Abstract

This paper tests whether the exchange rates of the Czech koruna, the Hungarian forint, and the Polish zloty were anchored by market expectations concerning their euro locking rates in the period from January 4, 2005, to March 6, 2007. First, the process of the exchange rate is derived as a function of the processes of the following three factors: the latent exchange rate, market expectations concerning euro locking rate, and market expectations concerning locking date. Then the market expectations concerning the locking date and euro locking rate are filtered from the yield curves and the time series of the exchange rates. The time-varying variances of the state variables are estimated from cross-sectional data on option prices. Here identification is ensured by the fact that the long end of the option term structure is influenced more by the variance of the locking rate. The main finding of the paper is that the relatively stable market expectations concerning the locking rates have substantial stabilizing effect on the exchange rates in the three analyzed countries.

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The views expressed are those of the authors and do not necessarily reflect the official view of the Magyar Nemzeti Bank. This disclaimer is particularly important in the case of the future euro locking rate of the Hungarian forint. The filtered market expectations concerning the euro locking rate do not necessarily coincide with the preferred euro locking rate of the Magyar Nemzeti Bank.

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Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?

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1 Introduction

This paper investigates the stabilizing feature of the market expectations concerning the euro locking rate. I apply the analysis to three countries, Czech Republic, Hungary and Poland, which are expected to join the Economic and Monetary Union (EMU) in a couple of years.

First, I extend the conventional asset-pricing exchange rate model with the final locking assumption. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks. Similarly, in the model with final locking the exchange rate is a linear combination of the fundamental and the market expectations concerning the euro locking rate. The relative weights of the linear combination depend on the market expectations concerning the locking date. The asset-pricing model with final locking is a three-factor model, where the factors are the market expectations concerning the locking rate and date. The third factor is the fundamental, which I will refer to as the latent exchange rate. It is defined as the exchange rate that would prevail if the currency was never going to be locked against the euro.

Then, in the empirical part of the paper, I filter out the three factors from the historical exchange rate, interest rate and currency option data. I estimate the expected date of locking from the euro and domestic yield curves by following the method suggested by Bates (1999). The Kalman filter technique is used to filter out the other two factors from the historical exchange rate data.

In order to Kalman filter the factors some parameters need to be estimated or calibrated. To estimate the time-varying variances of the innovations of the factors, I use a theoretical option pricing model derived in the paper and cross-sectional data on option prices with different maturities. The identification of the time-varying variances is based on the followings. Option prices both with long and short maturities are functions of the
variances of innovations of the latent exchange rate and the market expectation concerning the locking rate. However, the long end of the option term structure is influenced more by the variance of the market expectation concerning the locking rate.

By investigating the filtered market expectation concerning the euro locking rate I make inferences on the exchange rate stabilizing effect of the locking. I find that the market expectation concerning the euro locking rate is less volatile than either the historical exchange rate or the filtered latent exchange rate. This result has the important implication that market expectations concerning the locking rate have a stabilizing effect on the exchange rate in all three analyzed countries. Moreover, this stabilizing effect is substantial despite the uncertainty concerning the locking date.

Rather than filtering market expectations, I could, theoretically, have obtained the market expectations from an alternative source (Reuters polls) or with an alternative method (estimating equilibrium exchange rates). However, filtering has some advantages over its alternatives, for the reasons outlined below.

Reuters regularly surveys the expectations of analysts concerning the dates of EMU and ERM II entries and concerning the central parities in ERM II of all three analyzed countries. The central parity expected by respondents may be considered as the market expectations concerning the final conversion rate. And the reported expectations concerning either the date of EMU or ERM II entries may be considered as the market expectations concerning the locking date. Yet extracting market expectations from daily historical exchange rate and interest rate data may yield more accurate and more up-to-date information than the monthly or quarterly Reuters polls. Moreover, the higher frequency of the filtered expectations enables us to investigate the stabilizing effect of the prospect of locking has on the exchange rate.

As the EMU candidate countries aim at having their irrevocable conversion rates set equal to their equilibrium exchange rates, reliable estimates on the latter also reflect market expectations concerning the final conversion rate. In the context of our research this concept poses, however, at least three kinds of problems. First, economists use a number of concepts to define and a number of methods to estimate the equilibrium real exchange rate. Second, these estimates refer to the real rather than the nominal exchange rate. Third, market expectations might deviate from the estimated nominal equilibrium exchange rate, especially if the choice of the final conversion rate is based not only on observable fundamentals, but also on unobservables. For instance, if not only economic, but also political considerations do play a role in the negotiations over the locking rates. Another example is, when the locking rate is determined by the market as being set equal to the spot exchange rate just before the locking. In that case, speculative price pressure can push away the exchange rate from its equilibrium level and the bubble will never

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1. The frequency of these surveys are monthly for Hungary and quarterly for Czech Republic and Poland.

2. Egert, Halpern and MacDonald (2006) survey a number of issues relating to the equilibrium exchange rates of transition economies. They conclude that “deriving a precise figure for the equilibrium real exchange rates in general and also for the transition economies is close to mission impossible as there is a great deal of model uncertainty related to the theoretical background and to the set of fundamentals chosen.”

3. Williamson (1994) gives an overview of the widely used FEER, BEER, NATREX methods.

4. The personal view of the author is that the chance for the analyzed currencies to be subject to
burst due the locking.

The novelty of this paper is that it filters the subjective market expectation concerning the final conversion rate from exchange rate, interest rate and currency option data, which should mirror all the information that the market use at forming their expectations and at pricing bonds, options and currencies. This set of information is likely to be broader than the one that is used in the literature of estimating equilibrium exchange rate.

An important contribution of the paper to the theoretical exchange rate literature is that it finds empirical support for the conventional asset pricing exchange rate model. By comparing the filtered market expectations concerning the locking rate with the survey-based expectations I find that the difference between the two is small for all three countries. Further, if we consider the survey-based expectations to be unbiased estimates on the general view of the markets, then the model used for filtering should be judged successful as the filtered expectations are close to the reported expectations of the surveys. Moreover, the filtered locking rate for Hungary correlates somewhat more with the survey-based average expectations than the historical exchange rate. This finding can be interpreted, as the model helped to denoise the exchange rate, which is a rough proxy of the expected future locking rate. This test on the model is similar in spirit to the one applied by Engel and West (2005). They use the functional relationship of the model between the current fundamental and the discounted future fundamental on one hand and the exchange rate on the other hand to forecast future fundamental. They find weak evidence for forecasting ability, what can be interpreted as a general support for the conventional asset-pricing exchange rate models and also for this specific model.

The paper is structured as follows. Section 2 presents the exchange rate model. Section 3 derives an option pricing formula, which is utilized for parameter estimation in the empirical part of the paper. Section 4 shows some stylized facts supporting the stabilizing effect. Section 5 presents the applied method and results of filtering the factors. Section 6 concludes.

## 2 Exchange rate model

The exchange rate model is the conventional asset-pricing exchange rate model extended with the assumption of final locking. In the conventional asset-pricing model the exchange rate is the linear combination of the fundamental and the expected present discounted value of future shocks. Another extended version of the asset-pricing model is the target-zone model of Krugman (1991), which shows some similarities to this model.

First, in Krugman (1991) the target-zone exchange rate is derived from the fundamental. Similar to Krugman’s approach, our starting point is that the exchange rate subject to future locking is a function of the fundamental \( v \). Second, in Krugman’s model, the logarithm of the target-zone exchange rate is equal to the fundamental plus a term proportional to the conditional expected change of the logarithm of the exchange rate. Moreover,

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5 Our confidence on this result should depend, however, on how reliable are the survey data. The survey data usually tend to show systematic bias in the reported expectations as it has been documented by the early papers of Frankel and Froot (1987) and Froot and Frankel (1989).

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the exchange rate would be equal to the fundamental if there would be no target-zone. Similarly, in our model, the log exchange rate subject to future locking is equal to the fundamental plus a term proportional to the conditional expected instantaneous change of the log exchange rate. In the absence of future locking, the term proportional to the conditional expected instantaneous change of the exchange rate would be zero, so that the exchange rate would be equal to the fundamental. Given this relationship, I refer to the fundamental as the latent exchange rate, i.e. the exchange rate that would prevail if the currency was never going to be locked against the euro. Third, the implicit relationship between the exchange rate subject to future locking and the latent exchange rate in this model is the same as the relationship between the target-zone exchange rate and the fundamental in Krugman’s model. In general, this relationship between the exchange rate and the fundamental is common across all models with the asset-pricing approach to exchange rate. I can formulate this relationship as follows in a reduced form:

\[ s_t = v_t + c \frac{E_t(ds_t)}{dt}. \]  

Here, \( s_t \) is the log exchange rate, and \( v_t \) is the log latent exchange rate. The constant \( c \) is the time scale. The term \( E_t(ds_t) \) is the expected instantaneous change of the exchange rate. As I will derive it, the expected instantaneous change of the exchange rate depends on the log latent exchange rate \( v_t \), the market expectation concerning the log final conversion rate \( x_t \) and concerning the time of locking \( T_t \).

While Krugman investigates the stabilizing feature of the target-zone with a floating regime as a benchmark, I explore the stabilizing effect of future locking with the "no locking" as a benchmark.

In the following, I define the latent exchange rate as a function of some macro variables, as explained in footnote 6:

\[ v_t = -\alpha y_t + q_t + c \psi_t - p_{t^*}^t + m_t + c i_{t^*}. \]  

In this respect, \( y \) denotes the domestic real output, \( q \) is the real log exchange rate, \( \psi \) is the risk premium, \( p^* \) is the eurozone log price, \( m \) denotes the domestic nominal money supply and \( i^* \) is the euro interest rate.

As mentioned above, it is aimed to lock the exchange rates at their equilibrium levels. I use a concept of equilibrium exchange rate under which the strong law of purchasing power parity (PPP) holds for the locking rate, i.e. the log nominal exchange rate at the

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6Engel and West (2005) and Svensson (1991) presents the Money Income Model as one possible structural model for the reduced form (1):

(1f) \( m_t - p_t = \alpha y_t - c i_t \quad \alpha > 0 \quad c > 0 \) money market equilibrium

(2f) \( q_t = s_t + p_{t^*}^t - p_t \) real exchange rate

(3f) \( \psi_t = i_t - i_{t^*} - \frac{E(ds_t)}{dt} \) risk premium

(4f) \( v_t = -\alpha y_t + q_t + c \psi_t - p_{t^*}^t + m_t + c i_{t^*} \) fundamental/latent exchange rate.

