)}{4\Omega}
\end{align*}
\]  

(17)

Where \( \Omega = 9t^2 - 2(2\alpha_E + \alpha_F)(\alpha_E + 2\alpha_F) > 0 \), \( \Psi = 3t^2 - 2\alpha_E (2\alpha_E + \alpha_F) > 0 \) and \( \Omega > \Psi \) by concavity condition (Assumption A3).

Some intuitions about equilibrium prices can be provided. In the firms’ side, the price charged by each platform may depend either positively or negatively on the number of users who already subscribe to a given platform. This result is clearer once we understand what is going on in the users’ side pricing and participation behavior.

For what concerns users, we should distinguish between old and new customers. Prices charged to old consumers depend positively on the market share inherited from period 1 while prices for switchers go exactly to the opposite direction. Indeed:

\[
\begin{align*}
    \frac{\partial p^{AA}_{E2}}{\partial n_{E1}} &= -\frac{\partial p^{BB}_{E2}}{\partial n_{E1}} = \frac{t}{2} + \frac{3\Psi t}{2\Omega} > 0 \\
    \frac{\partial p^{AB}_{E2}}{\partial n_{E1}} &= -\frac{\partial p^{BA}_{E2}}{\partial n_{E1}} = -\frac{3t}{2} + \frac{3\Psi t}{2\Omega} < 0
\end{align*}
\]  

(18)  

(19)

Thus we can find the minimal market share of period 1 for which a given platform offers a discounted price to new customers. Below this threshold the price will be lower for old customers.

Consider platform A’s optimal equilibrium prices in users’ side, \( p^{AA}_{E2} \) and \( p^{AB}_{E2} \). Discounted prices to rival’s customers are offered as long as \( p^{AA}_{E2} > p^{AB}_{E2} \) or simply if \( n_{E1} > \frac{1}{3} \). Thus platform A finds it optimal to offer two different prices to old and new users, with a discount for the latter, only if his inherited market share is at least \( \frac{1}{3} \).

When the number of previous subscribers to A is instead lower that \( \frac{1}{3} \), then platform A offers a lower price to old customers compared to the price to new
customers. In the very particular case in which \( n_{E1} = \frac{1}{3} \), a uniform price is charged in users’ side by platform \( A \). The same argument hold for platform \( B \), considering \( 1 - n_{E1} \) instead of \( n_{E1} \). The results obtained so far are summarized in the following lemma.

**Lemma 1.** *(Price behavior)* Consider two symmetric two-sided platforms competing along a unit Hotelling segment for firms on one side and users on the other. Suppose that \( n_{E1} \) users have already subscribed to platform \( A \) in the past, then at equilibrium:

1. the price charged to the firms’ side may be decreasing or increasing in \( n_{E1} \).
2. if the number of users who subscribed in the past is unbalanced enough towards a platform \((n_{E1} \in \{0, \frac{1}{3} \} \text{ or } n_{E1} \in (\frac{2}{3}, 1)\)) , then the platform with a low number of old customers rewards loyalty while the rival offers discounted prices to new ones.
3. if the number of users who have subscribed to both platforms is close enough to the half of the whole population\((n_{E1} \in (\frac{1}{3}, \frac{2}{3})\)), then both platforms offer discounted prices to rival previous users.

To grasp the intuition, consider the case in which \( n_{E1} > \frac{2}{3} \), so that platform \( A \) is the dominant firm in the users’ side. In order to attract the residual number of users, \( A \) should charge them with a very low price, as they are very far away from \( A \)’s location. In a two-sided market, the incentive to induce switching of these distant users is stronger because and additional user produces an externality on firms’ side, making firms more willing to pay or making more firms willing to join platform \( A \). For these considerations, the difference \( p_{E1}^{AA} - p_{E1}^{AB} \) turns out to be very large.

On the other hand, platform \( B \) is in the opposite condition. Since it knows that incentives to offer discounted prices in its turf are very strong for \( A \), it is worried about being able to retain old users. Moreover, many \( A \)’s previous subscribers have a relative preference towards \( B \) and \( p_{E1}^{AA} \) is relatively high. According to that, platforms \( B \) charges higher prices to new users.
Given the equilibrium prices in (17), the indifferent user between switching to B and subscribe again with A turns out to be:

\[ x^A_2 = \frac{1}{12} + \frac{n_{E1}}{2} + \frac{9t^2(1 - 2n_{E1})}{12\Omega} \tag{20} \]

while the indifferent user between switching to A and subscribe again to B turns out to be:

\[ x^B_2 = \frac{5}{12} + \frac{n_{E1}}{2} + \frac{9t^2(1 - 2n_{E1})}{12\Omega} \tag{21} \]

First of all, notice that for any number of subscribers inherited from the past the threshold in (20) is always below the one in (21), meaning that the indifferent agent between switching and staying in A’s inherited turf is always below the one in B’s turf.