7I consider two different types of expectations in the paper. One is the subjective market expectation, and the other is the mathematical expected value of a random variable. Here, I refer to the latter one. In order to distinguish between the two, I refer to the first type of expectation as the market expectation. However, under rational expectation the two are the same.
time of locking is equal to the difference between the domestic and eurozone log prices:
\[ s_T = p_T - p_T^*. \]
Under rational expectation the market expects the log final conversion rate at time \( t \) to be \( x_t = E_t(s_T) \), which gives
\[ x_t = p_t - p_t^* + \int_t^{T_t} E_t(\pi_\tau - \pi_\tau^*)d\tau. \]
(3)

Where \( \pi \) and \( \pi^* \) denote domestic and eurozone inflation rates respectively.

Neither the definitions (3) and (2) nor the corresponding macro data are used directly in the empirical part of the paper - mainly because of the low frequency of these data, but also because of a possible misspecification of the underlying macro models. For instance, the examples of the current EMU countries show that the locking rate can deviate from its PPP value. Still, it is the equilibrium real exchange rate that should be the key determinant of the locking rate chosen on the basis of economic considerations. Therefore, while not used directly in the rest of the paper, these definitions motivate the calibration of the model, the choice of the processes of the underlying factors and the interpretation of the results.

2.1 Dynamics

In this subsection I specify the processes of the factors. These processes will be used to derive the process of the exchange rate.

I start by assuming that the factors follow Brownian motions. This assumption can be decomposed into an assumption on the martingale property of the processes and into the Gaussian distribution of the innovations. The Gaussian distribution of the innovations is assumed for technical reason. The martingale property of these processes, however, can be easily explained:

- Under rational expectation, the expectation of the market participants concerning the log final conversion rate is the expected value of the log final conversion rate given all the information available at the time the expectation is formed by the market \( (x_t = E_t(s_T)) \).

And also the market expectation concerning the time of locking is the expected value of the true time of locking \( T^* \) given all the information available at the time the expectation is formed \( (T_t = E_t(T^*)) \). The law of iterated expectations implies that the process of both \( T_t \) and \( x_t \) are martingales, since \( E_t(E_{t+1}(s_T)) = E_t(s_T) \) and \( E_t(E_{t+1}(T^*)) = E_t(T^*) \).

- The martingale property of the process of the log latent exchange rate can be derived from the Money Income Model under the assumption that the right hand side variables of equation (2) have martingale processes. The martingale property of \( v_t \) allows us to focus entirely on the dynamics caused by the future final locking, as opposed to the effects of predictable future changes in the latent exchange rate.

The process of the market expectations concerning the log euro locking rate \( x_t \) can be derived from equation (3). In a discrete time framework it is
\[ \Delta x_t = [\pi_{t+1} - E_t(\pi_{t+1})] + \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i) - E_t(\pi_i)] - [\pi_{t+1}^* - E_t(\pi_{t+1}^*)] - \sum_{i=t+2}^{T_t} [E_{t+1}(\pi_i^*) - E_t(\pi_i^*)]. \]
If we assume that both the expectation errors and the change of expectations are independent and normally distributed with zero mean then this process of \( x_t \) can be rewritten
in a continuous time framework as
\[ dx_t = \begin{cases} \sigma_{x,t} dz_{x,t} & \text{if } t < T_t \\ 0 & \text{otherwise} \end{cases} \]
(4)

where \( dz_{x,t} \) is a Wiener process.

Second, if the right hand side variables of equation (2) follow Brownian motion \(^8\), then so does \( v_t \). Hence, its process can be written as
\[ dv_t = \sigma_{v,t} dz_{v,t} \]
(5)

Where \( dz_{v,t} \) is a Wiener process, which is not necessarily independent from \( dz_{x,t} \). I assume linear, contemporaneous relationship between the two shocks, what can be captured by their time-varying correlation denoted by \( \rho(dz_{v,t}, dz_{x,t}) \).

The third factor, the market expectation concerning locking time \( T_t \) is modeled as a stochastic variable. In the empirical part of the paper I filter the factors not only with stochastic \( T_t \), but also with constant \( T_t \) in order to see to what extent does the dynamics of \( T_t \) influence the dynamics of the other factors. One can think of the constant locking time as having stochastic locking time with volatility \( \sigma_{T,t} \) restricted to zero. In the followings, I pose three restrictions ((6), (7), (8)) on the stochastic process of \( T_t \), which do not apply in the constant case.

The assumed process of the market expectations concerning locking time is the following martingale,
\[ dT_t = \begin{cases} (T_t - t) \sigma_{T,t} dz_{T,t} & \text{if } t < T_t \\ 0 & \text{otherwise} \end{cases} \]
(6)

Where \( dz_{T,t} \) is a Wiener process which may correlate with the other two shocks \( dz_{v,t} \) and \( dz_{x,t} \).

Along the following lines one can make restrictions on the correlations.

In this model it can be easily detected whether the inflationary or the real criteria of the Maastricht Treaty are more binding for an EMU accession country. In case of having higher \( v_t \) than \( x_t \), then one can conclude that it is the poorly performing real economy what can be blamed for a later EMU entry. Whereas in the opposite case, a later EMU entry is also due to the high current and future expected inflation. Consequently, the difference between the covariances \( Cov(dT_t, dv_t) - Cov(dT_t, dx_t) \) should be monotone increasing function of the difference of \( (v_t - x_t) \). Accordingly, I pose the following linear restriction \(^9\):
\[ Cov(dT_t, dv_t) - Cov(dT_t, dx_t) = B(v_t - x_t) \]
(7)

\(^8\)The assumption that the growth rate of GDP and the real appreciation rate are normally distributed follows from two additional equations of the model. One is the supply curve equation and the other captures the Balassa-Samuelson effect:

(5f) \( y_t - y_{t-1} = \beta(\pi_t - E_{t-1}(\pi_t)) \) \( \beta > 0 \) supply curve
(6f) \( dq_t = -\gamma dy_t \) \( \gamma > 0 \) Balassa-Samuelson effect (real appreciation).

The growth rate and the real appreciation rate should have normal distribution, because both are linear functions of the normally distributed expectation error \( \pi_t - E_{t-1}(\pi_t) \).

\(^9\)Equation (7) can be rearranged to get \( \sigma_{T,t} \) as a function of correlations \( \rho \)'s.
\[ \sigma_{T,t} = \frac{c}{(T_t - t)^{v_t-x_t}} \left[ \sigma_{v,t} \rho(dz_{T,t}, dz_{v,t}) - \sigma_{x,t} \rho(dz_{T,t}, dz_{x,t}) \right]. \]

7
where B should be non-negative. I set \( B = \frac{\sigma^2_{T,t}(T_t-t)}{c} \).

My primary motivation for choosing a linear restriction of (7) and for setting \( B \) as above is to get a closed form solution for the function \( s_t = f(t, v_t, x_t, T_t) \). As we will see in the next Subsection 2.2, the function \( s_t = f(t, v_t, x_t, T_t) \) can be expressed by an easily interpretable, closed form formula under restriction (7).

The choice for parameter \( B \) can also be motivated by some economic considerations along the following lines. If the numerator of \( B \) is zero then there is no uncertainty concerning the locking time, either because the country is just about to lock its exchange rate \( T_t = t \) or no future shock can modify the expectations \( \sigma^2_{T,t} = 0 \). If the numerator is positive, then there is chance for any of the Maastricht criteria to be binding in the future, i.e. it can not be ruled out that the locking will be postponed due to the failure of the country to meet the criteria in time. Moreover, the higher is the uncertainty relating the time of locking \( \sigma^2_{T,t} \) or the longer is the time until locking \( (T_t - t) \) the Maastricht criteria are more likely to be binding at a given horizon. If \( \sigma^2_{T,t}(T_t-t) > 0 \) and \( v_t > x_t \) \((v_t < x_t)\) then the real (inflationary) criteria are more difficult for the country to meet. In that case, the shocks to \( v_t \) \((x_t)\) should have a stronger effect on the market expectation concerning the locking date, than the shocks to \( x_t \) \((v_t)\). Moreover, the higher \(^{10}\) is \( c \), the more effective is the monetary policy at boosting the economy at the expense of higher inflation or at disinflating by sacrificing real growth. A highly effective monetary policy can contribute to the most binding criterion to be easier meet.

Besides equation (7), I pose another restriction on the correlations. I assume that there is no such shock that affects both \( v_t \) and \( x_t \) but not \( T_t \), because \( v_t \) and \( x_t \) are interrelated through \( T_t \) as the correlation between nominal and real shocks helps to make the more binding Maastricht criteria to earlier meet. Or in other words, shocks to \( x \) and \( v \) that are orthogonal to \( dz_{T,t} \) are assumed to be independent from each other as well. This assumption can be formalized as

\[
\rho(dz_{x,t}, dz_{v,t}) = \rho(dz_{T,t}, dz_{v,t}) \rho(dz_{T,t}, dz_{x,t})
\]

(8)

### 2.2 Functional relationship between the exchange rate and underlying factors

Here, I derive the functional relationship between the exchange rate on the one hand and the latent exchange rate, the market expectations concerning the locking time and locking rate on the other hand. Subsection 2.2.1 presents the derivation under the assumption of having constant locking time. Then I release this assumption in Subsection 2.2.2 to derive the function in the general case with stochastic locking time.

The derivation has the following two steps in both cases with constant and stochastic locking time. First, I derive the process of the log exchange rate \( s_t \) from the processes of the factors by using Ito’s stochastic change-of-variable formula. Second, I obtain that the only function satisfying the derived process and the terminal condition \( s_{T^*} = x_{T^*} \) and equation (1) is given by

\[
s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{T_t-t}{c}}\right) v_t + e^{-\frac{T_t-t}{c}} x_t
\]

(9)

\(^{10}\)A higher \( c \) parameter can be interpreted in the Money Income Model as having higher interest rate semi-elasticity of money demand.
2.2.1 Constant locking time

Here I assume that the locking time \( T \) is constant. According to Ito’s formula, the function \( f(t, v_t, x_t, T) \) should satisfy (10).

\[
\begin{align*}
\frac{df}{dt} &= \left[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t} \mu_{v,t} + \frac{\partial f}{\partial x_t} \mu_{x,t} + \frac{\partial^2 f}{\partial t^2} \mu_{T,t} + \frac{\partial^2 f}{\partial v_t^2} \sigma_{v,t}^2 + \frac{\partial^2 f}{\partial x_t^2} \sigma_{x,t}^2 + \frac{\partial^2 f}{\partial t \partial v_t} \right] dt + \\
&\quad + \frac{\partial f}{\partial v_t} \sigma_{v,t} dz_{v,t} + \frac{\partial f}{\partial x_t} \sigma_{x,t} dz_{x,t}.
\end{align*}
\]

The different \( \mu \)’s denote the drift terms, whose values are zero in equations (4) and (5).

At time \( T^* \) the exchange rate \( s_{T^*} \) is equal to the market expectation concerning the final conversion rate \( x_{T^*} \), because at the time of locking the market already knows the final conversion rate. Consequently, the function \( f(t, v_t, x_t, T) \) should satisfy the terminal condition

\[
f(T^*, v_{T^*}, x_{T^*}, T) = x_{T^*}.
\]

The solution is given by (9). The proof is provided in the general case with stochastic locking time in Appendix A.

2.2.2 Stochastic locking time

Here I assume that the locking time \( T_t \) is stochastic and its process is given by (6). The function \( f(t, v_t, x_t, T_t) \) is derived under the assumption of stochastic locking time similarly to the deterministic case. The solution is again given by (9), however, this finding depends on the exact form of the restriction on the covariances. The restriction on the covariances (7) was chosen so to have exactly the same solution with stochastic locking time as with constant one.

The Ito’s formula can be used again to find the function \( f(t, v_t, x_t, T_t) \). By using Ito’s stochastic change-of-variable formula, we will get a similar expression for \( df \) as previously with constant locking time, however some new terms appear in the formula.

\[
\begin{align*}
\frac{df}{dt} &= \left[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial v_t} \mu_{v,t} + \frac{\partial f}{\partial x_t} \mu_{x,t} + \frac{\partial^2 f}{\partial t^2} \mu_{T,t} + \frac{\partial^2 f}{\partial v_t^2} \sigma_{v,t}^2 + \frac{\partial^2 f}{\partial x_t^2} \sigma_{x,t}^2 + \\
&\quad + \frac{\partial^2 f}{\partial T_t^2} \sigma_{T,t}^2 \right] dt + \\
&\quad + \frac{\partial f}{\partial v_t} \sigma_{v,t} dz_{v,t} + \frac{\partial f}{\partial x_t} \sigma_{x,t} dz_{x,t}.
\end{align*}
\]

The different \( \mu \)’s denote the drift terms, whose values are zero in equations (4), (5) and (6).

I obtain again that the only function satisfying the derived process (12), the terminal condition (11) and the equation (1) borrowed from Krugman (1991) \(^{11}\) is given by (9).

\(^{11}\)Similarly to the solution in the Krugman (1991) paper, equation (9) also satisfies a smooth pasting condition \( \frac{dx_{T^*}}{dt} = 0 \).
Equation (9) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights are changing over time; if the time until locking is infinite, or in other words, if the currency will never be locked, then the weight of the latent exchange rate is one, and the weight of the expected final conversion rate is zero. As the time until locking decreases, the weight of the expected final conversion rate increases. Finally, as the time until locking approaches zero, the weight of the expected final conversion rate approaches one.

In order to examine the exchange rate dynamics of the model, I substitute (9), (4), (5) and (6) into equation (12).

\[
\begin{align*}
    ds_t &= \frac{1}{c} e^{-\frac{T_t}{c}} (x_t - s_t) dt + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} dz_{v,t} + \frac{1}{c} e^{-\frac{T_t}{c}} \sigma_{x,t} dz_{x,t} - \frac{1}{c} e^{-\frac{T_t}{c}} (x_t - s_t) (T_t - t) \sigma_{T,t} dz_{T,t}.
\end{align*}
\]

Equation (13) shows that the dynamics of the exchange rate is such that it converges to the actual market expectation concerning the final conversion rate. Moreover, the closer the time of locking, the faster the convergence is.

Equations (4), (5), (6) and (9) define a three-factor model. One factor is the market expectation concerning the final conversion rate; another factor is the market expectation concerning the time of locking; the third factor is the latent exchange rate.

### 3 Option pricing

In this section I show a pricing formula for European-type options. The pricing formula is consistent with the exchange rate model with constant locking time and factor volatilities. Although not being fully consistent with the stochastic locking time, the pricing formula is used also in the general framework with stochastic locking time to estimate parameters. The parameters to be estimated are the variances of the innovations of the factors \(v_t\) and \(x_t\). The historical option prices are given in terms of implied volatility; consequently, I derive the pricing formula in terms of volatility as well.

In the theoretical model the uncertainty is present due to the stochastic innovations \(dz_{v,t}, dz_{x,t}, dz_{T,t}\) of the factors; consequently, the price of an option is a function of the variances and covariances of these normally distributed innovations. From equation (13) and (9), we can derive, that the instantaneous variance of returns at time \(t\) is

\[
\begin{align*}
    \sigma_{s,t}^2 &= \left(1 - e^{-\frac{T_t}{c}}\right)^2 \sigma_{v,t}^2 + \left(e^{-\frac{T_t}{c}}\right)^2 \sigma_{x,t}^2 + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \sigma_{x,t} T_t (T_t - t) \sigma_{T,t}^2 + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \rho (dz_{v,t}, dz_{x,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \rho (dz_{T,t}, dz_{v,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{x,t} \rho (dz_{T,t}, dz_{x,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \rho (dz_{v,t}, dz_{x,t}, dz_{T,t}) + \\
    &+ 2 \left(1 - e^{-\frac{T_t}{c}}\right) \left(e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \sigma_{x,t} T_t (T_t - t) \sigma_{T,t}^2 + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \rho (dz_{v,t}, dz_{x,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \rho (dz_{T,t}, dz_{v,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{x,t} \rho (dz_{T,t}, dz_{x,t}) + \\
    &- 2 \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} (x_t - v_t) (T_t - t) \sigma_{T,t} \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} \rho (dz_{T,t}, dz_{v,t}) + \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{x,t} \rho (dz_{T,t}, dz_{x,t}) + \\
    &- 2 \left(1 - e^{-\frac{T_t}{c}}\right) \sigma_{v,t} (x_t - v_t) (T_t - t) \sigma_{T,t} \sigma_{x,t} \rho (dz_{T,t}, dz_{x,t}).
\end{align*}
\]
In order to derive a closed form option pricing formula, I work with the simple constant volatility model. I assume that the covariance matrix of the three shocks is constant over the life of the option. Until now, I allowed $\sigma_{v,t}$, $\sigma_{x,t}$ and $\sigma_{T,t}$ and the correlations to change over time. While I do not rule out the possibility, that the covariance matrix used to price option can vary over time, I do not model the dynamics of the covariance matrix. Obviously, the price of options in the stochastic volatility framework, is different from the one of the constant volatility framework, however the latter is a good approximation for the theoretical value of at-the-money options with a maximum of one-year maturity.\footnote{As it is pointed out by Hull (1997, p. 620): "For options that last less then a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases."}

In order to have a closed form option pricing formula, I fix not only the covariance matrix over the life of the option, but also the market expectation concerning the locking date $T_t$.

By applying this final simplification and by calculating the integral we obtain the option pricing formula

$$\begin{align*}
g(t, m, \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho (dz_{v,t}, dz_{x,t}), \rho (dz_{T,t}, dz_{x,t}), \rho (dz_{T,t}, dz_{v,t})) &= \left[ \int_t^{t+m} \sigma_{s,t}^2 d\tau \right]^{\frac{1}{2}} = \\
&= \sigma_{v,t}^2 m + \Gamma_1 \left[ \sigma_{v,t}^2 + \sigma_{x,t}^2 - 2\rho (dz_{v,t}, dz_{x,t}) \sigma_{v,t} \sigma_{x,t} + \right. \\
&\left. - \rho^2 (dz_{T,t}, dz_{x,t}) \sigma_{v,t}^2 - \rho^2 (dz_{T,t}, dz_{x,t}) \sigma_{x,t}^2 + 2\rho (dz_{T,t}, dz_{x,t}) \sigma_{x,t} \rho (dz_{T,t}, dz_{v,t}) \sigma_{v,t} + \right. \\
&\left. + \Gamma_2 \left[ -2\sigma_{v,t}^2 + 2\rho (dz_{v,t}, dz_{x,t}) \sigma_{v,t} \sigma_{x,t} + 2\rho^2 (dz_{T,t}, dz_{x,t}) \sigma_{v,t}^2 + \right. \\
&\left. - 2\rho (dz_{T,t}, dz_{x,t}) \sigma_{x,t} \rho (dz_{T,t}, dz_{v,t}) \sigma_{v,t} \right] \right)^{\frac{1}{2}}. \quad (15)
\end{align*}$$

The option is sold at time $t$ and the maturity of the option is denoted by $m$. The $\Gamma_1 = \frac{c}{2} e^{-\frac{1}{2}(T_t - t - m)} - \frac{c}{2} e^{-\frac{1}{2}(T_t - t)}$ and the $\Gamma_2 = ce^{-\frac{1}{2}(T_t - t - m)} - ce^{-\frac{1}{2}(T_t - t)}$.

4 Stylized facts on the stabilizing effect

This section investigates the stabilizing effect of locking by looking at some stylized facts on the term structure of options and by comparing estimated constant factor volatilities.