Moreover bi-directional switching does not occur for any inherited number of subscribers to each platform, but only if there is enough symmetry. Indeed, only if a platform has a high enough number of subscribers, the rival can steal some of them.

To see why bi-directional switching cannot always occur, consider the threshold in (20). This threshold is below \( n_{E1} \) if and only if:

\[ n_{E1} > \bar{n}_{E1} \equiv \frac{1}{6} + \frac{t^2}{\Omega + 3t^2} \tag{22} \]

Thus, if the number of subscribers to A is lower than the threshold above, then only switching towards A can occur at equilibrium. For the same reasons, only switching towards B happens whenever \( n_{E1} > 1 - \bar{n}_{E1} \).

As it can be easily intuited, the likelihood of having two-direction switching (i.e. \( n_{E1} \in (\bar{n}_{E1}, 1 - \bar{n}_{E1}) \)) depends on the strength of externalities in both sides, through the effect of \( \alpha_E \) and \( \alpha_F \) have on the term \( \Omega \). Specifically, the higher the externalities are, the narrower the interval of inherited number of users allowing two-direction switching to occur.

Indeed, since the term \( \Omega \) is clearly decreasing in both \( \alpha_E \) and \( \alpha_F \), \( \bar{n}_{E1} \) moves up as externalities increase. If we keep fixed an externality parameter \( \alpha_j \) referred to group \( j \), the threshold \( \bar{n}_{E1} \) gets bigger as \( \alpha_{i\neq j} \) increases, therefore reducing the actual possibilities of two-direction switching. Moreover, since the externality parameters are bounded by \( t \) (from above) and 0 (from below) \( n_{E1} \) always lays
on the interval $(0, \frac{1}{4})$.\textsuperscript{11}

Another important and slightly surprising result is that, whenever a platform starts period 2 with a relatively small number of subscribers, then it will attract more than the half of the subscribers in the second period. Roughly speaking, it overturns the first period result becoming the most present platform in users’ side.

To see why, consider the case in which $n_{E1}$ is slightly below $\frac{1}{2}$. In this case, bi-directional switching occurs so that $A$ loses some past subscribers and gains some others. According to (20) and (21), its new total number of subscribers will be $n^A_{E2} = x^A_2 + x^B_2 - n_{E1} = \frac{1}{2} + \frac{9t^2(1-2n_{E1})}{6t^2}$, which is more than $\frac{1}{2}$ as long as $n_{E1} < \frac{1}{2}$.

According to this result, the number of firms joining platform $A$ depends as well on the number of previous subscribers. In particular, the number of firms joining a given platform is decreasing in the number of previous subscribers has from the past. Indeed, consider as an example platform $A$ number of firms joining at equilibrium, $n^A_{F2}$ (notice that the number of firms joining network $B$ are $1 - n^A_{F2}$):

$$n^A_{F2} = \frac{1}{2} + \frac{t(1 - 2n_{E1})(2\alpha_E + \alpha_F)}{2\Omega} \quad (23)$$

The number of firms joining platform $A$ is exactly $\frac{1}{2}$ when the initial number of subscribers is perfectly split. When instead, the initial number of subscribers from period is unbalanced towards platform $A$, the number of firms joining platform $A$ in period 2 will be lower.

This is an indirect effect of externalities: a high number of previous subscribers makes it less likely to retain loyal subscribers as well as to attract new of them. Moreover, the number of users that switch to the rival is not compensated by the number of new users attracted. Since each user carries an externality to firms’ side, the number of firms tends to decrease as the number of previous users increases.

\textsuperscript{11}To be precise we only consider in our analysis the case of positive externalities in both sides, since for a platform existence to make sense, agents should be interested in the interaction with the other side. In the case of media, though, readers/viewers can be considered as disturbed by the presence of ads. This assumption may change substantially our results.
In Lemma 2, the results about switching behavior are summarized.

**Lemma 2.** (Participation behavior) According to the equilibrium price behavior described in Lemma 1:

1. If and only if the number of users who have subscribed to both platforms lays on the interval \((\bar{n}_E, 1 - \bar{n}_F)\), then there will be some users who switch in one direction and some others to the opposite one (two-directions switching).

2. If point 1 is satisfied, then the total number of firms joining platform A will be

\[ n^A_F = \frac{1}{2} + \frac{t(2n_E - 1)(2n_E + \alpha_F)}{21\Omega} \]

while \(1 - n^B_F\) will join platform B.