The stabilizing effect of locking can be detected by comparing the volatility of the exchange rate with future locking $\sigma_s$ on the one hand and the volatility of exchange rate without locking $\sigma_v$ on the other hand. Obviously, if the former is smaller than the latter, then we can infer that the prospect of locking stabilizes the exchange rate.

Under the assumption of having constant locking time and independent factors with constant volatilities, the instantaneous variance of the exchange rate $\sigma_{s,t}^2$ is the weighted
average of the variances of the factors $x$ and $v$ (see equation (14)). Consequently, it is sufficient to show $\sigma_v > \sigma_x$ in order to prove the existence of stabilizing effect. It is important to notice, that the condition $\sigma_v > \sigma_x$ is sufficient only under the highly restrictive assumptions of having constant locking time and independent factors with constant volatilities. If these restrictive assumptions are not fulfilled, then it is possible to have no exchange rate stabilization $\sigma_s > \sigma_v$ despite of having more volatile latent exchange rate than locking rate $\sigma_v > \sigma_x$. For instance, high uncertainty relating the locking time can increase the volatility of the exchange rate with future locking even to be more than the volatility of the latent exchange rate.

In the next sections, these restrictive assumptions will be released and the magnitude of the stabilizing effect will be estimated by controlling for the uncertainty of the locking date, the dynamics of the market expectation concerning the locking date, correlations between the factors, and time-varying volatilities of the factors.

Unlike next sections, here it is assumed to have constant locking time and independent factors with constant volatilities. The advantage of this framework, is that it is consistent with the option pricing formula (16), therefore estimates on the factor volatilities are not subject to bias due to model misspecification. Moreover, even the term structure of implied volatilities can provide us insight into the stabilizing feature of locking. In this framework the stabilizing effect is analyzed by applying two simple approaches. First, we look at the term structure of options. Second, we compare estimated volatilities of the factors.

In this framework, the option pricing formula (15) reduces to

$$ g(t, m, \sigma_x, \sigma_v) = \left( \sigma_v^2 (m + \Gamma_1 - 2\Gamma_2) + \sigma_x^2 \Gamma_1 \right)^{\frac{1}{2}}. \tag{16} $$

The $\Gamma_1 = c e^{-\frac{1}{2}(T-t-m)} - \frac{c}{2} e^{-\frac{1}{2}(T-t)}$ and the $\Gamma_2 = ce^{-\frac{1}{4}(T-t-m)} - ce^{-\frac{1}{4}(T-t)}$.

The option pricing formula (16), but also our intuition suggest that longer options are more exposed to shocks occurring in the far future than options with shorter maturities. Or in other words, $\sigma_x$ has higher relative weight in a longer option, then in a shorter one. And the opposite holds for $\sigma_v$. Consequently, if the term structure of options is downward sloping, then $\sigma_v > \sigma_x$. Therefore, a decreasing term structure can be interpreted as evidence for the stabilizing feature of locking. This relationship between the term structure and the stabilizing effect is not an if and only if relationship, because even if the term structure is upward sloping, it is possible to have $\sigma_v > \sigma_x$.

Table 1 shows the average implied volatilities for each of the six maturities for the three countries. The six options are ATM options and have different maturities. In case of Czech koruna and Polish zloty the maturities are one-month, two-months, three-months, six-months, nine-months and one-year. Whereas in case of the Hungarian forint the currency options have one-week, one-month, two-months, three-months, six-months and one-year m(6) maturities.

The average term structure of options is clearly downward sloping for Czech koruna and Polish zloty, but not for Hungary. In order to have a more detailed picture on the Hungarian term structure, I divide the sample into two equal sized subsamples and calculate the average implied volatilities for the two subsamples separately. In the second half of the sample, the Hungarian term structure is downward sloping as well, but not in the first one.
All in all, the average term structure of these countries suggest, that the prospect of locking has stabilized the Czech koruna and Polish zloty, and also the Hungarian forint in the second half of the sample. Purely based on the investigation of the term structure, we can not rule out, that locking have stabilized the forint even in the first half on the sample. In order to judge the stabilization in that period and to make inferences on the magnitude of the stabilizing effects, I present estimates on the factor volatilities.

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Table 1: The term structure of implied volatilities

The option pricing formula (16) and historical option prices can be used to estimate the constant volatilities $\sigma_v$, $\sigma_x$ of the factors. I assume that the pricing errors, i.e. the difference between the historical prices and the theoretical prices, are independent identically distributed (IID) with Gaussian distribution. Under this assumption, the volatilities $\sigma_v$, $\sigma_x$ can be estimated by maximum likelihood method. The constant market expectation concerning the locking date $T$ is calibrated to 1.Feb.2008, 13.Feb.2010 and 8.Jun.2010. for Czech Republic, Hungary and Poland respectively. The calibration is based on the filtered $\hat{T}_t$ where $t_0 = 5.Jan.2005.$ is the beginning of the sample (see Subsection 5.1 on the filtering of $T_t$).

Table 2 presents the estimated factor volatilities. These point estimates support the existence of the stabilizing effect in all three countries as having $\hat{\sigma}_v > \hat{\sigma}_x$. Moreover, the differences between the volatilities are significant. Based on the likelihood ratio test, we can reject the null that the volatilities are equal at 10% significance level for each country and for both periods for Hungary. And we can also reject the null even at 1% significance level in three cases out of the four.

This section have shown some empirical evidences on the stabilizing effect under the highly restrictive assumptions of having constant locking time and independent factors with constant volatilities. In the followings, I will release these assumptions and investigate whether our findings on the stabilizing effect remains still valid even under more general conditions.
Table 2: The estimated constant volatilities of the latent exchange rate $v$ and the market expectation concerning the locking rate $x$ – with independent factors and constant locking time

5 Filtering factors

I apply the Kalman filter technique to extract the time series of the factors from the time series of the observable exchange rate. However, filtering all three factors from only one series would be overambitious and unlikely to provide robust results. Moreover, the Kalman filter technique can be applied only if the model is linear in all the factors to be filtered. In fact, the log exchange rate $s_t$ is linear in two of the factors, namely the latent exchange rate $v_t$ and the market expectation concerning the final conversion rate $x_t$, but not in the expected date of locking $T_t$. However, the expected date of locking $T_t$ can be filtered independently from the other two factors.

The next Subsection 5.1 introduces the applied method and the results of filtering $T_t$. Then, I set up the filtering problem for the remaining two factors in Subsection 5.2. Subsection 5.3 describes how the parameters of the filtering problem are set. Finally, Subsection 5.4 presents the filtered market expectation concerning the locking rate $x_t$ and its estimated stabilizing effect on the exchange rate.

5.1 Filtering the expected locking date

The expected date of locking can be estimated independently from the other two factors from the euro and domestic yield curves. The yield curve method has been popularized by Bates (1999) and it has been applied for Hungary by Csajbók and Rezessy (2006). The yield curve method is based on the fact that after adopting the euro, the interest

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13To filter all three factors from the time series of the exchange rate, one should apply a different technique than the Kalman filter, because the model is not linear in $T_t$. The Extended Kalman filter and the particle filter are possible candidates.
rate differential will become marginal, but not before. The higher probability is attached by the market participant to the scenario that a country is already a full member of the eurozone by a given year, the lower is the forward differential for that particular year. Along these lines the market expectation on the date of locking can be estimated from the forward differentials. Appendix B describes the method in details.

Figure 1 shows the smoothed and the original filtered market expectations concerning the locking dates. The smoothed time series is the moving average of the original time series where the window size is 21. In the followings, I consider the smoothed time series to be the filtered $\hat{T}_t$, because it is cleaned from daily noises.

The Reuters polls queries the analysts about their opinion on the most likely year of EMU and ERM II entry of the three analyzed countries. These are the only references for the filtered $T_t$ to be compared with. Consequently, I put also these survey data on Figure 1. The filtered time series and the survey-based one show similar trends. One can see that analysts expectations were relatively stable until the autumn of 2005 and have changed subsequently between the quarterly polls of August and November for the Czech Republic and Poland, and between September and October 2005 for Hungary. Until the autumn of 2005 the three countries had been expected to enter ERM II in the year 2007. Thereafter, the expectations changed dramatically, as reflected by the monthly and quarterly Reuters polls, pointing to a postponement of ERM II entry to 2008 for the Czech Republic, to 2009 for Poland, and to 2010 for Hungary. Similar trend can be detected in the expected EMU entry date. The filtered locking dates suggest that expectations were subject to both positive and negative shocks during 2006 and the first quarter of 2007 in all three countries. The positive shocks shifting the expected locking date of Hungary and Poland substantially upwards have started in July 2005 according to the filtered locking date, whereas the survey-based expectations have changed only a few months later. This finding suggests that the estimates on the expected locking date provide us earlier warnings on the changes in the market sentiment than the surveys. In the first half of 2006 the filtered expected locking dates have downward trends in Hungary and Poland, which is either due to some favorable news on the EMU outlook, or due to a correction of the previous overshooting of the expectations.

The locking of the exchange rate can precede the EMU entry, but not vice versa. For instance, the exchange rates of almost all the countries that entered the ERM II system in recent years are almost fixed: the volatility of the Estonian kroon, the Lithuanian lita, the Slovenian tolar, the Cyprus pound and the Maltese lira dropped below 1% after entering the ERM II regime. These facts implies that practically locking does not necessarily take place when a country enters the monetary union, but rather when it enters the ERM II regime. The filtered locking date of the Czech Republic has fluctuated between the narrow band given by the survey-based expected EMU and ERM II entry dates, which is in accordance with the sequence of locking and EMU entry. Whereas in case of Hungary and Poland the survey-based expected EMU entry dates precede sometimes the yield curve based locking dates. Or in other words, the survey-based expectations are more optimistic relative to the yield curve based estimates. Since I have more confidence in the filtered expectations then in the survey-based expectations, I attribute the difference to systematic bias in the survey data. This systemic bias can be

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14The Estonian kroon, the Lithuanian lita and the Slovenian tolar joined ERM II on 27 June 2004. On 2 May 2005 three other Member States joined ERM II: Cyprus, Latvia and Malta.
explained, for instance, with the skewed distribution of the entry date. If market analysts report the most likely entry date of a country, i.e. the mode of the distribution, instead of the expected value and if the subjective distribution of the entry date is skewed towards a later entry date, then the survey-based expectations are downward biased estimates on the expected locking dates.

Once we have estimates on $T_t$, we can filter the other two factors from the historical exchange rate by Kalman filter. In the Kalman filter I treat $T_t$ as being exogenously given.

5.2 Filtering problem

Here, I set up the filtering problem for $x_t$ and $v_t$ under the general case of modeling $T_t$ as a stochastic variable. I do not derive the filtering problem for the case with constant locking time separately, because it is just a special case of the general one with the zero restriction for the correlation parameters and $\sigma_{T_t}$. Under the assumption of constant locking time, the shocks to the factors are independent, since the parameters $\rho(dz_{T,t},dz_{x,t})$ and $\rho(dz_{T,t},dz_{v,t})$ are zero and so is $\rho(dz_{v,t},dz_{x,t})$ due to restriction (8).

In contrast to the case with constant locking date, the shocks to the factors do correlate under the assumption of having stochastic locking date. Since the third factor $T_t$ is exogenous and it is not independent of the other two factors, I have to use the conditional distributions of $x_t$ and $v_t$, where I condition on the realization of $T_t$. The conditional expected innovations of $x_t$ and $v_t$ are $\sigma_{x,t}\rho(dz_{T,t},dz_{x,t})dz_{T,t}$ and $\sigma_{v,t}\rho(dz_{T,t},dz_{v,t})dz_{T,t}$ respectively. The expected values of the innovations are taken into account in the model by having a constant as a third state variable. Moreover, the system covariance matrix $Q(t)$ is also conditional on $T_t$.

The filtering problem can be written in the usual form:

$$\Lambda(t + 1) = A(t)\Lambda(t) + w_1(t + 1)$$  (17)

$$\Omega(t) = C(t)\Lambda(t) + w_2(t)$$  (18)

$$E\left[ \begin{pmatrix} w_1(t + 1) \\
 w_2(t) \end{pmatrix} \begin{pmatrix} w_1(t + 1) \\
 w_2(t) \end{pmatrix} \right] = \begin{pmatrix} Q(t) & 0 \\
 0 & R \end{pmatrix}$$  (19)

In our problem, the vector of states is $\Lambda'(t) = (v_t \ x_t \ 1)$. The system matrix is

$$A(t) = \begin{pmatrix} 1 & 0 & \sigma_{v,t}\rho(dz_{T,t},dz_{v,t})\frac{dT_t}{T_t - t} \\
 0 & 1 & \sigma_{x,t}\rho(dz_{T,t},dz_{x,t})\frac{dT_t}{T_t - t} \\
 0 & 0 & 1 \end{pmatrix}.$$  

The vector $w_1(t)$ is assumed to be a Gaussian vector white noise. The observable variable is the log exchange rate $\Omega(t) = s_t$. Equation (9) implies that the observation matrix is $C(t) = \begin{pmatrix} 1 - e^{-\frac{T_t - t}{c}} & e^{-\frac{2(T_t - t)}{c}} & 0 \end{pmatrix}$.

The system covariance matrix can be written as

$$Q(t) = \begin{pmatrix} Q_{1,1}(t) & Q_{1,2}(t) & 0 \\
 Q_{1,2}(t) & Q_{2,2}(t) & 0 \\
 0 & 0 & 0 \end{pmatrix}.$$  

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where the covariance is conditional on the observed $T_i$, therefore
\[
Q_{1,1}(t) = \sigma^2_{v,t} \left[ 1 - \rho^2(dz_{T_i,t}, dz_{e,t}) \right],
\]
\[
Q_{1,2}(t) = \sigma_{v,t} \sigma_{x,t} \left[ \rho(dz_{x,t}, dz_{e,t}) - \rho(dz_{T_i,t}, dz_{v,t}) \rho(dz_{T_i,t}, dz_{x,t}) \right] = 0,
\]
\[
Q_{2,2}(t) = \sigma^2_{x,t} \left[ 1 - \rho^2(dz_{T_i,t}, dz_{x,t}) \right].
\]

The conditional covariance $Q_{1,2}$ between $dz_{x,t}$ and $dz_{e,t}$ is zero due to the restriction (8). In contrast to the conditional covariance, the unconditional covariance between $dz_{x,t}$ and $dz_{e,t}$ is not zero, which is captured by the system matrix $A(t)$.

I assume that the error term $w_2(t)$ is zero. In other words, the exchange rate is assumed to be observed without error and the model (9) perfectly describes the relationship between the factors and the exchange rate. Hence, the variance of the observation error term $R$ is set to zero. The Kalman filter remains valid even in this case.

In our problem, the observation matrix $C(t)$, the system matrix $A(t)$ and the system covariance $Q(t)$ are changing over time.

The parameters $c$, $\sigma_{v,t}$, $\sigma_{x,t}$, $\sigma_{T,t}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$ of the system covariance $Q(t)$ and of the system matrix $A(t)$ need to be either calibrated or estimated. Moreover, the initial values $x_{t_0}$ and $v_{t_0}$ of the factors belonging to the beginning of the sample period, $t_0 = 5Jan.2005$, need to be set as well. The next Subsection describes how these parameters are estimated or calibrated.

### 5.3 Parameters

This Subsection shows how the parameters $c$, $\sigma_{T,t}$, $x_{t_0}$, $v_{t_0}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{v,t})$, $\sigma_{v,t}$ and $\sigma_{x,t}$ are calibrated or estimated in the general case with stochastic locking time. The calibration of parameters is almost the same with the constant locking time as with the stochastic locking time. The calibration in the constant case deviates from that with stochastic locking time only by having the constant market expectation concerning the locking date $T$ set equal to the initial value of the filtered time series $T_{t_0}$ and the parameters $\sigma_{T,t}$, $\rho(dz_{v,t}, dz_{x,t})$, $\rho(dz_{T,t}, dz_{x,t})$ and $\rho(dz_{T,t}, dz_{v,t})$ are zero.

By following Engel and West (2005), I calibrate the parameter $c$ based on estimates on the interest rate semi-elasticity of money demand. Frankel (1979), Stock and Watson (1993, 802, table 2, panel 1) and Bilson (1978) estimate the semi-elasticity approximately 29, 40, 60 respectively. These estimates are either on quarterly data or rescaled to quarterly basis. By dividing the quarterly semi-elasticity parameter by 4 we get the annualized $c$ parameter. It should be in the range of 7.25–15. I calibrate $c$ to the average ($\bar{c} = 10.75$) of the estimates of the three cited studies and I use the two extreme values of the range for sensitivity analyses.

One can interpret the parameter value of $c$ around 10.75 as follows. If a country will lock its exchange rate to the euro in four years, then the elasticity of the exchange rate with respect to market expectations concerning the final conversion rate ($e^{-\frac{T-1}{c}} = e^{-\frac{15}{10.75}}$) is almost 70%. If locking is expected to occur in two years, then this elasticity or the relative weight of the log final conversion rate in the log exchange rate is more than 80%.

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15See Harvey (1990) page 108 for a detailed discussion.

16I used the Kalman filter toolbox for Matlab written by Kevin Murphy. Its advantage is that it allows for time-varying parameters. (See http://www.ai.mit.edu/~murphyk/Software/kalman.html for details.)
Figure 2 shows the relative weight of the locking rate component of the log exchange rate in the investigated period. Under the assumption of constant locking date, it is a monotonous exponential increasing function of time. More interesting is the case with stochastic locking date, where positive shocks to $T_t$ can even decrease the relative weight of $x$. In that case, the relative weights are stable for the Czech Republic, just like the expected time until locking. For Hungary and Poland, the largest change in the relative weights took place after July 2005, when the market sentiment on the EMU entry chances has started to become more and more pessimistic. Thereafter, the upward shift in the relative weight of $x$ in the first half of 2006 is due to the decreasing trend of the market expectation concerning the time until locking.

The market expectation concerning the locking date $T_t$ is estimated from the yield curves as it is discussed in Subsection 5.1. Under the assumption of stochastic locking date, its volatility $\sigma_{T,t}$ is estimated from the filtered $\hat{T}_t$. Since estimates on $\sigma_{T,t}$ may be sensitive to the window size of the moving average used to smooth the original filtered locking time, I calculated the stabilizing effect of future locking with different window sizes as part of the sensitivity analysis. In case of over-smoothing the original time series, the estimated volatility becomes downward biased, whereas failure to average out some errors leads to upward bias. Each value of the time-varying volatility is estimated from 11 data points

$$\hat{\sigma}_{T,t} = \sqrt{\sum_{\tau=t-5\text{days}}^{t+5\text{days}} \left( \frac{dT_{\tau}}{T_{\tau} - \tau} \right)^2}. \quad (20)$$

For a given value of $c$, one can calibrate the initial values of the factors $x_{t0}$ and $v_{t0}$. What makes these calibrations somewhat difficult is that I have no direct information on the latent exchange rate. For the calibration of the initial states, $x_{t0}$ and $v_{t0}$ I use the Reuters polls. I assume that $x_{t0}$ is equal to the log of averaged expectations on the central parity reported by the last Reuters polls of 2004. The initial value of $v_{t0}$ is calculated by plugging $s_{t0}$, $\hat{c}$ and $\hat{T}_{t0}$ into equations (9).

The calibrated initial values are utilized in the Kalman filter not only as the initial state variables, but also to estimate the correlations according to equation (7). These estimated correlations are plugged into the system matrix and the system covariance matrix used to filter the next period state variables. And these filtered state variables are used again for estimating the correlations between the next period’s shocks. These steps of estimation and filtering follow each other until the end of the Universe or until the end of the sample.

By rearranging (4) and (6) we obtain that the correlation between the shocks to market expectation concerning the locking rate and locking date is equal to $\rho(dz_{T,t}, dz_{x,t}) = \frac{d\hat{x}_t}{dT_t} \frac{\sigma_{T,t} \hat{T}_{t-t}}{\sigma_{x,t}}$. This formula suggests that it is sufficient to calibrate or estimate $T_t$, $\sigma_{T,t}$, $\frac{d\hat{x}_t}{dT_t}$ and $\sigma_{x,t}$ in order to obtain the correlation.