As a matter of fact, lemma 2 implies that when we move from the interval in point 1 (i.e. when the inherited number of subscribers is not symmetric enough), the profit functions considered so far are not coherent with what would happen at equilibrium. Indeed for the prices in equation (17) to be an equilibrium, platforms should believe bi-directional switching to occur. However, when \(n_E\) is too close to one of the two end-points platforms should believe that bi-directional switching is not a possible outcome. These expectations entail that they maximize a different profit function.\(^\text{12}\)

To complete this section, equilibrium profits may be potentially different between platforms. Which is the platform that receives the higher profit depends on the inherited number of subscribers, which affects switching in users side and, in turn, number of firms and total profits. We report below time 2’s equilibrium profits when \(n_E\) is kept as given:

\[
\pi^A_2(n_E) = \frac{90t^3(1+2(n_E-1)n_E)-18(t-a_E-a_F)(2a_E+a_F)(a_E+2a_F)}{18t}\n + \frac{t[81t^2+18t((n_E-1)a_E+(n_E-1)a_F)]}{18t}\n - \frac{(2a_E+a_F)(a_E(22+36n^2_2-42n_E)+a_F(35-66n_E+72n^2_2))}{18t}\n\]

\(^\text{12}\)Consider as an example the case in which \(n_E\) is very small, in particular smaller that \(\bar{n}_E\): in these case platforms should expect that switching occurs only from platform B to platform A, so that \(n^A_F = x^B\). The profit function to be maximized should change accordingly.
\[ \pi_2^B(n_{E1}) = \frac{90t^3(1+2(n_{E1}-1)n_{E1})-18(t-\alpha_E-\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{18t} \]
\[ + \frac{t[81t^2+18t((n_{E1}-5)\alpha_E)+(n_{E1}-4)\alpha_F]}{18t} \]
\[ - (2\alpha_E+\alpha_F)(\alpha_E(16+18n_{E1}^2-30n_{E1}))+\alpha_F(41-78n_{E1}+72n_{E1}^2)) \]

When we let platforms compete from period one by endogenizing \( n_{E1} \), then profits in (24) and (25) enter in the first time inter-temporal maximization problem of platforms. Indeed, they anticipate the effects on time 2 profits of their price setting in time 1. The inter-temporal equilibrium is provided in the following subsection.

### 4.2 Period one game.

This section is devoted to the analysis of the first period decisions. Specifically, we consider the second period equilibrium and we endogenize the number of subscribers \( n_{E1} \), which depends on price competition in time 1.

We assume customers in both sides to be myopic. An agent is said to be myopic if, when taking decisions in period \( t \), he only looks at that period outcomes, regardless the effects on time \( t+k \)’s utility. In our setting, a myopic customer simply decides which platform to join in time 1 according to the utility he gets in time 1, without taking into account that tomorrow he could switch to the rival, possibly enjoying a discounted price.

Following the notation of the second period game, we denote by \( n_{E1} \) (respectively \( n_{F1} \)) the user (resp. firm) indifferent between joining platform A and joining platform B. This thresholds are defined by:

\[ n_{E1} = \frac{1}{2} + \frac{\alpha_E}{2t} \left( n_{F1}^A - n_{F1}^B \right) + \frac{1}{2t} \left( p_{E1}^B - p_{E1}^A \right) \]  

(26)

\[ n_{F1} = \frac{1}{2} + \frac{\alpha_F}{2t} \left( n_{E1}^A - n_{E1}^B \right) + \frac{1}{2t} \left( p_{F1}^B - p_{F1}^A \right) \]  

(27)

Because of assumption A1, \( n_{E1}^A = n_{E1}, n_{E1}^B = 1 - n_{E1} \) and \( n_{F1}^A = n_{F1}, n_{F1}^B = 1 - n_{F1} \) are the total numbers of customers joining each platform in side E and F respectively. Putting together (26) and (27), we obtain the number of customers in each side depending only on prices:

\[ n_{E1} = \frac{1}{2} + \frac{\alpha_E \left( p_{F1}^B - p_{F1}^A \right)}{t^2 - \alpha_E \alpha_F} + t \left( p_{E1}^B - p_{E1}^A \right) \]  

(28)
As said before, first period prices chosen by platforms have an effect not only on current profits but also on second period profits, since the market share of period 1 determines whether platforms choose to offer discounted prices to rivals’ previous subscribers as well as whether switching may actually occur. Indeed, having a high number of previous subscribers today reduces the possibilities both to steal customers from the rival and to retain old customers overcoming the poaching attempted by the rival.

We consider only the maximization problem of platform A, since the problem is symmetric for B. Platform A sets prices for firms and users in order to maximize inter-temporal profits. Formally:

$$\max_{p_{E1}, p_{F1}} p_{E1} n_{E1} + p_{F1} n_{F1} + \delta \pi_A^2(n_{E1}(p_{E1}, p_{F1}, q_{E1}, q_{F1}))$$

From the first order conditions of this problem, we simply obtain the following first period equilibrium prices.

$$p_{E1}^A = p_{E1}^B = t - \alpha_F + \frac{\delta t(3t - 2\alpha_E - \alpha_F)(\alpha_E - \alpha_F)}{3\Omega}$$

$$p_{F1}^A = p_{F1}^B = t - \alpha_E$$

Because of the symmetry, in each side the price charged by both platforms turns out to be the same at equilibrium. Firms pay the same price that they would have paid if the price in users’ side had been uniform in the second period. In users’ side, the effect of BBPD is captured by the third term in (30). Comments on these results are provided in the following section.