Among these terms $\frac{d\hat{x}_t}{dT_t}$ is calibrated based on the inflation dynamics. Equation (3) of the exchange rate model assumes that the market expectation concerning the log locking rate $x_t$ equals to the difference between the current domestic and eurozone log prices plus the cumulated expected excess domestic inflation rate over the eurozone inflation rate. Based on a plausible view on the inflation dynamics until locking, we can calibrate $\frac{d\hat{x}_t}{dT_t}$. This term is likely to depend more on the reaction of the domestic inflation path to a change in the expected locking date due to real shocks, rather than on the reaction of
the expected locking date to inflationary shocks. The calibration of $\frac{dx_t}{dT}$ depends on which of these effects is considered to be more relevant. Both effects and the corresponding calibrations are discussed below.

My view on the first effect is that the targeted path of the inflation of the domestic central bank depends on the outlook for the locking date. Following a shift in the expected locking date, the targeted path of inflation is assumed to be adjusted so that the inflationary Maastricht criterion will be met only at the new expected locking date. Further, I assume, that the targeted inflation at the locking date is 2 % no matter when the locking will be realized and whether the country had higher or even lower inflation rate before. Moreover, the eurozone inflation rate is expected to be constant 2 % as well until the locking date.

My assumption on the second effect is that if the locking is postponed due to an inflationary shock, then the rate of disinflation will remain unchanged.

This strategy of the central bank is illustrated by Figure 3 under the scenario of having higher initial inflation rate then the targeted inflation rate. The initial inflation rate $\pi$ of Figure 3 is expected to decrease linearly to the targeted inflation rate $\pi^*$ until time $T$. Hence, the initial cumulated excess inflation rate is the checkboard area. The economy can be hit either by inflationary or by real shocks.

If the expected locking date shifts to $T^s$ due to a real shock, then the cumulated inflation rate will increase by the dotted area. Under this scenario the term $\frac{dx_t}{dT}$ is half of the excess inflation $\frac{\pi - \pi^*}{2}$.

If the expected locking date shifts to $T^s$ due to an inflationary shock increasing the inflation rate to $\pi_s$, then the cumulated inflation rate will increase not only by the dotted area, but also by the striped one. Under this scenario, the term $\frac{dx_t}{dT}$ is the product of the time until locking and the constant rate of disinflation $\nu$.

I consider the real shocks to be more relevant, because EMU entry delays were mainly due to fiscal slippages in Hungary. Moreover, in Czech Republic and Poland the inflation rates were already moderate in the analyzed period. Table 3 presents the inflation rates by country and by year and also the average inflation rate used for calibration. I calibrate $\frac{dx_t}{dT}$ equal to half of the average excess inflation rate over 2%: -0.085 %, 1.15% and -0.125 % for Czech Republic, Hungary and Poland respectively.

As part of the sensitivity analysis I use the alternative approach of calibration, where $\frac{dx_t}{dT}$ is the function of the disinflation rate $\nu$ and the time until locking $T - t$. Table 4 reports the disinflation rate calculated as the ratio of the average excess inflation rate over 2% and the average filtered time until locking. With this approach, the average calibrated values of $\frac{dx_t}{dT}$ are twice as big as with the other calibration approach.

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2006</th>
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<th>Average (2005–2007Q1)</th>
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<td></td>
<td>YoY</td>
<td>YoY</td>
<td>QoQ</td>
<td></td>
</tr>
<tr>
<td>CZ</td>
<td>1.60%</td>
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<td>1.83%</td>
</tr>
<tr>
<td>HU</td>
<td>3.48%</td>
<td>4.04%</td>
<td>8.81%</td>
<td>4.32%</td>
</tr>
<tr>
<td>PL</td>
<td>2.18%</td>
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<td>1.96%</td>
<td>1.75%</td>
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</table>

Table 3: Harmonized consumer price index HCPI (source IFS)


<table>
<thead>
<tr>
<th>Average time until locking</th>
<th>Disinflation rate$^{18}$ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>CZ</td>
<td>3.21 years</td>
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<tr>
<td>HU</td>
<td>5.57 years</td>
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<td>6.06 years</td>
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Table 4: The average time until locking and the disinflation rate

Once I have calibrated or estimated $c$, $T_t$, $\sigma_{T,t}$, $\frac{dx}{dT}$ and the factors $x$ and $ν$ have already been filtered until time $t$, the only terms that need to be estimated are the volatilities $\sigma_{x,t}$ and $\sigma_{v,t}$. To see this, I show that the correlations can be expressed as functions of some already calibrated or estimated parameters and the volatilities $\sigma_{x,t}$ and $\sigma_{v,t}$. The calibrated or estimated variables are denoted by hats.

\[
\rho(dz_{T,t}, dz_{x,t}) = \frac{\hat{d}x_t \hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{x,t}}. \tag{21}
\]

By substituting (21) into equation (7), we obtain

\[
\rho(dz_{T,t}, dz_{v,t}) = \frac{(\hat{v}_t - \hat{x}_t)\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t} \hat{c}} + \frac{\hat{d}x_t \hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t} \hat{c}}. \tag{22}
\]

Finally, by substituting (21) and (22) into equation (8), we obtain

\[
\rho(dz_{v,t}, dz_{x,t}) = \frac{\hat{d}x_t \hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{x,t}} \left[ \frac{(\hat{v}_t - \hat{x}_t)\hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t} \hat{c}} + \frac{\hat{d}x_t \hat{\sigma}_{T,t}(\hat{T}_t - t)}{\sigma_{v,t} \hat{c}} \right]. \tag{23}
\]

The time-varying volatilities $\sigma_{x,t}$ and $\sigma_{v,t}$ are estimated from six implied volatilities $\sigma_{i,t}^{imp}$ for each time $t$ by ordinary least squares (OLS). The six options have different maturities $m(i)$. The basic idea of the estimation is to minimize the distance between the theoretical and the historical option prices.

The OLS estimates of $\sigma_{v,t}$ and $\sigma_{x,t}$ satisfy the minimization problem

\[
\min_{\sigma_{v,t}, \sigma_{x,t}} \sum_{i=1}^{6} \left[ g(t, m(i), \sigma_{x,t}, \sigma_{v,t}, \sigma_{T,t}, \rho(dz_{v,t}, dz_{x,t}), \rho(dz_{T,t}, dz_{x,t}), \rho(dz_{T,t}, dz_{v,t})) - (\sigma_{i,t}^{imp}) \right]^2. \tag{24}
\]

The option pricing formula for $g()$ is given by (15). All the input parameters of the pricing formula should be substituted by their estimated or calibrated values. The only exceptions are the correlations, which are substituted by the expressions of (21),(22) and (23), where the correlations are functions of the volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$.

$^{17}$The average inflation rate is calculated as the weighted average of the yearly and quarterly inflation rates, where the weight of the quarterly inflation rate is one-forth of that of the yearly rates.

$^{18}$The disinflation rate is calculated as the ratio of the excess average inflation rate over 2% and the average expected time until locking. For instance, for Czech Republic $ν = \frac{1.88\%-2\%}{3.21} = -0.05\%$. The average time until locking is the mean of the filtered time until locking.
Figure 4 shows the historical option prices with the shortest and longest maturities and also the fitted values. One can see from Figure 4 that the implied volatilities of the options with the shortest maturity fluctuate around the implied volatilities of the options with the longest maturity. In those periods, when the short implied volatility exceeds the long one, the fitted short volatility exceeds the fitted long one as well. This is reflected by the increased estimated volatility of $v_t$ and the decreased estimated volatility of $x_t$ in these periods (see Figure 5). The higher than average $\sigma_{v,t}$ and the substantially lower than average $\sigma_{x,t}$ in these periods can be explained by the followings: first, the relative option prices suggest that uncertainty concerning shocks in the short horizon increases relatively to that in the long horizon; second, $\sigma_{v,t}$ can be thought of as a proxy for the short-run uncertainty, whereas $\sigma_{x,t}$ reflects more the expected magnitude of shocks occurring in the far future.

Figure 5) shows, that on average $\bar{\sigma}_{v,t} > \bar{\sigma}_{x,t}$ for all three countries, providing further support for the stabilizing effect of locking. In the followings, I will calculate not only the factor volatilities estimated from option prices, but also the volatilities of the filtered factors. Moreover, I will measure the stabilizing effect as the relative difference between the realized volatility and the volatility of the filtered latent exchange rate. The advantage of this measure of the stabilizing effect is that it better reflects the magnitude of the ex post stabilization as being calculated from the realized volatility.

Figure 5 shows not only the estimated volatilities of $v_t$ and $x_t$, but also that of the third factors $T_t$. The estimated volatility of $T_t$ is higher than average in those periods when there are big changes in the filtered locking date. Among the three countries, Czech Republic has the highest average volatility of $T_t$, despite of the fact that the filtered locking date $\hat{T}_t$ is the most stable for Czech Republic (see Figure 1). The magnitude of the volatility can be explained by the followings. First, Czech Republic is likely to be the first among these countries that will be eligible to join the EMU. Second, the volatility measures the variation of the relative change of the expectations and not the absolute. The relative change of the expectation is a decreasing function of the market expectation concerning the time until locking.

5.4 The filtered expected locking rate and the stabilizing effect of locking

Figure 6 shows the historical exchange rates of the koruna, the forint and the zloty against the euro, the filtered states and analysts’ average expectations concerning the central parity as polled by Reuters. Market expectations about the final conversion rate may be thought to be close to the expected central parity of the ERM II regime. In that case, the expected central parity is a good reference for the filtered expected final conversion rate to be compared with. Here, we compare the filtered market expectations with the average expectations reported by the Reuters polls.

Respondents’ views on central ERM II parities vary a lot in each poll (see: Figure 8). There is at least 6% difference between the two extreme views of the analysts, and even differences of more than 20% being nothing rare. These differences indicate that uncertainty around the reported expectations is likely to be big, which one needs to bear in mind when taking the average reported expectations as the general view of the market on central parity.
As is evident from Figure 6, the patterns of the filtered expected final conversion rate and of the reported expected central parity are similar for each of the three countries. The difference between the filtered and the survey-based expectation is always less than 5% for Czech Republic. For Hungary and Poland this difference is more than 5% in a few times, but never exceeds 10%. Although being higher than for Czech Republic, still the difference can be considered to be moderate for Hungary and Poland as well.

According to a naive approach, the spot exchange rate can be considered as noisy estimates on the locking rate. By taking this approach as a bench-mark, the criterion for filtering the expected locking rate successfully, is that the filtered locking rate moves closer together with some alternative measures of the locking rate than the historical exchange rate. Table 5 shows that the innovations in the filtered log locking rate for Hungary correlate somewhat more with the changes in the survey-based log locking rate than the exchange rate returns. This finding can be interpreted, as the model helped to denoise the naive estimates on the locking rate. For the other two countries, the filtering can not be judged to be successful on the same basis, however, there the substantially fewer number of observations disables us to make any firm conclusion.

It is worth to compare the filtered locking rate $\hat{x}_t$ not only with the survey-based expectation, but also with the filtered latent exchange rate $\hat{v}_t$. In this model higher $\hat{v}_t$ than $\hat{x}_t$ suggest that inflation is less of a problem for the economy. Whereas in the opposite case, a later EMU entry is partly due to the high current and future expected inflation.

The filtered locking rate is weaker than the filtered latent exchange rate in Hungary, supporting the view that not only the criteria on the real variables, but also the inflationary Maastricht criterion is binding. In contrast to Hungary, the filtered locking rate is stronger than the filtered latent exchange rate in Poland, what is due to the low inflation rate in the analyzed period. For Czech Republic, there is no significant difference between the filtered locking rate and the latent exchange rate.

We have some important findings on the volatility of the expected final conversion rate as well. The volatility of market expectations concerning the final conversion rate is important, because relatively stable market expectations can stabilize the exchange rate. The locking rate is often referred to as the nominal anchor of the exchange rate that can decrease the volatility of the exchange rate through the expectation channel.

<table>
<thead>
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<th>HU</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(X_{\text{survey}})$</td>
<td>2.4%</td>
<td>3.93%</td>
<td>4.31%</td>
</tr>
<tr>
<td>$\sigma(X_{\text{filtered}})$</td>
<td>4%</td>
<td>5.7%</td>
<td>7.48%</td>
</tr>
<tr>
<td>$\rho(ds, dx_{\text{survey}})$</td>
<td>72.43%</td>
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<td>42.35%</td>
</tr>
<tr>
<td>$\rho(dx_{\text{filtered}}, dx_{\text{survey}})$</td>
<td>33.33%</td>
<td>24.22%</td>
<td>11.96%</td>
</tr>
<tr>
<td>Frequency</td>
<td>Quarterly</td>
<td>Montly</td>
<td>Quarterly</td>
</tr>
<tr>
<td>Num. obs. of $X_{\text{survey}}, X_{\text{filtered}}, S$</td>
<td>7</td>
<td>19</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5: The volatility of filtered and survey-based locking rate ($X$) and the correlations of their percentage changes with the exchange rate returns ($ds$) – all data are on the frequency of the survey data.
Regarding the volatilities, one can see from Table 5 that the filtered $x$ is more volatile than the survey-based expectation both calculated from data on the same frequencies. This finding might adversely modify our previous view based purely on the Reuters polls on the stabilizing feature of locking. Still, if the volatility of $x$ is lower than that of $s$, then market expectations concerning the final conversion rate might have some stabilizing effect on the exchange rate. The advantage of filtering the market expectations over using the survey-based expectations is that no reliable estimate can be given on the volatility of $x$, and on its relative weight in the exchange rate from the low frequency Reuters polls data, therefore no estimate can be given for the stabilizing effect of the locking either purely from these data.

The big picture on the stabilizing feature of locking in the entire sample period is provided by Table 6. The stabilizing effect of locking is calculated as the relative difference between the volatilities of the historical exchange rate and the filtered latent exchange rate. Across the sample period, the stabilizing effect of locking is found to be the highest in Czech Republic. And somewhat lower, but still high in Poland and Hungary. Even in these two countries, the stabilizing effect is substantial, the volatility without future locking would be approximately twice as large.

The estimated stabilizing effect is robust to the calibration of the parameter $c$. The expectations concerning the locking decrease the volatility at least by 30% even under a smaller parameter of 7.25 for each of the countries (see Table 7 in Appendix C). The stabilizing effect is increasing in $c$, for $c = 15$ it is close 60% (see Table 8 in Appendix C).

Appendix C presents the results of some other sensitivity analysis as well. The estimates are robust not only to the calibration of $c$, but also to the calibration of $\frac{dx}{dT}$ (see Table 9). Moreover, it is robust to the window size used to smooth the original filtered $T_t$ (see Tables 10 and 11).

The sensitivity of the stabilizing effect to the locking date is also investigated. Instead of assuming stochastic locking time, one may assume to have either constant or deterministic locking time. In the constant case the market expectation concerning the locking date is set equal to the initial value of the filtered locking time ($T_t = T_{t_0}$ where $t_0 = 5. Jan. 2005$.). Whereas in the deterministic case, we have time-varying locking time with zero volatility, i.e. the market expectation concerning the locking date is set equal to the filtered one and it is assumed to be independent from the other factors. In order to see, to what extent does the stabilizing effect depend on the dynamics of the volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$, I calculate the stabilizing effect also with constant volatilities. These constant volatilities are set equal to the averages of their time-varying estimates. Tables 12, 13 and 14 show that the magnitude of the stabilizing effect is robust even to the locking date and the dynamics of the volatilities.

This model is rich in dynamics: we have time-varying volatilities $\sigma_{v,t}$, $\sigma_{x,t}$ and $\sigma_{T,t}$, and stochastic $T_t$. Each of these dynamics may have important effect on the filtered time series of the factors. In order to see what drives the dynamics of the filtered states I shut down each of these effects one by one. First, I filter the states with deterministic $T_t$, i.e. with zero $\sigma_{T,t}$, but with time-varying $T_t$. Second, I filter the states with constant $T_t$, by shutting down not only the dynamics due to $\sigma_{T,t}$, but also the one due to time-varying $T_t$. Finally, I look at the effect of time-varying volatilities on the dynamics of the filtered $x_t$ by filtering with zero $\sigma_{T,t}$, and with constant volatilities $\sigma_{v,t}$ and $\sigma_{x,t}$. These constant volatilities are set equal to the averages of their time-varying estimates. Figure 7 clearly shows that most of the dynamics of the filtered $x$ is due to the dynamics of the exchange
Table 6: The volatility of the exchange rate ($S$) and the latent exchange rate ($V$) – with stochastic $T_t$ and time-varying volatilities, window size for smoothing the original filtered $\hat{T}_t$ is 21, $c=10.75$. $\frac{dx}{dt}$ equals to half of the average excess inflation rate over 2%: -0.085 %, 1.15% and -0.125 % for Czech Republic, Hungary and Poland respectively.

rate as other sources of dynamics do not seem to change the patterns of the filtered factors substantially.
This paper investigated market expectations concerning the locking rate of EMU candidate countries based on a theoretical model for exchange rate subject to future locking. The model is the conventional asset pricing exchange rate model extended with the future locking assumption. In this model, the exchange rate converges to the actual market expectation concerning the locking rate in expected term. The asset-pricing model with final locking is a three-factor model, where the factors are the market expectations concerning the locking rate, the locking date and the latent exchange rate. In the empirical part of the paper, I used the model to Kalman filter the subjective expectation of market participants concerning the locking rate for Czech Republic, Hungary and Poland.

An important contribution of the paper to the theoretical exchange rate literature is that it finds empirical support for the conventional asset pricing exchange rate model. If we consider the survey-based expectation to be unbiased estimates on the expectation of the markets, then the exchange rate model used to filter the market expectation should be judged successful as the filtered expectations are close to the reported expectations of the surveys. This test on the model is similar in spirit to the one applied by Engel and West (2005). Engel and West provide also some empirical support for this type of models based on the forecasting ability of the future fundamentals.19

Another important finding of the paper is that the stabilizing effect of the future locking is substantial. This finding is robust to the calibration of the parameters. Moreover, this qualitative result is in line with our view based on the term structure of option prices and also with our view purely based on Reuters polls data. In contrast to the low frequency Reuters polls data, our high frequency data on the filtered state variables enables us to measure the magnitude of the stabilizing effect. The estimated effect is that the volatility without future locking would be approximately twice as large as its current level in all three countries. The magnitude of the stabilizing effect depends on two determinants - the stability of market expectations concerning the locking rate, and the importance of expectations in determining the exchange rate. In case of an earlier euro area entry, the stabilizing effect is likely to be more substantial, because market expectations concerning the locking rate are themselves more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this intuitive argument, the prospect of locking should contribute to the stabilization of the koruna the most. Actually, the estimated stabilizing effect of locking is found to be the highest in Czech Republic.

Future research will aim at examining the exchange rates of the main currency pairs by using the same idea presented in this paper. The exchange rate of the main currency

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19For many years, the standard criterion for judging the exchange rate models has been based on the forecasting power relative to the random-walk model. This criterion was first proposed by the seminal paper of Meese and Rogoff (1983). Recently, Engel and West (2005) has questioned the standard criterion. They have demonstrated on a similar model to the one in this paper that if fundamentals are integrated of order one and the discount factor is close to unity then the exchange rate will approximately follow random-walk. The important implication of this result is that the standard criterion is not useful at judging the performance of exchange rate models. Engel and West (2005) have proposed an alternative criterion, which is based on the forecasting ability of the future fundamental.
pairs can also be expressed as a weighted average of a short and a long term component in accordance with the asset pricing view. The long term component of the exchange rate can be justified not only by a possible final locking, but also by the long run reversion of the exchange rate to its PPP level. To complete the analogy between the locking and the no locking case, we can see that uncertainty relating the half-lives of exchange rate deviations from PPP plays a similar role in case of the main exchange rates as the uncertainty relating the locking time of the analyzed currencies.