Looking to the segments of the demands, since equilibrium prices are equal for both platforms, we obtain a perfectly split market in both sides, i.e. $n_{E1} = n_{F1} = \frac{1}{2}$.

Bringing this result to period two equilibrium, both platforms charge lower prices for new users and bi-directional switching occurs. Indeed, substituting $n_{E1} = \frac{1}{2}$ in the second period prices, platforms converge exactly to the following equilibrium prices:

$$p_{E2}^{AA} = p_{E2}^{BB} = \frac{2t}{3} - \alpha_F$$

$$p_{E2}^{AB} = p_{E2}^{BA} = \frac{t}{3} - \alpha_F$$

$$p_{F2}^A = p_{F2}^B = t - \alpha_E$$

22
These prices imply bi-directional switching and, since both platforms charge exactly the same prices, the number of users switching from $A$ to $B$ is the same as the number of switchers from $B$ to $A$. In particular, users laying on the interval $(\frac{1}{3}, \frac{2}{3})$ switch from platform $B$ to platform $A$ and agents in $(\frac{1}{3}, \frac{2}{3})$ switch towards the opposite direction.

Therefore $x^A_2 = \frac{1}{3}$ and $x^B_2 = \frac{2}{3}$. In firms’ side, nothing changes: since the total number of agents joining each platform in time 2 is still $\frac{1}{2}$, no switching in this side may happen and so $n^A_{F2} = n^B_{F2} = \frac{1}{2}$. Finally, the inter-temporal equilibrium profits for both firms are given by:

$$\Pi^A = \Pi^B = \Pi = \frac{9(2t-\alpha_E-\alpha_F)\Omega+(45t)^2-18(t-\alpha_E-\alpha_F)(2\alpha_E+\alpha_F)(\alpha_E+2\alpha_F)}{18t} + \frac{t(81(t)^2-18(4\alpha_E+5\alpha_F)-(2\alpha_E+\alpha_F)(13\alpha_E+17\alpha_F))\delta}{18t}$$

(33)

5 Discussion

The main purpose of this paper was to understand which are the effects of the combination of cross-group externalities with the implementation of a within group price discrimination strategy on competition and platforms’ profits. In particular, in a two period model in which second period strategies differ from and depend on first period ones, both ex-ante and ex-post competition may be affected because of the interplay between externalities and price strategies.

Indeed, externalities affect ex-ante competition because prices should take into account that bringing one firm (or one user) in the platform means also bringing some users (firms) and, consequently, they affect the possibility to poach users tomorrow (ex-post competition).

On the other hand, discrimination between old and new users affects ex-post competition because platforms compete fiercely to steal rival’s users and to retain their own. In turn, when platforms compete ex-ante they take into account the possible poaching tomorrow.

The way to investigate about these effects is to compare the within group price discrimination regime with the benchmark case in which platforms do not (or cannot) offer any differentiated price to users according to the past purchases. The results are summarized in the following proposition and we comment them with some intuitions in the course of this section.
Proposition 3. Consider the case in which 2 symmetric two-sided platforms compete in a two-period Hotelling segment for users on one side and firms on the other side. Suppose that two regimes can arise: within group uniform price regime and BBPD regime. Under assumptions A1, A2 and A3, if the customers are myopic, then:

1. first and second period prices in firms’ side are the same under both regimes.
2. second period prices for users are lower under the BBPD regime.
3. Depending from the externalities, first period prices for users are either lower or higher under the BBPD regime.
4. inter-temporal profits are unambiguously lower under the BBPD regime

Proof.

See appendix.

To understand what is going on in the model, it is worth to spend a few words on the pricing rule in two sided markets. As already hinted in section 3, cross-group externalities involve price discrimination among sides because each agent on one side is rewarded according to the number of agents on the other side he indirectly attracts by himself joining a platform. This reward is simply given by the externality parameter of the other side. Indeed, $F$ (respectively $E$) represents the number of firms (users) that follow the joining decision of a user (firm). Because of that, a user (firm) will pay a price lower than the transportation cost by an amount $F (E)$.