7 Acknowledgements

The author gratefully acknowledges comments and suggestions from Christian Gourieroux, András Fülöp, Péter Benczúr, Attila Csajbók, Balázs Égert, Péter Kondor, Csilla Horváth, Ádam Reiff and István Kónya.
Appendix A

Here, I prove, that the derived function $s_t = f(t, v_t, x_t, T_t)$ of (9) satisfies the dynamic condition given by (12), the terminal condition (11) and (1).

I prove by substitution that (9) satisfies (12) as follows.

$$ds_t = \left[ \frac{1}{c} e^{-\frac{\tau_{x,t}}{c}} (x_t - v_t) + \frac{1}{2} \frac{1}{c^2} e^{-\frac{\tau_{x,t}}{c}} (x_t - v_t) \sigma_{T,t}^2 (T_t - t)^2 + \right.$$

$$\left. - \frac{1}{2} \frac{1}{c} e^{-\frac{\tau_{x,t}}{c}} \rho (d z_{T,t}, d z_{x,t}) (T_t - t) \sigma_{T,t} \sigma_{x,t} + \frac{1}{2} \frac{1}{c} e^{-\frac{\tau_{x,t}}{c}} \rho (d z_{T,t}, d z_{v,t}) (T_t - t) \sigma_{T,t} \sigma_{v,t} \right] dt +$$

$$+ \left( 1 - e^{-\frac{\tau_{x,t}}{c}} \right) \sigma_{v,t} dz_{v,t} + e^{-\frac{\tau_{x,t}}{c}} \sigma_{x,t} dz_{x,t} - \frac{1}{c} e^{-\frac{\tau_{x,t}}{c}} (x_t - v_t) (T_t - t) \sigma_{T,t} dz_{T,t}. \quad (25)$$

By substituting and (9) into (25), we get that the expected instantaneous change of the exchange rate is

$$E_t (ds_t) dt = \frac{1}{c} (s_t - v_t). \quad (26)$$

By substituting (26) into (1), we obtain an identity. This proves that (9) satisfies (1). Thus, (9) satisfies the terminal condition (11) as well. Hence, the function in (9) is the solution I was looking for.
This appendix shortly introduces the yield curve method applied to estimate the market expectation concerning the locking date. The paper by Bates (1999) and Csajbók and Rezessy (2006) provide a more detailed description on this method.

The market expectation, formed at time $t$, on the date of locking $T_t$ can be estimated from the forward differentials as follows. The expected value is calculated from the subjective probability distribution of the year the country enters the EMU.

$$T_t = \sum_{i=t}^{T} p_t(EMU_i)i$$

where $p_t(EMU_i)$ is the probability that the market attaches at time $t$ to the scenario in which the country becomes a full member of eurozone in the $i$th year. The distribution is assumed to have finite support, the country will join the eurozone in the year $\hat{T}$ at the latest. The marginal probability $p_t(EMU_i)$ can be calculated from the cumulative probability distribution $P_t(EMU_i)$. The interpretation of $P_t(EMU_i)$ is straightforward, this is the probability that the market attaches at time $t$ to the scenario in which the country is in the EMU by the $i$th year. The cumulative probability can be derived from the pricing equation of the one-year forward interest differential.

$$FS_{t,i} = (1 - P(EMU_i))FS_{t,i}^{non-EMU_i} + P(EMU_i)FS_{t,i}^{EMU_i}$$

where $FS_{t,i}$ is the one-year forward interest rate differential for year $i$ observed at time $t$. The $FS_{t,i}^{non-EMU_i}$ and $FS_{t,i}^{EMU_i}$ are the expected interest rate differentials under the two alternative scenarios, i.e. the accession country is either in or out the eurozone by year $i$. By rearranging (28) we obtain

$$P(EMU_i) = \frac{FS_{t,i}^{non-EMU_i} - FS_{t,i}}{FS_{t,i}^{non-EMU_i} - FS_{t,i}^{EMU_i}}$$

Among the right-hand-side variables of (29) only $FS_{t,i}$ is observable. However, by assuming that the analyzed countries will surly enter the EMU in nine years, and have almost zero chance to become an EMU member in one year, we can set $FS_{t,i}^{non-EMU_i} = FS_{t,t+1}$ and $FS_{t,i}^{EMU_i} = FS_{t,t+9}$. 

28
Appendix C
This appendix presents the results of some sensitivity analysis. The parameters which are not subject of the analysis are calibrated exactly the same as in the baseline case used in Table 6.

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<th></th>
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<td>( \sigma_S )</td>
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<td>10.34%</td>
</tr>
<tr>
<td>( \sigma_{\hat{V}} )</td>
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</table>

Table 7: The volatility of the exchange rate \( (S) \) and the latent exchange rate \( (V) \) – Parameter \( c \) is calibrated to 7.25

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<td>10.34%</td>
</tr>
<tr>
<td>( \sigma_{\hat{V}} )</td>
<td>16.31%</td>
<td>22.61%</td>
<td>26.79%</td>
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<td>( \frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}} )</td>
<td>-66.45%</td>
<td>-58.03%</td>
<td>-61.41%</td>
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</tbody>
</table>

Table 8: The volatility of the exchange rate \( (S) \) and the latent exchange rate \( (V) \) – Parameter \( c \) is calibrated to 15

<table>
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<th></th>
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</thead>
<tbody>
<tr>
<td>( \sigma_S )</td>
<td>5.47%</td>
<td>9.49%</td>
<td>10.34%</td>
</tr>
<tr>
<td>( \sigma_{\hat{V}} )</td>
<td>12.81%</td>
<td>17.65%</td>
<td>21.28%</td>
</tr>
<tr>
<td>( \frac{\sigma_S - \sigma_{\hat{V}}}{\sigma_{\hat{V}}} )</td>
<td>-57.29%</td>
<td>-46.24%</td>
<td>-51.43%</td>
</tr>
</tbody>
</table>

Table 9: The volatility of the exchange rate \( (S) \) and the latent exchange rate \( (V) \) – Parameter \( \frac{dx_t}{dt} \) is calibrated under the assumption that changes in the expected locking date are due to inflationary shocks
### Table 10: The volatility of the exchange rate ($S$) and the latent exchange rate ($V$) – The window size used to smooth the original filtered $T_t$ is 41

<table>
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<td>10.41%</td>
</tr>
<tr>
<td>$\sigma_V$</td>
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<td>17.65%</td>
<td>20.5%</td>
</tr>
<tr>
<td>$\frac{\sigma_S - \sigma_V}{\sigma_V}$</td>
<td>-58.18%</td>
<td>-46.31%</td>
<td>-49.24%</td>
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### Table 11: The volatility of the exchange rate ($S$) and the latent exchange rate ($V$) – The window size used to smooth the original filtered $T_t$ is 11

<table>
<thead>
<tr>
<th></th>
<th>CZ</th>
<th>HU</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_S$</td>
<td>5.45%</td>
<td>9.5%</td>
<td>10.31%</td>
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<tr>
<td>$\sigma_V$</td>
<td>13.19%</td>
<td>18.13%</td>
<td>21.5%</td>
</tr>
<tr>
<td>$\frac{\sigma_S - \sigma_V}{\sigma_V}$</td>
<td>-58.66%</td>
<td>-47.61%</td>
<td>-52.06%</td>
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</tbody>
</table>

### Table 12: The volatility of the exchange rate ($S$) and of the latent exchange rate ($V$) – with deterministic locking time

<table>
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<tr>
<th></th>
<th>CZ</th>
<th>HU</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_S$</td>
<td>5.47%</td>
<td>9.49%</td>
<td>10.34%</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>13.03%</td>
<td>17.7%</td>
<td>21.04%</td>
</tr>
<tr>
<td>$\sigma_S - \sigma_V$</td>
<td>-7.56%</td>
<td>-8.21%</td>
<td>-10.7%</td>
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<tr>
<td>$\frac{\sigma_S - \sigma_V}{\sigma_V}$</td>
<td>-58.01%</td>
<td>-46.4%</td>
<td>-50.87%</td>
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</tbody>
</table>

Table 10: The volatility of the exchange rate ($S$) and the latent exchange rate ($V$) – The window size used to smooth the original filtered $T_t$ is 41

Table 11: The volatility of the exchange rate ($S$) and the latent exchange rate ($V$) – The window size used to smooth the original filtered $T_t$ is 11

Table 12: The volatility of the exchange rate ($S$) and of the latent exchange rate ($V$) – with deterministic locking time
Table 13: The volatility of the exchange rate ($S$) and of the latent exchange rate ($V$) – with constant locking time ($T_t = T_{t_0}$ where $t_0 = 5.Jan.2005.$)

<table>
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<tr>
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<th>CZ</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_S$</td>
<td>5.47%</td>
<td>9.49%</td>
<td>10.34%</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>12.46%</td>
<td>23.72%</td>
<td>21.87%</td>
</tr>
<tr>
<td>$\sigma_S - \sigma_V$</td>
<td>-6.99%</td>
<td>-14.23%</td>
<td>-11.53%</td>
</tr>
<tr>
<td>$\frac{\sigma_S - \sigma_V}{\sigma_V}$</td>
<td>-56.09%</td>
<td>-59.99%</td>
<td>-52.73%</td>
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</table>

Table 14: The volatility of the exchange rate ($S$) and of the latent exchange rate ($V$) – with deterministic locking time and constant volatilities $\sigma_{x,t} = \bar{\sigma}_{x,t}$, $\sigma_{v,t} = \bar{\sigma}_{v,t}$

<table>
<thead>
<tr>
<th></th>
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<th>HU</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_S$</td>
<td>5.47%</td>
<td>9.49%</td>
<td>10.34%</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>11.68%</td>
<td>13.34%</td>
<td>18.6%</td>
</tr>
<tr>
<td>$\sigma_S - \sigma_V$</td>
<td>-6.21%</td>
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</tr>
<tr>
<td>$\frac{\sigma_S - \sigma_V}{\sigma_V}$</td>
<td>-53.14%</td>
<td>-28.84%</td>
<td>-44.42%</td>
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</tbody>
</table>

References


Figures
Figure 1: The average expectation of the analysts concerning the EMU and the ERM II entry date and the filtered locking date $T_t$.
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