The main conclusion is that the price rule followed by the platforms is such that the low value group (the side exhibiting lower $\alpha$) is a loss leader or break even segment. This group is subsidized (or at least it enjoys a lower price) in order to attract the high value group (the side with $\alpha$ relatively high), which becomes the profit making segment for platforms. The key to compete in these markets is to charge one group with a very low price and recoup the losses by charging a high price to the side more interested in the interaction. The first quite intuitive result is that within group price discrimination in users’ side does not affect equilibrium prices charged to firms, both in time one and two.\footnote{This result holds once we assume that asymmetric equilibria in the first period do not arise. We follow the idea of Fudenberg and Tirole (2000), who consider only the cases in which the market is symmetric enough in their backward reasoning.}
time one, the result is quite obvious, since users are equally split between the two platforms. Thus, the price of both platforms should be the same as if we were in the benchmark model, since the strategy used is the same and the externality effect remains the same (users split between platforms exactly in number $\frac{1}{2}$ for each).

Moreover, the outcome is not so different in the second period. The number of users who switch from $A$ to $B$ is exactly the same as the number of users moving towards the opposite direction, keeping the total number of users joining each platform equal. Intuitively, the identity of users subscribing to a given platform changes, but the total number (what matters when pricing firms) remains the same. For these simple reasons, prices in firms’ side are the same in both regimes and reflect the general two-sided markets’ price rule just described above.

More interesting and puzzling are the effects on competition in the users’ side. In this side, ex-ante and ex-post competition are basically driven by two different effects, the poaching effect and the externality effect.

**(i) Poaching.** With poaching effect we refer to the fact that platforms compete fiercely in the second period, lowering prices to steal consumers from the rival’s inherited turf and/or to retain old consumers in their own turf. This strategy has a clear ex-post effect, while the impact on ex-ante competition is ambiguous.

Ex-post competition is very strong, since the incentives to steal some users to the rival as well as the fear to lose some others make prices go down, both for new and old users. This effect on prices and then competition is simply measured by the differences $t - \frac{4}{3}$ for switchers and $t - \frac{2}{3}$ for loyalists that we can observe comparing $\bar{p}_E$ with prices for users in (32). Since the level of prices is lower both for loyalists and switchers (and equal in the firms’ side) under within group price discrimination and the total number of agents in each side remains the same, second period profits will be lower.

As stated above, the effect of this strategy on ex-ante competition is ambiguous. Indeed, being aggressive in the first period price competition entails two different effects. On one hand, if a platform is aggressive in pricing users, then it can enjoy a high number of subscribers and consequently either attract a relatively high number of firms or charge them more.

On the other hand, this approach has some drawbacks: it reduces the likelihood both to attract new users and retain old users tomorrow. In particular, from the analysis of the second period switching and price behavior, we have an
overturn in the relative advantage in the number of subscribers. This negative
effect on future profits makes the competition ex-ante less strong.

Which one of the two forces on ex-ante competition prevails depends crucially
on the externalities of each side of the market, as we explain in point (ii).

(ii) Externality. With the externality effect we refer to the fact that a price
cut in one side of the market involves an effect on the number of joiners on the
other side. In the ex-post competition, it is clear that externalities have exactly
the same effects both in the BBPD regime and the benchmark case. They only
reduce the price for users by an amount equal to the externality parameter of the
firms’ side.

The effect on ex-ante competition depends on whether the users are the low
or the high value group. Looking at the equilibrium prices for users in (30) and
comparing them with the benchmark case, we can conclude that if users are the
high value group, first period prices are higher under within group discrimination
regime and then ex-ante competition is relaxed. On the other hand, when users
are the low value group, competition is intensified in the first period.

One question spontaneously arises: why the optimal first period price differ in
the two regimes? The key is the balance between pros and cons of being aggressive
in the first period competition discussed in point (i). Suppose that one platform
set the price as in the benchmark case. If users care relatively much about the
interaction with firms (more than how much firms care), the rival would have an
incentive to set a higher price because the gain tomorrow in future subscriptions
is higher than the loss today. On the other hand, if firms are the high value
group, platforms tend to be more aggressive.

The intuition behind is strongly linked to the concept of subsidizing/subsidized
segment typical of a two sided market: suppose that users are the high value
group, i.e. \( \alpha_E > \alpha_F \). In this case, users are attracted basically offering price cuts
to firms in order to have a critic mass of them and increase the willingness to pay
of users. Then, the basic strategy for a platform is to charge more users than
firms.

Moreover, the strategy of within group price discrimination in time 2 pushes
platforms to charge users even more. The most part of platforms’ profits are
made on users. In this side, platforms have another possibility to attract them
in the future.

Thus, externalities change the competition in the first period when platforms
use BBPD in users’ side. Suppose that one platform sets the benchmark price
in $\bar{p}_E$ in users’ side. Then, the rival best response turns out to be: (i) set a higher price for users (which do not impose a big loss on firms’ participation) with a lower number of subscribers but higher margins, (ii) set different prices and overturn the result in the second period.

On the other hand, if users are the low value group, the main aim of platforms is to attract a big number of users, subsidizing them and make profits on the firms’ side. Here, the incentive to attract a very high number of users is stronger than before, since attracting a high number of users also means to be very strong in the firms segment. Thus, price competition between platforms becomes aggressive.

Putting together these conclusions about prices, we can infer the effects on platforms’ profits. Ex-post competition clearly increases and profits decrease, since prices go down for all users and nothing changes in the firms’ side. This is the same result that is common in the literature of BBPD, e.g. Fudenberg and Tirole (2000). Ex-ante, two different instances may arise.

If price discrimination is used in the high value group, then competition is strengthened and first period profits are higher when platforms discriminate prices compared to the one that would have been attained under a within group uniform price.

So, we can conclude that when users are very interested in reaching firms, cross-group externalities emphasize the negative effects on inter-temporal profits of BBPD, decreasing also first period profits. If platforms discriminate among agents belonging to the low value group instead, cross-group externalities mitigate the negative effects on inter-temporal profits of BBPD, relaxing the first period competition. Although, even if the first period competition is relaxed, the negative total effect on profits is confirmed.

6 Conclusions

We have provided a model of two-sided platforms which compete in two periods for firms on one side and users on the other side of the market. We allow platforms to discriminate prices among users, according to the fact that BBPD is often used in subscription markets. The main finding of the analysis is that cross-group externalities do involve some effects on prices and competition when platforms discriminate prices in users’ side.
First of all, when we consider the second period keeping the first period inherited market share as given, externalities have an effect on the concrete possibility for two-direction switching to occur. In particular we can find that the stronger externalities are, the more likely is the case of two-direction switching, since the minimal (maximal) market share for two-directions switching to occur depends negatively (positively) on the externalities.

The natural next step of the analysis should be to investigate about under which conditions on parameters switching occurs one-direction towards one of the two platform and to compare these scenarios with the one of two-directions switching. In presence of externalities, the inter-temporal equilibrium found so far may be not unique, i.e. asymmetric equilibria may arise.

In the case analyzed so far, when we consider the two-period model as a whole, prices turn out to be the same for both platforms and both users and firms equally split between the two competing networks.

Along the equilibrium path, platforms face a prisoners’ dilemma in the second period. Each one alone has the incentive to offer discounted prices to rival’s previous users but if both of them do it, then they are both worse off. The general level of prices goes down under discrimination and each platform steals the same number of agents, leaving platforms with a lower level of profits in the second period.

In the first period, the general level of prices under discrimination may be either below or above the within group uniform price regime. In other words, the combination of cross-group externalities and within group price discrimination may either mitigate or intensify ex-ante competition.

Specifically, ex-ante competition is intensified when users are the low value group, mitigated otherwise. In the markets we have in mind, often users represent the the low value group. Medias market is the most telling example: advertisers care a lot about how many readers/viewers subscribe to a newspaper/TV while on the opposite direction the externality is clearly lower. For these reasons, medias make profits on advertisers, while end-users are subsidized. What the model predicts is that BBPD intensifies ex-ante competition, hurting firms and benefiting consumers even more in the first period.

Ambiguous in terms of which side exhibits the strongest externality is the instance of the subscription program offered by Amazon, which provides contents (video and reading) to the subscribers and access to demand for providers. Since subscribers are interested in contents, the effect of an additional provider entering the platform is substantial, because the number of videos and e-books available
increases. If the externality of subscribers is higher than the one of providers, according to our model ex-ante competition is relaxed when platforms use within group BBPD.

This paper aims to be a first attempt to study within group price discrimination in a multi-period setting. As a first tentative, it still has some weaknesses as well as some disappointing features of the equilibrium.

On one hand, in the group of users inherited leadership is always reversed in the second period when the inherited market is not perfectly split. This result is quite surprising in a two-sided market context, in which the early stages of competition are very critical. Indeed, attracting a high number of customers in both groups at the very beginning is often decisive to keep a strong market position in the future.

On the other hand, externalities do not play any role other than the usual "reward" for each side for the agents joining the platforms on the other side, which is nothing than the usual pricing rule in two-sided markets. No any other effect of externalities arises in second period prices, keeping the results of BBPD in one-sided markets.

It is reasonable to believe that seeking for asymmetric equilibria with switchers going only towards one direction may be a way to have different results. In particular, if these kinds of equilibria exist, externalities may well entail different consequences on second period profits.

Another important assumption which the paper is based on is that both firms and users are allowed to join at most one platform (single-homing). In fact, it is common that at least one side decides to bear price and transportation cost twice in order to be present in both platforms or, using the jargon of two-sided market literature, to multi-home.

As pointed out by Armstrong (2006), when agents are mostly interested in the interaction with the other group rather than the product offered by the platforms themselves, they may take the decision to join both platforms in order to meet the other side. The main result is that multi-homing relaxes price competition between platforms, because the multi-homing side exhibits a lower elasticity to price. In our setting, considering multi-homing firms may change the results on platforms’ profits, overturning the results of the standard one-sided BBPD literature. The intuition behind is simply that platforms could recover the losses in the users’ side by charging higher prices to firms, which are strongly interested

\footnote{Think for example of medias, which offer products (contents) that are basically not differentiated in the eyes of advertisers.}
in reaching the whole population of users. Accordingly, we may eventually end up with situations in which BBPD in the users’ side and the consequent increase in competition in this side is simply financed by firms.

The last point to notice is that the only case of myopic customers is analyzed in our model. As shown by Fudenberg and Tirole (2000) and Villas-Boas (1999) in one-sided markets, if customers are assumed to be forward looking the ex-ante competition is relaxed. This result depends on the fact that ex-ante customers’ elasticity is reduced because they know that tomorrow they can switch, enjoying a discounted price. In our setting, which already assumes deep rationality, it can be interesting to see how ex-ante competition is affected by the fact that both firms and end-users expect platforms to use within group price discrimination and take it into account when taking their ex-ante decisions.
References


### 7 Appendix

**Concavity conditions.** Under ASSUMPTION 3, the profits functions are 
strictly concave.

**Proof.** Let compute the first derivatives of the profit w.r.t. prices and the 
Hessian matrix.

\[
H = \begin{pmatrix}
\frac{\partial^2 \pi_E}{\partial p_E^2} & \frac{\partial^2 \pi_E}{\partial p_E \partial p_F} & \frac{\partial^2 \pi_E}{\partial p_F^2} \\
\frac{\partial^2 \pi_F}{\partial p_E^2} & \frac{\partial^2 \pi_F}{\partial p_E \partial p_F} & \frac{\partial^2 \pi_F}{\partial p_F^2}
\end{pmatrix} = \begin{pmatrix}
t^2 - \alpha_E \alpha_F & \frac{\alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} & \frac{\alpha_E + \alpha_F}{2t^2-4\alpha_E \alpha_F} \\
\frac{\alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} & \frac{\alpha_E + \alpha_F}{2t^2-4\alpha_E \alpha_F} & \frac{\alpha_E + \alpha_F}{2t^2-4\alpha_E \alpha_F}
\end{pmatrix}
\]

**Conditions for strict concavity**

1. Fundamental principal minor of order 1 should be negative

\[
-t^2 - \alpha_E \alpha_F \frac{t}{t(2t^2-4\alpha_E \alpha_F)} < 0 \quad \text{if } t^2 > 2\alpha_E \alpha_F \text{ and } t > 0
\]

2. Fundamental principal minor of order 2

\[
\left| \begin{array}{cc}
-t^2 - \alpha_E \alpha_F & \frac{\alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} \\
\frac{\alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} & -\frac{\alpha_E + \alpha_F}{t(2t^2-4\alpha_E \alpha_F)}
\end{array} \right| = \left( \frac{t^2 - \alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} \right)^2 - \left( \frac{\alpha_E \alpha_F}{t(2t^2-4\alpha_E \alpha_F)} \right)^2 > 0
\]

Satisfied if and only if \( t^2 - 2\alpha_E \alpha_F > 0 \)
3. Fundamental principal minor of order 3 (Hessian matrix)

\[
\begin{vmatrix}
-t^2 - \alpha_E \alpha_F \\
-\frac{t(2t^2 - 4\alpha_E \alpha_F)}{\alpha_E \alpha_F} \\
-\frac{\alpha_E + \alpha_F}{2t^2 - 4\alpha_E \alpha_F}
\end{vmatrix} = \frac{2(\alpha_E + \alpha_F)^2 - t^2}{8t(t^2 - \alpha_E \alpha_F)} < 0
\]

Satisfied if and only if ASSUMPTION A3: \( t^2 > 2(\alpha_E + \alpha_F)^2 \) holds.

**Second period maximization problem** Platform A solve the following maximization problem:

\[
\max_{p^{AA}_{E2}, p^{AB}_{E2}, p^{A}_{F2}} \quad p^{AA}_{E2} x^A_2 + p^{AB}_{E2} (x^B_2 - n_{E1}) + p^{A}_{F2} n_{F2}^A \quad (34)
\]

Using the first order conditions of this problem, we obtain the best response function of platform A, represented by prices \( p^{AA}_{E2}, p^{AB}_{E2}, p^{A}_{F2} \) in function of prices charged by the rival platform:

\[
p^{AA}_{E2} (p^{BB}_{E2}, p^{BA}_{E2}, p^{B}_{F2}) = \frac{4t^3 - 2t^2(\alpha_E + \alpha_F) + (\alpha_E + \alpha_F)(p^{BB}_{E2} \alpha_E - p^{BB}_{E2} \alpha_F + 4\alpha_E \alpha_F)}{8t^4 - 4(\alpha_E + \alpha_F)^2} + \frac{2t(\alpha_E(p^{BB}_{F2} - n_{E1} \alpha_E) - (p^{BB}_{F2} + 3\alpha_E) \alpha_F + (1 - n_{E1} \alpha_E) p^{BA}_{E2}(4t^2 - (\alpha_E + \alpha_F)(\alpha_E + 3\alpha_F)))}{8t^4 - 4(\alpha_E + \alpha_F)^2} \quad (35)
\]

\[
p^{AB}_{E2} (p^{BB}_{E2}, p^{BA}_{E2}, p^{B}_{F2}) = \frac{t^2(4(1 + \alpha_E) - 2t(\alpha_E + \alpha_F) + (\alpha_E + \alpha_F)(p^{BB}_{E2} \alpha_E - p^{BB}_{E2} \alpha_F + 4\alpha_E \alpha_F))}{8t^4 - 4(\alpha_E + \alpha_F)^2} + \frac{2t(n_{E1} \alpha_E - \alpha_E \alpha_F - 3\alpha_E + \alpha_F + 9\alpha_E \alpha_F(p^{BB}_{E2} \alpha_E - p^{BB}_{E2} \alpha_F + 4\alpha_E \alpha_F))}{8t^4 - 4(\alpha_E + \alpha_F)^2} \quad (36)
\]

\[
p^{A}_{F2} (p^{BB}_{E2}, p^{BA}_{E2}, p^{B}_{F2}) = \frac{3(1 - \alpha_E)(4t^2 - 2t^2 - 4\alpha_E \alpha_F) + 2t(\alpha_E + \alpha_F)(p^{BB}_{E2} \alpha_E - p^{BB}_{E2} \alpha_F - 4\alpha_E + 2\alpha_E \alpha_F)}{8t^4 - 4(\alpha_E + \alpha_F)^2} + \frac{2t^2((-1 + n_{E1}) \alpha_E - n_{E1} \alpha_F + 2p^{B}_{F2}(t^2 - \alpha_E(\alpha_E + \alpha_F)))}{4t^2 - 2(\alpha_E + \alpha_F)^2} \quad (37)
\]

**Proposition 3** Proof. It is easy to prove the first two results simply comparing prices in (31) and (32) with \( \bar{p}_F \) and \( \bar{p}_E \). In particular, second period prices in inside \( E \) become lower than under within group uniform price regime when platform discriminate among users. This is true both for new customers (for who discriminatory prices are lower by an amount \( \frac{2t}{3} \)) and for old customers (for who
the difference is \( \frac{t}{3} \)). For the third result, look at first period prices under BBPD regime in (30) and first period prices under uniform price. First period prices in side \( E \) are higher under BBPD regime if and only if
\[
(3t - 2\alpha_E - \alpha_F)(\alpha_E - \alpha_F) > 0,
\]
provided that \( \delta \frac{\delta}{\delta t} > 0 \) by concavity conditions and \( 3t > 2\alpha_E + \alpha_F \) by assumption A2. This inequality is verified only if \( \alpha_E > \alpha_F \), otherwise we have exactly the opposite result, i.e. first period prices for users are lower under the BBPD regime.

The fourth point of proposition is proved as follows. The difference between price discrimination profits in (33) and benchmark profits in (6) is given by
\[
\Pi - \overline{\Pi} = \frac{\delta t(-36t^2 + 9t(\alpha_E - \alpha_F) + (2\alpha_E + \alpha_F)(5\alpha_E + 19\alpha_F))}{18\Omega} \tag{38}
\]
Since denominator is positive by concavity conditions, the difference in (38) is positive if and only if:
\[
9t(\alpha_E - \alpha_F) - 36t^2 + (2\alpha_E + \alpha_F)(5\alpha_E + 19\alpha_F) > 0 \tag{39}
\]
First thing to notice is that LHS is decreasing in \( t \), thus a lower \( t \) makes it more likely to be fulfilled. It means that if condition (39) is not fulfilled even when we consider transportation cost high just enough to satisfy assumptions A1 and A2, then it is never satisfiable.

We consider two cases. If \( \alpha_E > \alpha_F \), then the minimum value of \( t^2 \) is \( 2(\alpha_E + \alpha_F)^2 + \epsilon \) while \( t > \alpha_E + \epsilon \). Substituting these lowest possible values we obtain
\[
-61\alpha_E^2 - 53\alpha_F^2 - 102\alpha_E\alpha_F < 0
\]
The same result arises when \( \alpha_F > \alpha_E \), which implies that \( t > \alpha_E + \epsilon \). Substituting these lowest possible values we obtain
\[
-62\alpha_E^2 - 54\alpha_F^2 - 101\alpha_E\alpha_F < 0
\]
Thus, condition (39) cannot be satisfied under assumption A1 and A2, profits are lower under the BBPD regime.

\[\blacksquare\]